# Special Relativity

## History, Concepts and Experimental Verification

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ISU QuarkNet Seminar

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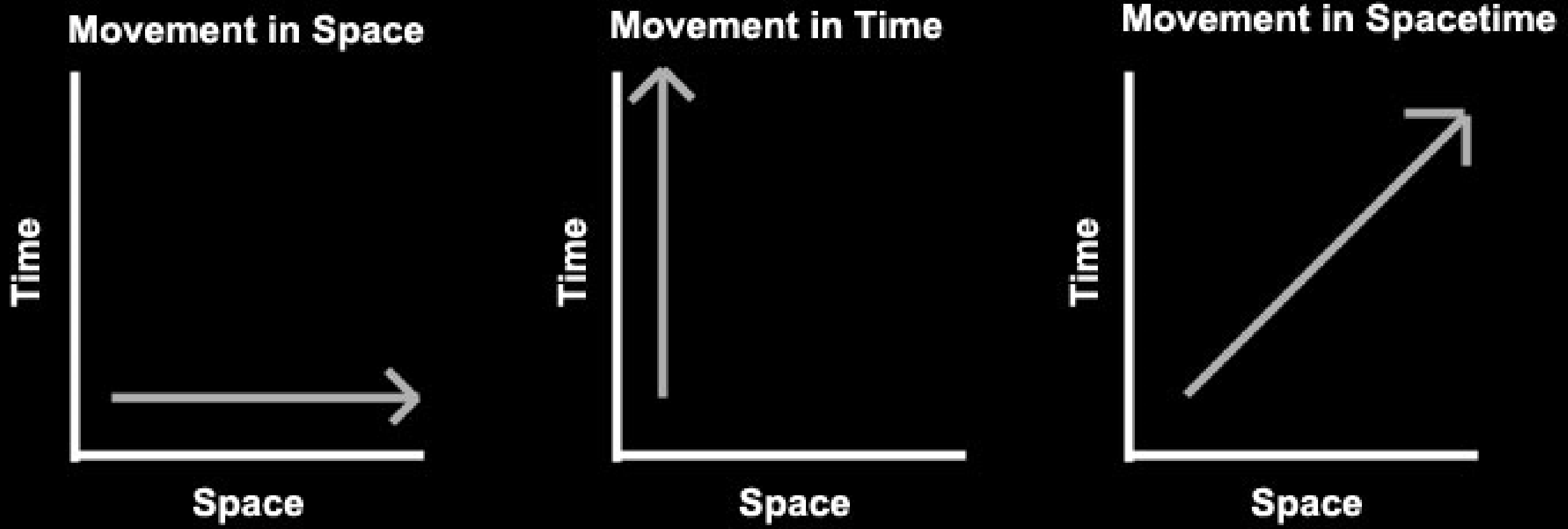


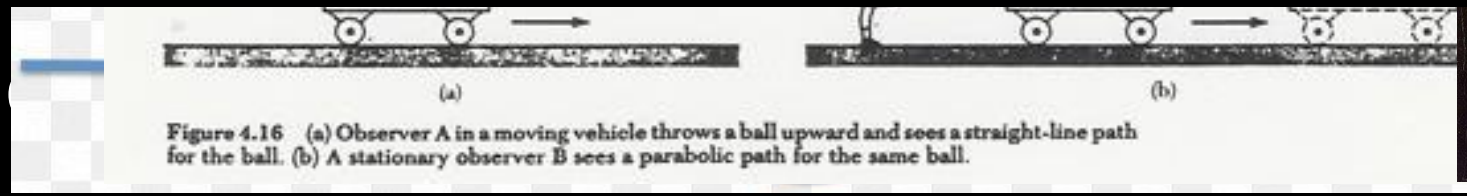
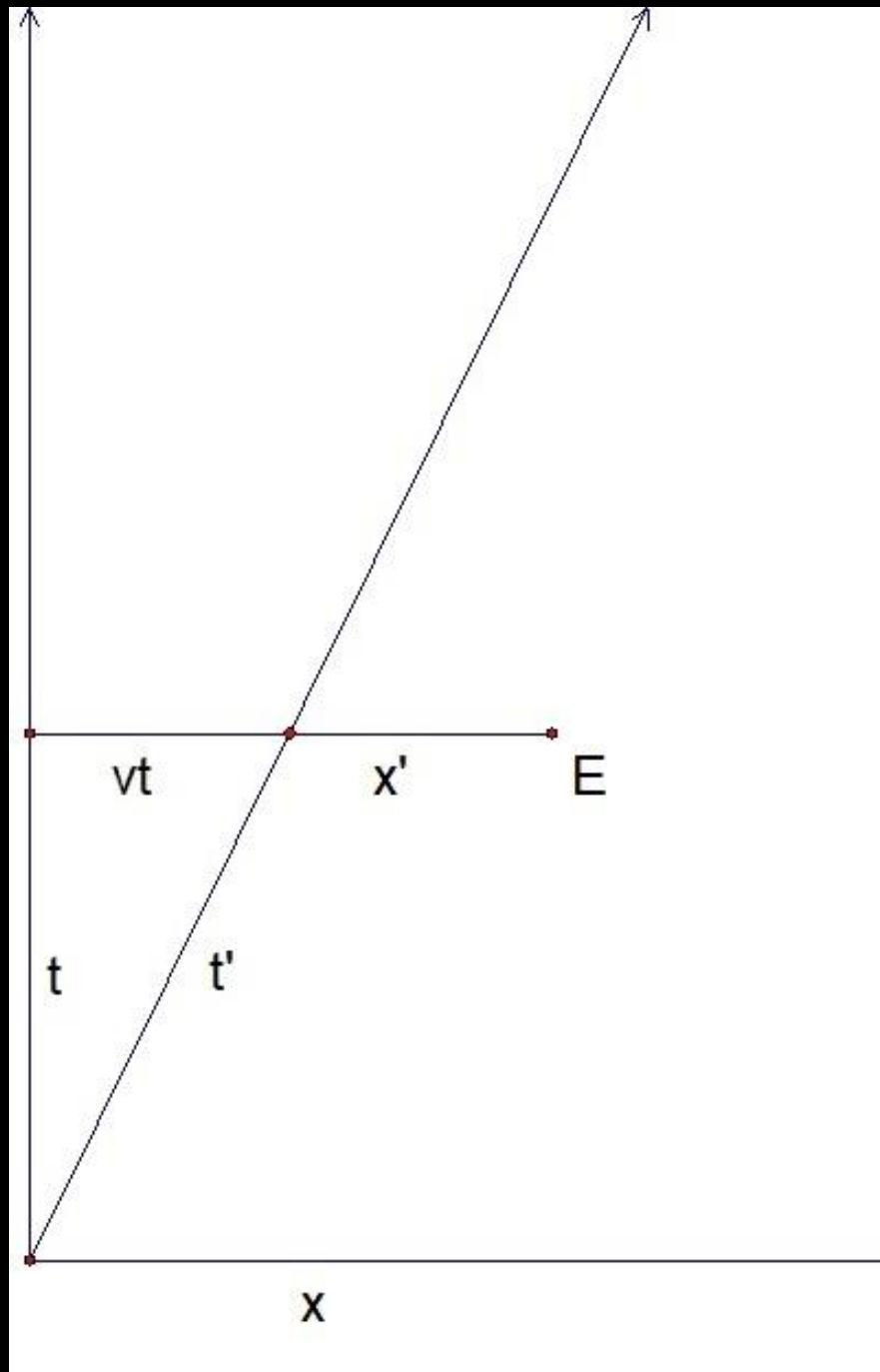




Figure 4.16 (a) Observer A in a moving vehicle throws a ball upward and sees a straight-line path for the ball. (b) A stationary observer B sees a parabolic path for the same ball.

# Galilean Transformations

- Assuming the simplest difference between the two reference frames allows us to construct the following spacetime diagram with both observers shown.
- Event E has coordinates $(x, t)$ in the rest frame, but has coordinates $(x', t')$ in the moving frame.
- Since both frames have the same temporal axis, note that $t = t'$.
- However, the origin of the primed frame is constantly moving, thus the $x$ and $x'$ axes point in the same direction but E does not have the same coordinate in each.
- Basic geometry shows that $x' + vt = x$.
- Thus, $t' = t \quad x' = x - vt \quad y' = y \quad z' = z$ are the Galilean transformations between these two frames.



