Introduction to Special Relativity

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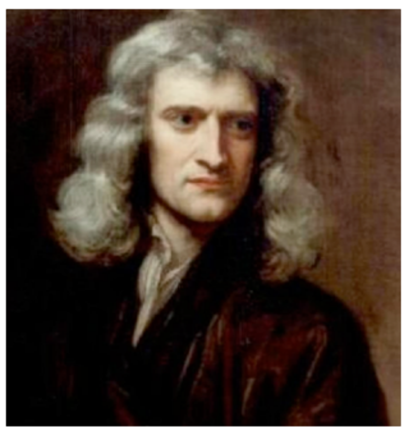
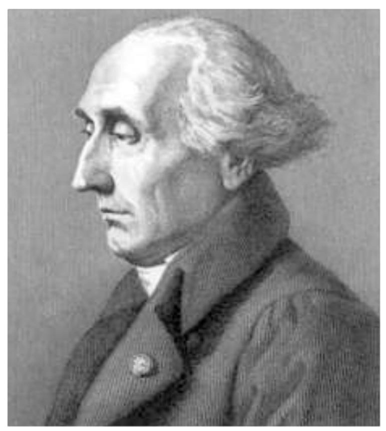
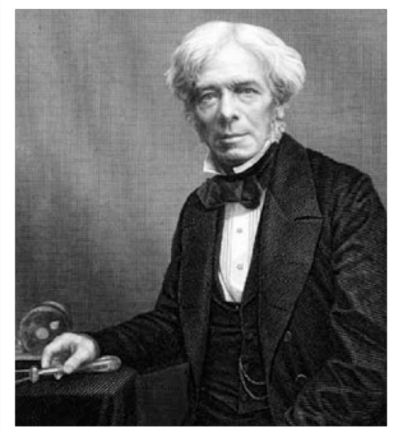
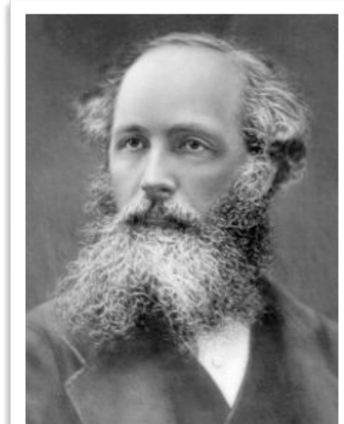
Quarknet @ UMD summer workshop

Physics complete in the XIX century?

- From the **XVII to the XIX centuries, extraordinary progress** in our fundamental understanding of the universe
  - ➡ Newton's Gravitation, Classical and Statistical Mechanics, Thermodynamics, Electromagnetism





$$F = G\frac{Mm}{R^2}$$

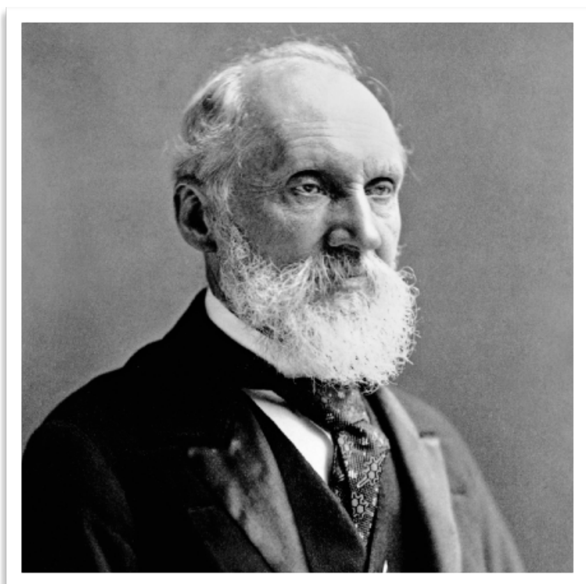
$$\boldsymbol{\nabla} \cdot \boldsymbol{E} = \frac{\rho}{\epsilon_0} \qquad \boldsymbol{\nabla} \cdot \boldsymbol{B} = 0 \qquad \boldsymbol{\nabla} \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \qquad \boldsymbol{\nabla} \times \boldsymbol{B} = \mu_0 \left(\boldsymbol{J} + \frac{\partial \boldsymbol{E}}{\partial t}\right)$$

Feeling at the end of the XIX century

> *"There is nothing new to be discovered in physics now, All that remains is more and more precise measurements."*

Lord Kelvin's clouds

"The beauty and clearness of the dynamical theory, which asserts heat and light to be modes of motion, is at present obscured by two clouds"



THE

LONDON, EDINBURGH, AND DUBLIN

PHILOSOPHICAL MAGAZINE

AND

JOURNAL OF SCIENCE

[SIXTH SERIES.]

JULY 1901.

I. *Nineteenth Century Clouds over the Dynamical Theory of Heat and Light* *. *By* The Right. Hon. Lord KELVIN, *G.C.V.O., D.C