# Velocity, Acceleration and Absolute Rest

- Differentiation of the $x'$ equation with respect to time gives us the relationship between the velocity of the event measured in the two reference frames as

$$v'_E = v_E - v$$

- When rearranged, this is the usual velocity addition rule, in the $x$-direction.
- Note, however, that one more time derivative gives us

$$a'_E = a_E$$

- This shows that both frames measure the same acceleration for the event and, consequently, Galilean relativity and Newton's laws hold.
- Note that there is nothing special about our "un-primed" system.
- The "primed" system could just as easily be considered at rest with the un-primed system moving, albeit with velocity $-v$, and the transformations would still hold.
- Thus, another way of stating Galilean relativity is to say that there is ***no*** absolute frame of rest, all inertial reference frames are equivalent!

kind of universal atmosphere, ... Undulatory Theory of Light.

space. It is so subtle that it glides among the molecules of bodies as the air does among the branches and the foliage of trees. It fills the pores of all substances, eludes all chemical tests, passes in through the receiver, and remains even in the vacuum of an air-pump. A luminous body sets in motion waves of ether, which go off in every direction. They move at the rate of 188,000 miles per second, and, breaking upon the eye, give the impression of sight. In the wave-motion of light, the vibrations are transverse (crosswise). Thus, if we suppose a star directly overhead, and a ray of light coming down to us, we should conceive that some of the particles which compose the waves are vibrating E. and W., others N. and S., and others toward all other possible points of the compass in succession.

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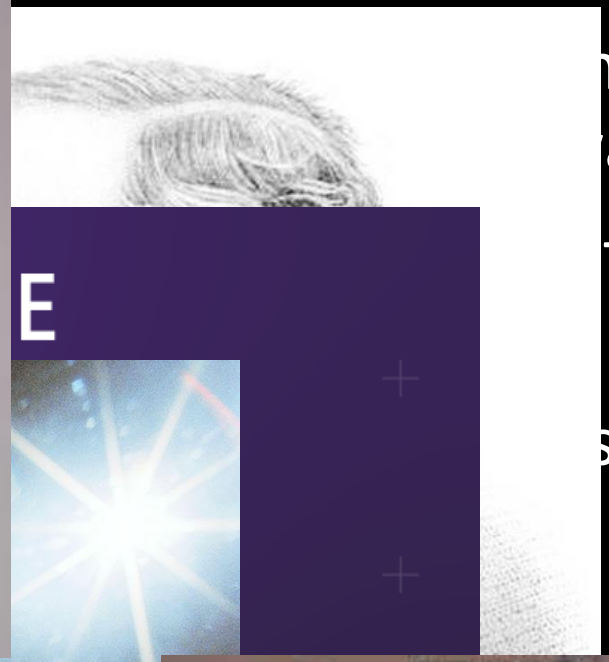

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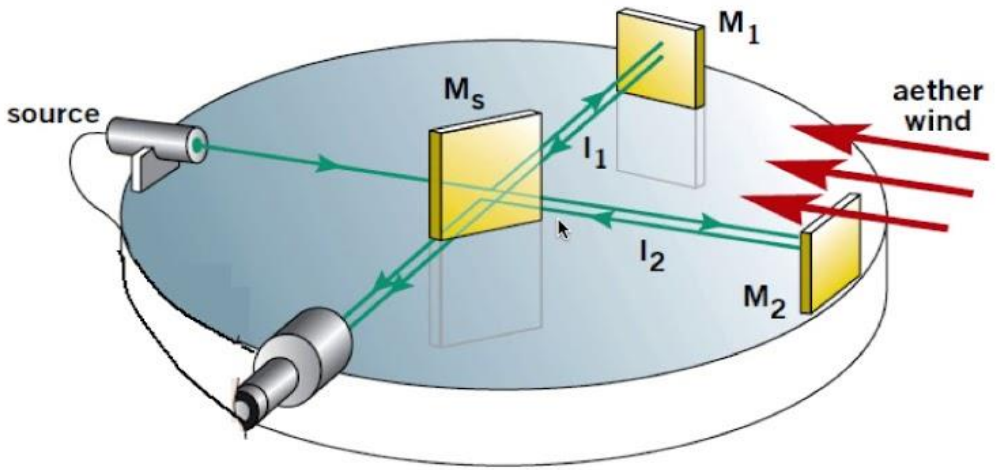
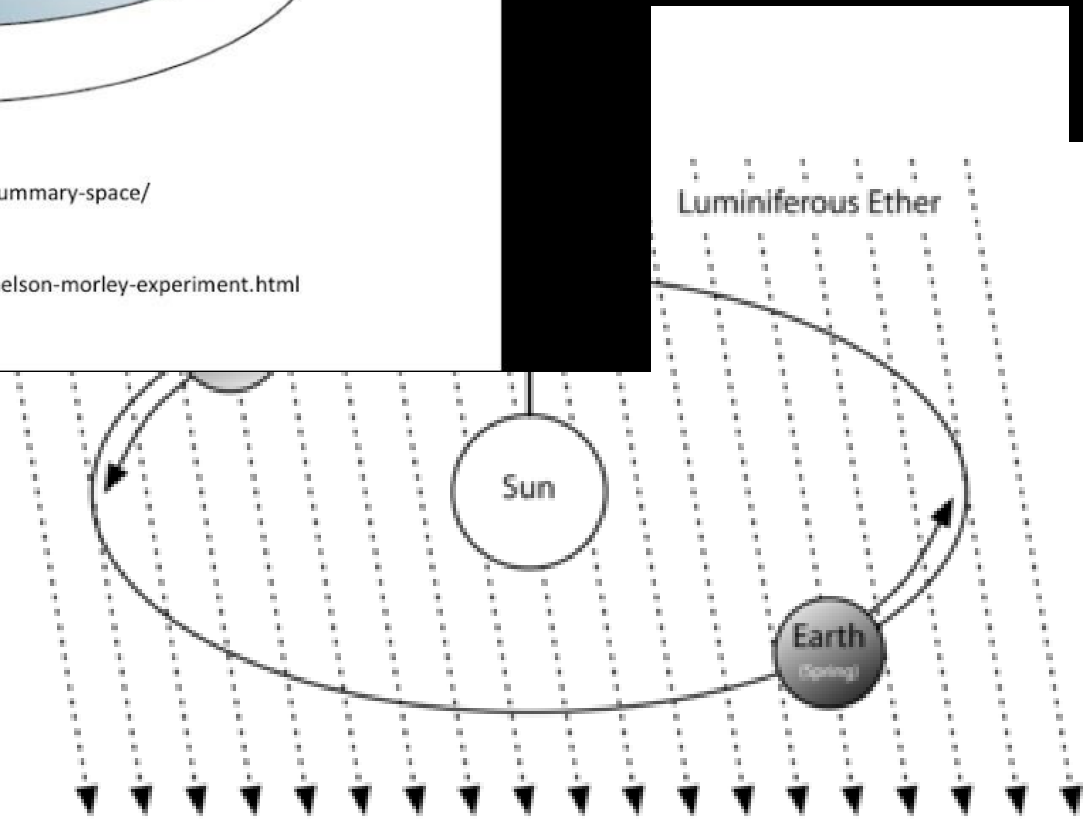
## Michelson-Morley Experiment (1887)




https://dc.edu.au/dot-point-summary-space/

http://novicemathandscience.blogspot.com/2012/03/michelson-morley-experiment.html

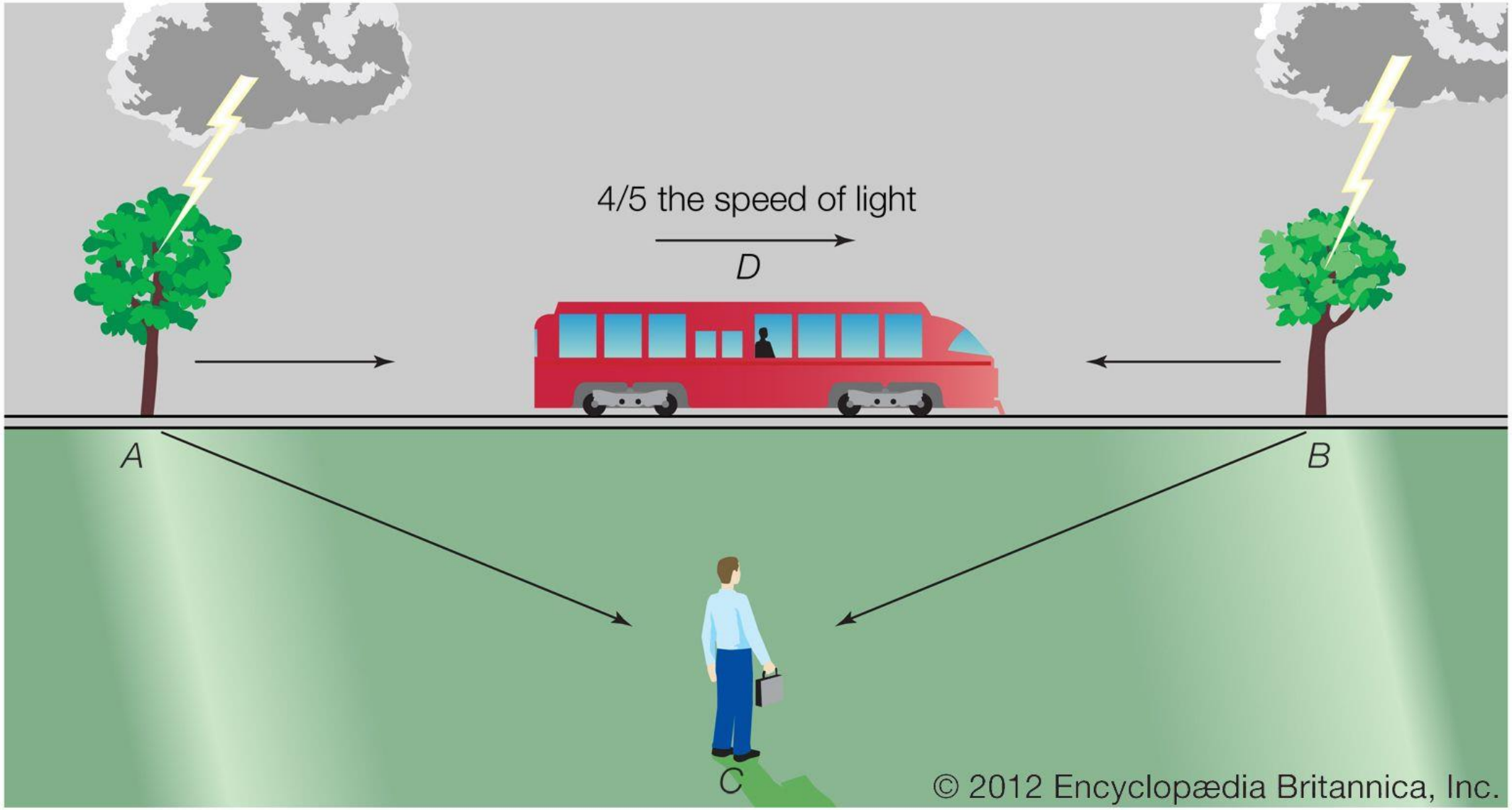





- What were physicists to do?

# Partial Answers

- In 1889, two years after Michelson and Morley's null result, George Fitzgerald wrote a letter to the editor of *Science* suggesting that if the length of the interferometer in the direction of motion somehow shortened it would explain their result.
- In 1892 Hendrik Lorentz made the same suggestion as Fitzgerald in terms of length contraction and then in 1895 showed that transformations between reference frames could be simplified by introducing a "local time". He combined these ideas together in what are known as the Lorentz transformations.
- Between 1900 and 1905, Henri Poincar*é*, a world-renowned mathematician, noted that Lorentz's transformation formed a group, gave his own replacement for the velocity addition rule and stressed how the constancy of the speed of light can be used to synchronize reference frames.

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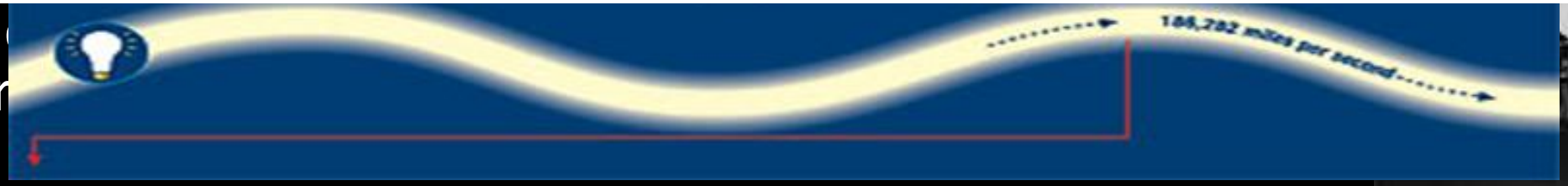
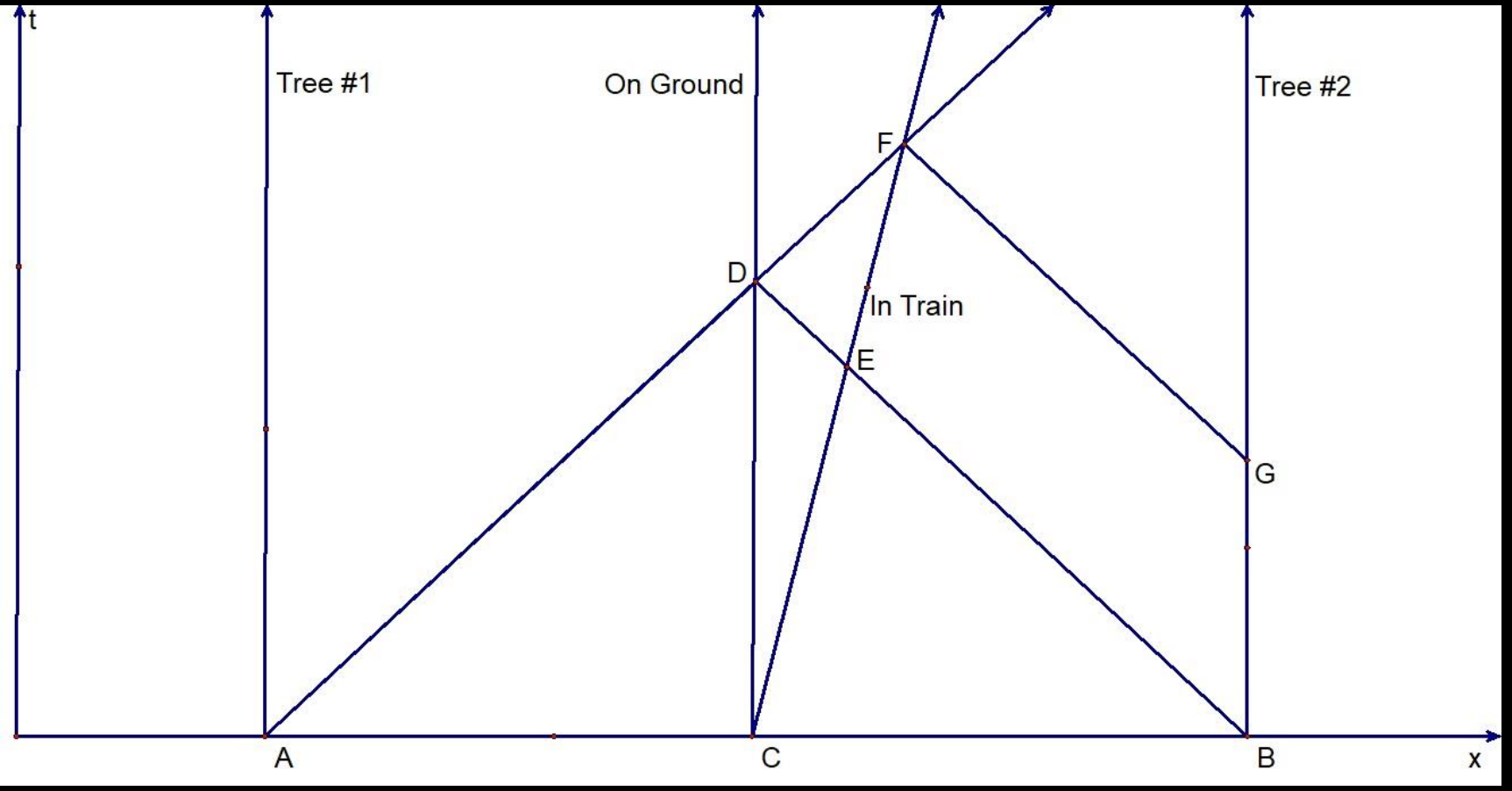


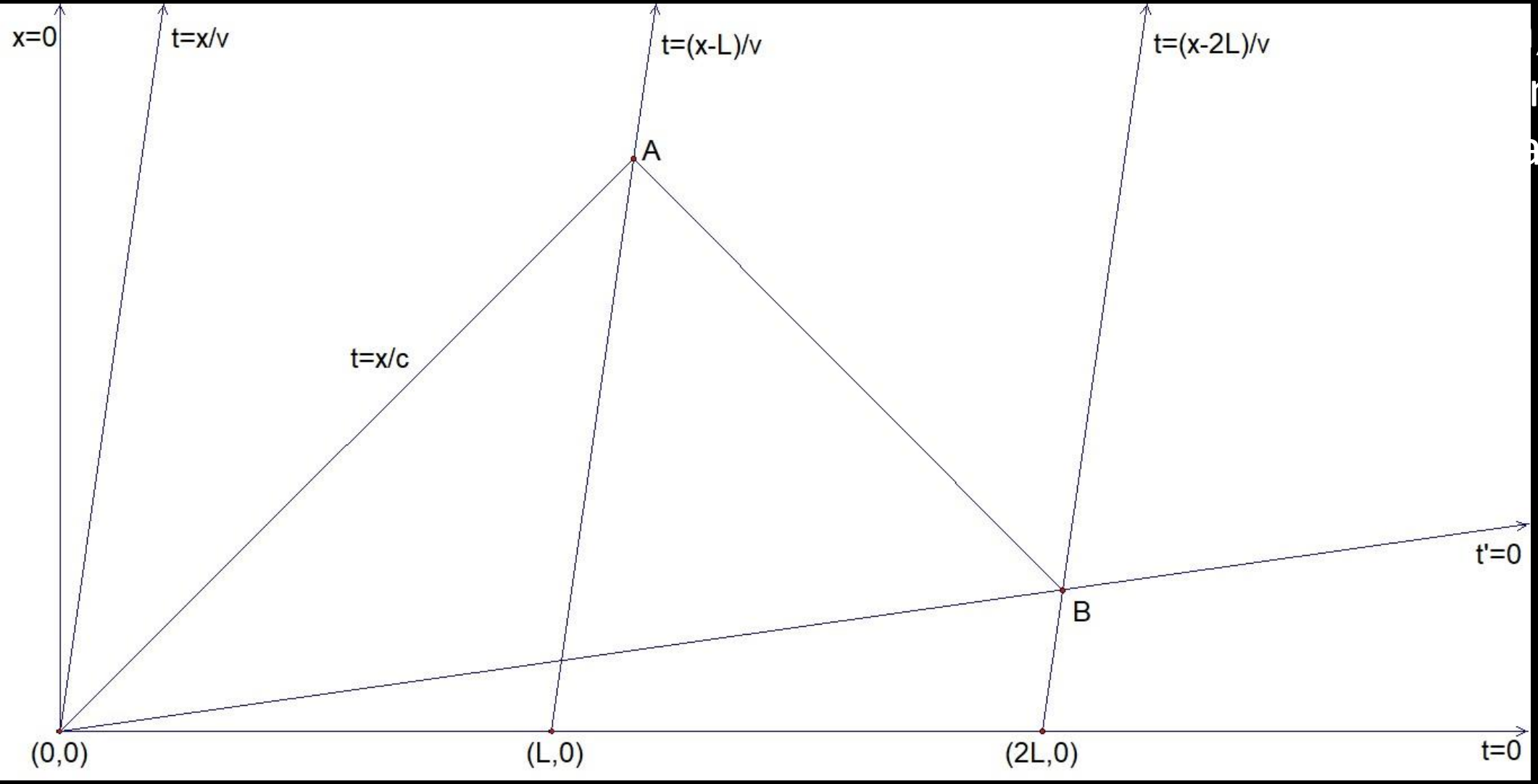

# The Loss of Simultaneity




- Thus, two observers will not necessarily agree that two events are simultaneous!
- Where then are the lines of simultaneity for the observer on the train?

- A-Light leaving Tree #1
- B-Light leaving Tree #2
- C-Train observer passes ground observer
- D-Light from Tree #1 and Tree #2 reaches observer on the ground
- E-Light from Tree #2 reaches observer on train
- F-Light from Tree #1 reaches observer on train
- G-Time when light must leave Tree #2 in order to be simultaneous with light from Tree #1 according to train observer

# Time in the “Moving” Frame

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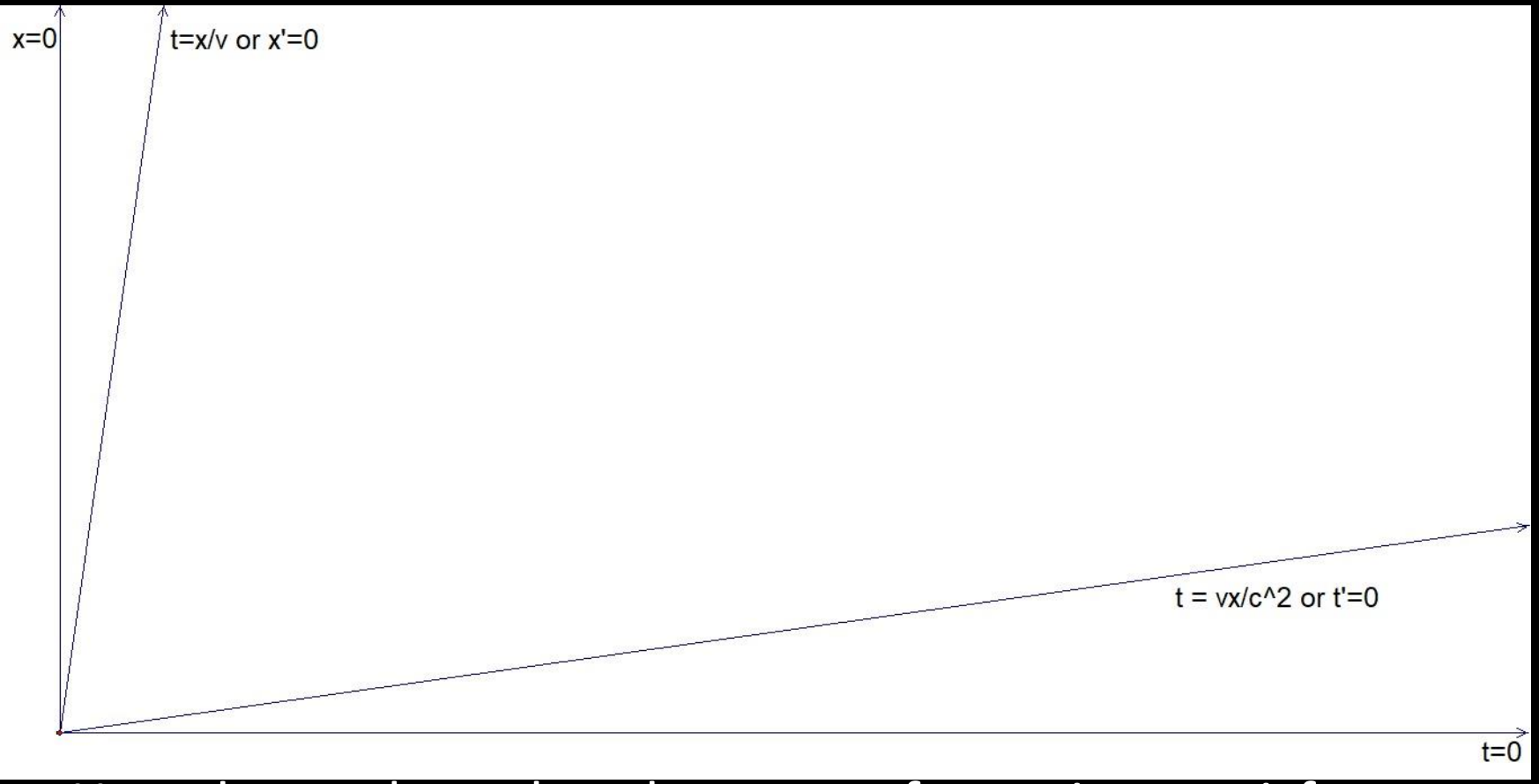
ain.

- It is found to be
$$t = \frac{v}{c^2} x$$
- Thus time in the two frames is different!

- If we can find the coordinates of point B, we can find the line of simultaneity of the flashlights for the train observer in the coordinates of the ground observer.*

# Deriving the Lorentz Transformations




- How do we show that these transformations satisfy Postulate 1?
- We consider the primed system to be at rest and the unprimed system to moving with a velocity $-v$, and require the transformations to still be valid.**

- In order to satisfy both equations describing the primed system lines we must require that

$$x' = x - vt$$

and

$$t' = t - \frac{v}{c^2}x$$

- To check if these work, we first demand that Postulate 2 holds, thus when

$$t = \frac{x}{c}$$

it must be true that

$$t' = \frac{x'}{c}$$

- Some simple algebra shows that this is true.*

# The Lorentz Transformations and Velocity Addition

- In order for both postulates to be satisfied, we must include an extra factor of $\sqrt{1-\frac{v^2}{c^2}}$ in both equations.
- Thus, the four Lorentz transformations in this coordinate system are given by

$$x' = \frac{x - vt}{\sqrt{1-\frac{v^2}{c^2}}} \quad y' = y \quad z' = z \quad t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

- These replace the Galilean transformations while tending to them when $v \ll c$.
- The Galilean velocity addition rule in the $x$-direction becomes

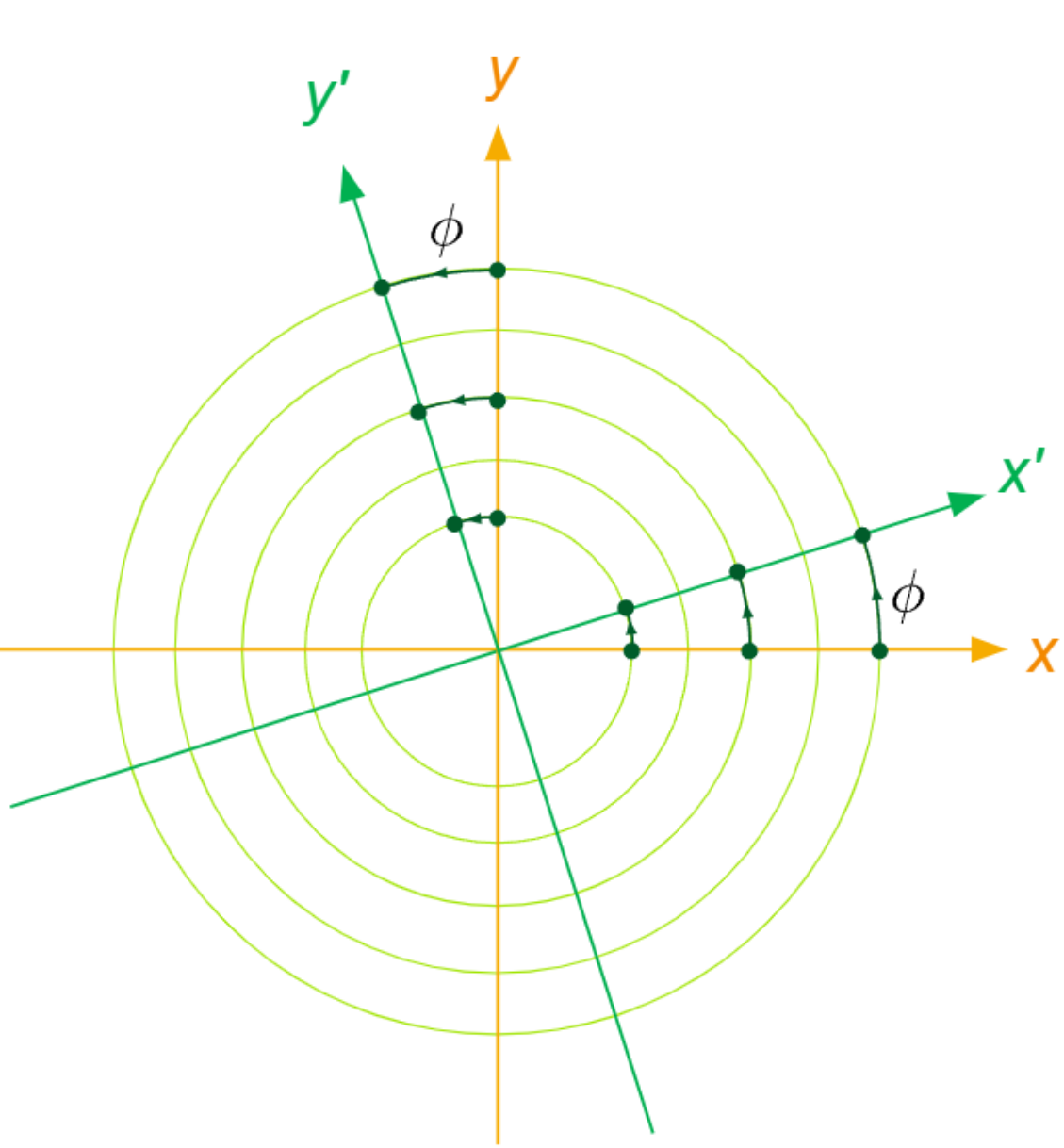
$$v_{AC} = \frac{v_{AB} + v_{BC}}{1 + \frac{v_{AB}v_{BC}}{c^2}}$$

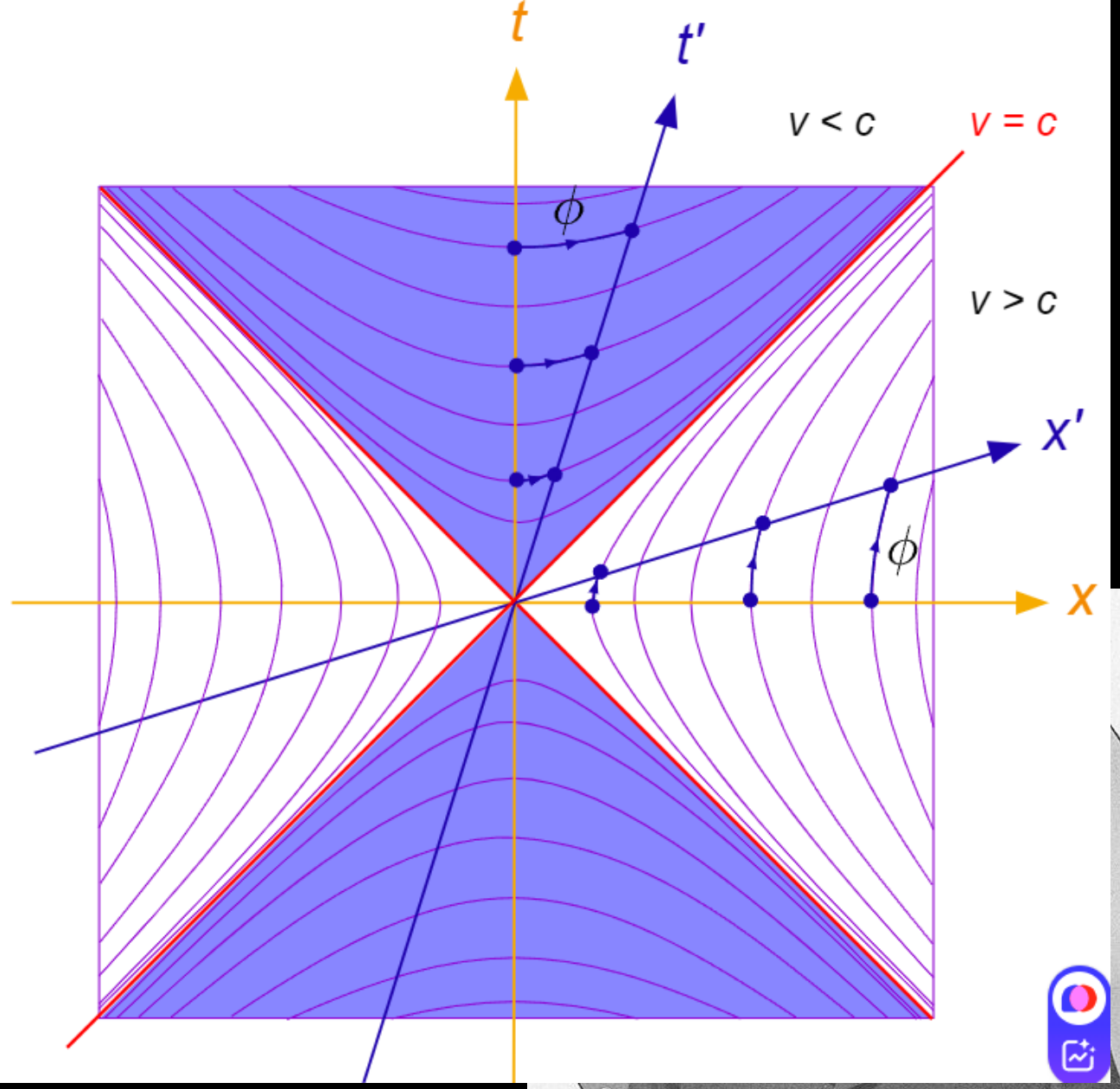
  where $A$ is the event, $B$ is the primed frame and $C$ the unprimed frame.
- From this equation we see that $c$ is essentially a universal speed barrier.
- Examples*

# Weird Predictions

- We can simplify the Lorentz transformations by using the auxiliary variables $\beta = \frac{v}{c}$, a dimensionless speed ratio and $\gamma = \frac{1}{\sqrt{1-\beta^2}}$, the Lorentz factor.
- Using these we can see how the Lorentz transformations predict new phenomena as well as modify many of the equations of classical kinematics and dynamics. Some* of these include (where motion is only in the $x$-direction):

| | | |
|---|---|---|
| • Length contraction | $L = \frac{L_0}{\gamma}$ | $L_0$ is the "proper length" |
| • Time dilation | $T = \gamma T_0$ | $T_0$ is the "proper time" |
| • Relativistic momentum | $p = \gamma m v$ | $m$ is the particle mass |
| • Relativistic Newton's 2nd Law | $F = \frac{dp}{dt} = \gamma^3 m a$ | |
| • Relativistic total energy | $E = \gamma m c^2$ | |
| • Rest energy | $E = m c^2$ | |
| • Relativistic kinetic energy | $E = m c^2 (\gamma - 1)$ | |
| • Energy-momentum relationship | $E^2 = p^2 c^2 + m^2 c^4$ | |

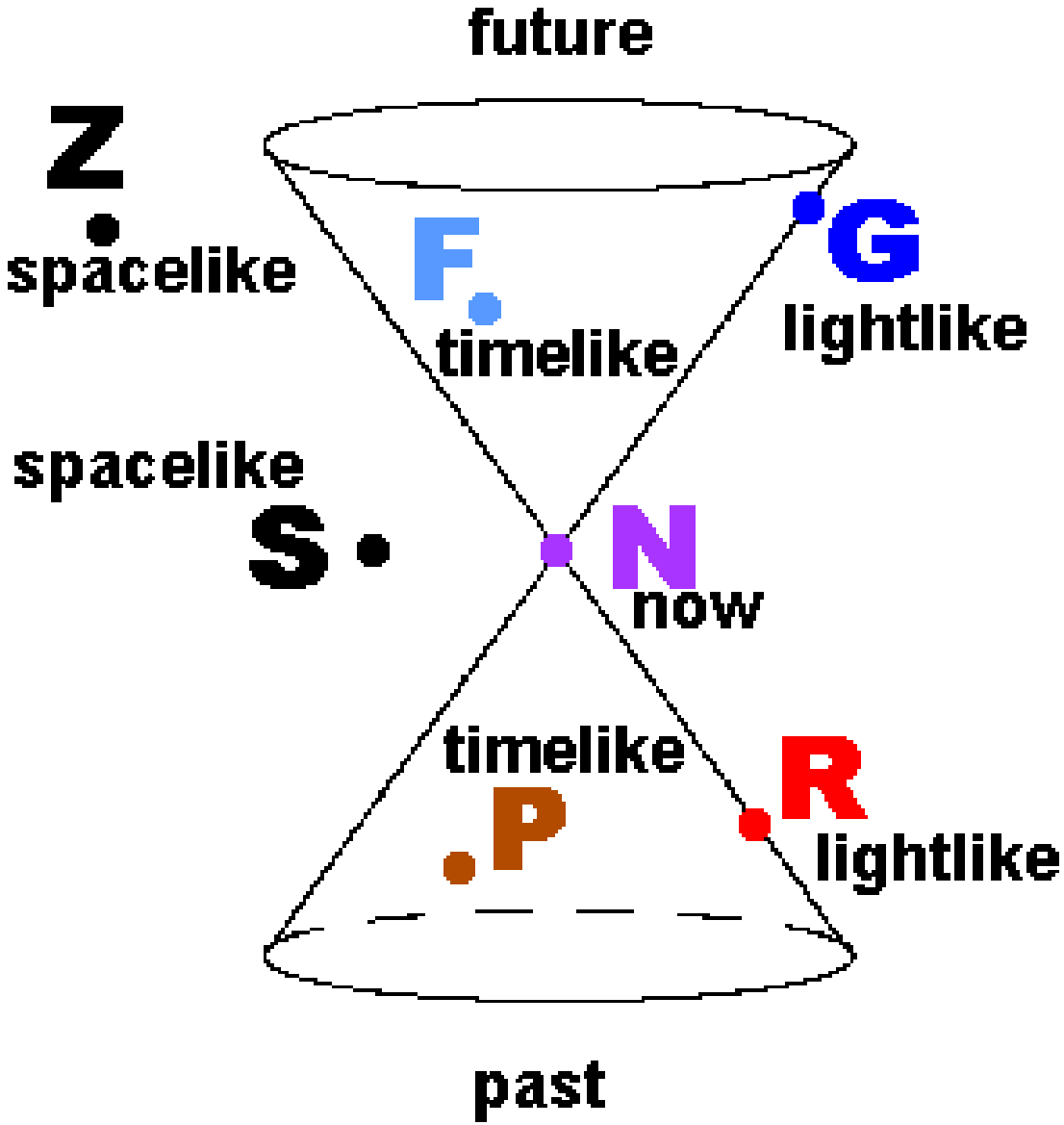



- This makes spacetime hyperbolic not circular. The hyperbolas sections show that an observer can't catch up to a light beam.

# Light Cones and Causality

- N, P and F-time-like interval $(ds < 0)$
- N, G and R-light-like interval $(ds = 0)$
- N, S and Z-space-like interval $(ds > 0)$
- All observers agree that P happened first, then N and then F, but not what the interval of time is between them.
- Some observers think N happened first, some S and some believe Z happened first!



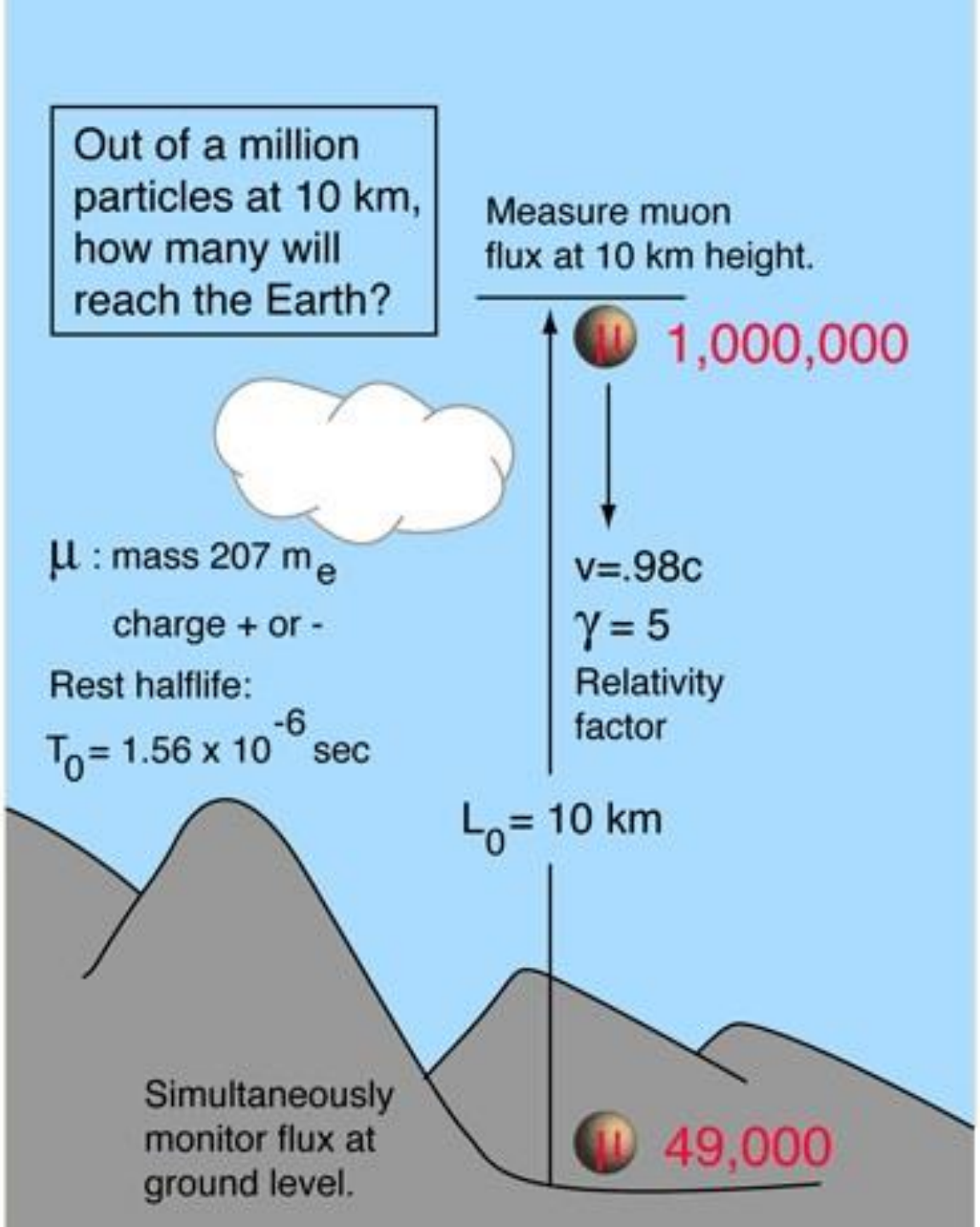
# 4-Vectors

- The combination of space and time as seen in the Minkowski metric can be shown in vector form, just with four components: the displacement 4-vector

$$(c\,dt, dx, dy, dz)$$

- There are other things that are "mixed" together as a result of the mixing of space and time and can be written as 4-vectors. They include:
- 4-velocity $\gamma(c, v_x, v_y, v_z)$
- 4-acceleration $\gamma(\frac{d\gamma}{dt}c, \frac{d\gamma}{dt}v_x + \gamma a_x, \frac{d\gamma}{dt}v_y + \gamma a_y, \frac{d\gamma}{dt}v_z + \gamma a_z)$
- 4-momentum $(\frac{E}{c}, p_x, p_y, p_z)$
- 4-force $\gamma(\frac{P}{c}, F_x, F_y, F_z)$
- 4-current density $(\rho c, j_x, j_y, j_z)$
- 4-potential $(\frac{\phi}{c}, A_x, A_y, A_z)$
- Using 4-vectors many equations are simplified including Maxwell's four equations which can be written as a single 4-vector equation which transforms correctly under Lorentz transformations, with electric and magnetic fields converting into one another.

# Experi


Distance: $L_0 = 10^4$ meters

Time: $T = \frac{2000\ m}{(0.98)(3 \times 10^8\ m/s)}$

$T = 6.8 \times 10^{-6}$ s = 4.36 halflives

Survival rate:

$$\frac{I}{I_0} = 2^{-4.36} = 0.049$$

Or about 49,000 out of a million.

The muon sees distance as length-contracted so that $L = L_0/\gamma = 0.2L_0 = 2$ km.

- Relativist

# Everyday life and paradoxes

- So if relativity has been around for over 100 years and continually agrees with experimental data, why don't we see evidence of it in our everyday life?
- We never travel anywhere close to c! The fastest thing humans have ever created is…a manhole cover?
- https://www.youtube.com/watch?v=NSeL5c65v-g
- This is only 0.0002c.
- At this rate, a meter stick would only appear 28 nm shorter and watches would tick off less than 1 extra second per year.
- Because these predictions are so counterintuitive, they can be hard to understand and present "paradoxes".

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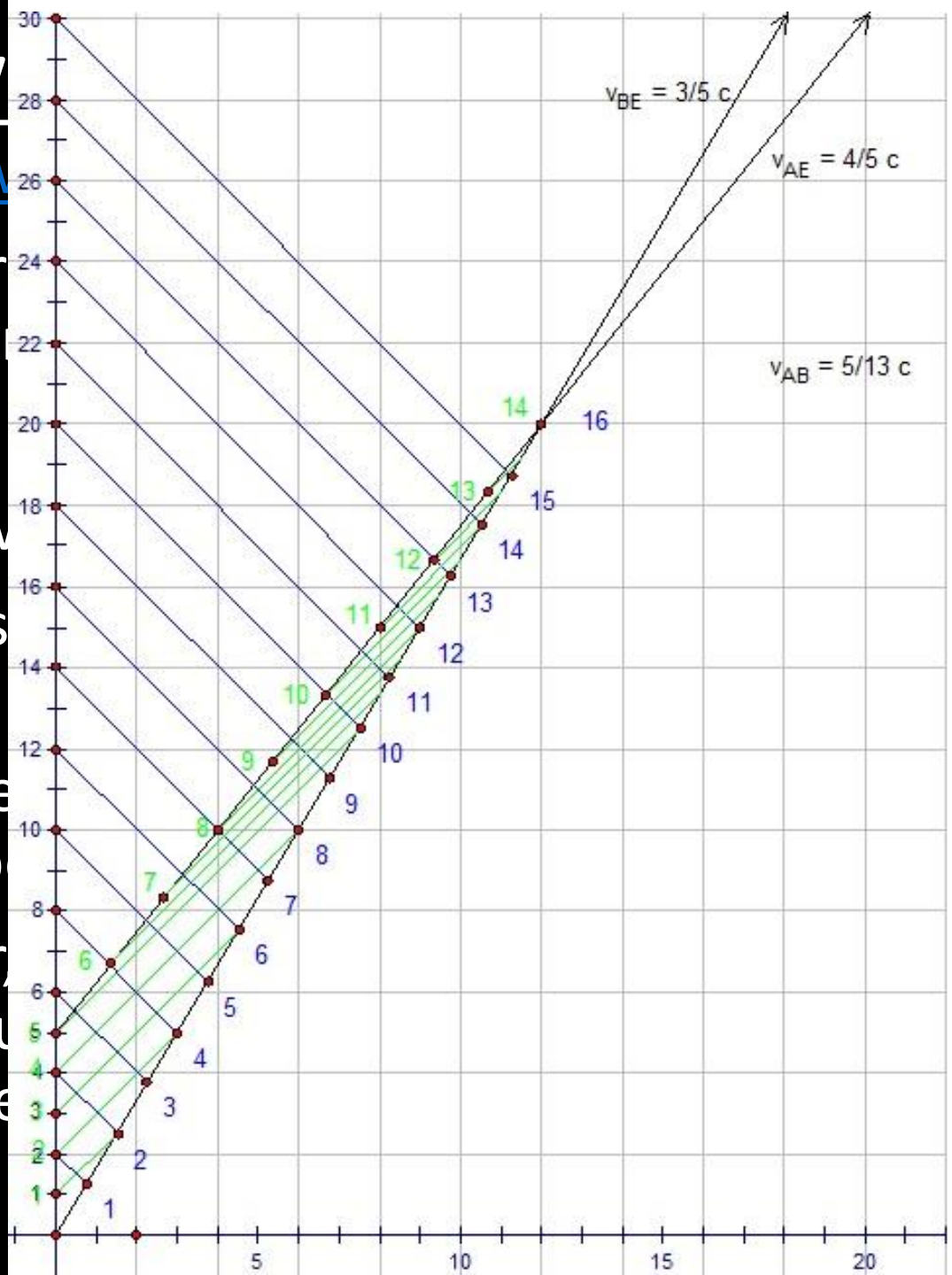

# Pole in the Barn Paradox

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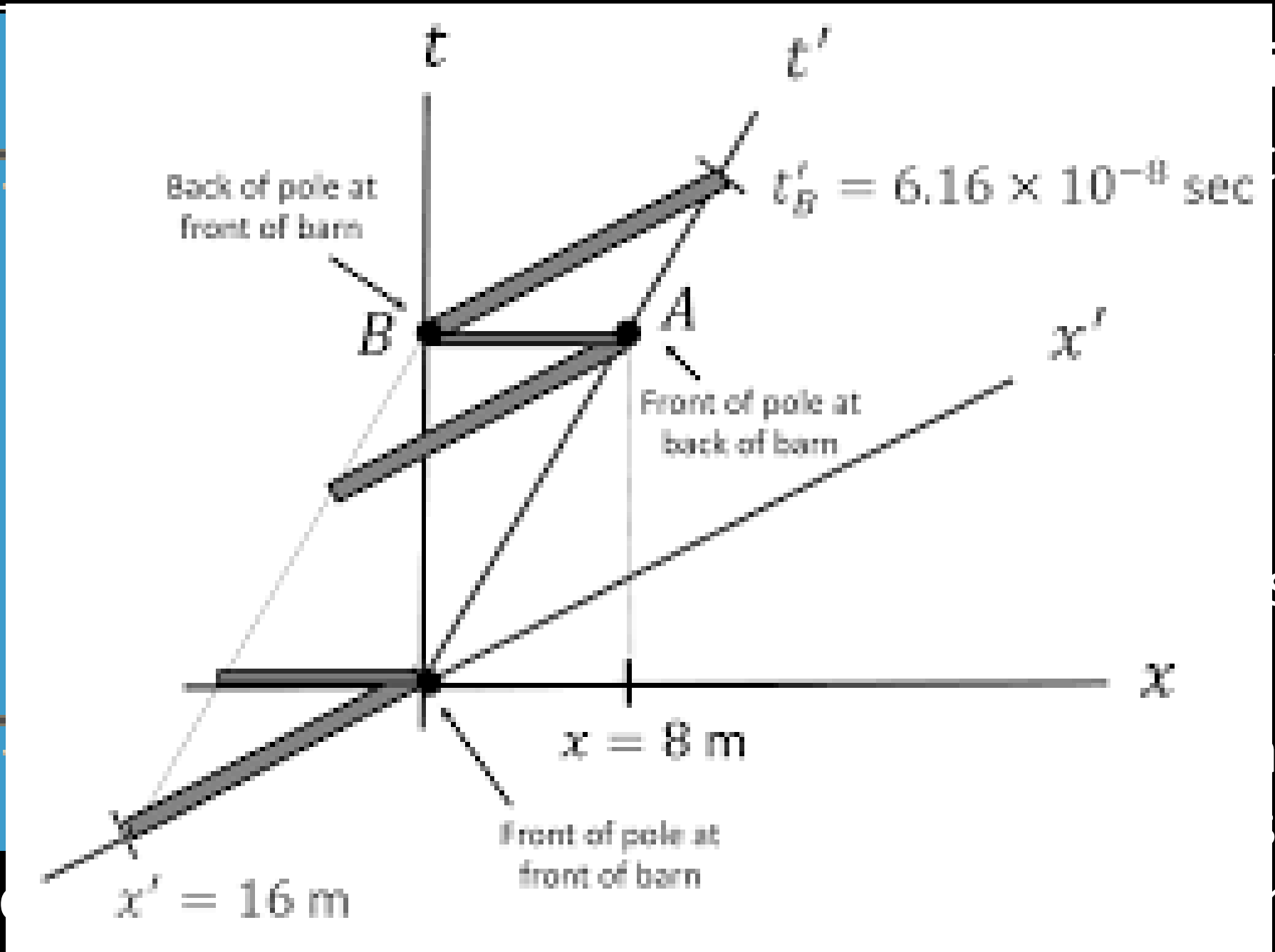
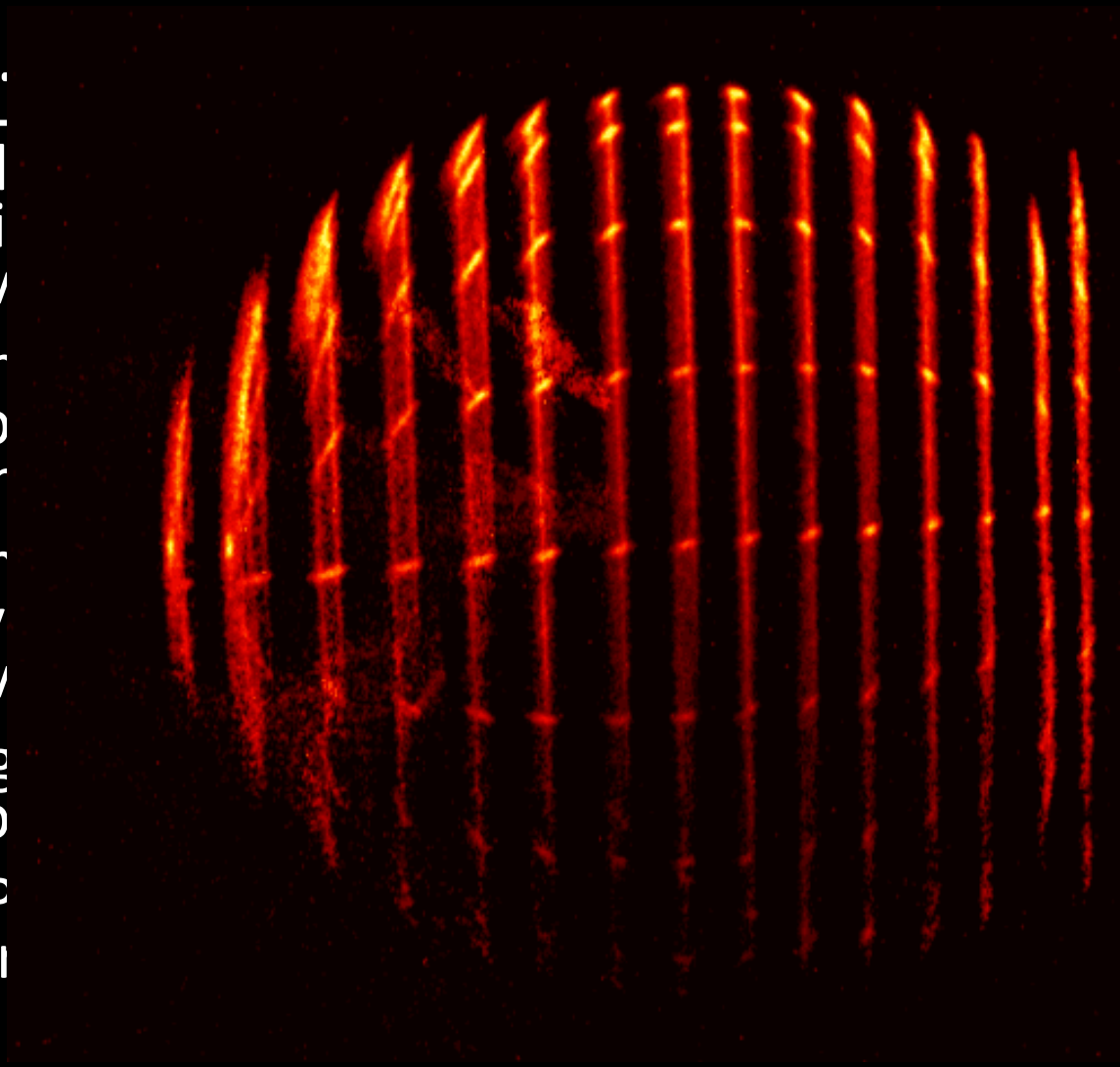


# The Lampa



- In 1924, over a d e chair of the theoretical physi d on how a rod moving at relativ
- However, it wasr ered again, independently, b t people showed interest in the ph
- This effect, comr n contraction occurs but is essentially taking longer to reach the observ
- Instead of seeing same size but be rotated so that p vould not be.
- In the words of P e observer from behind the spher moving out of its way".
- This counterintuitive effect was experimentally verified in 2024 by Peter Schattschneider by using femto-second laser pulses to reduce the speed of light to 2 m/s .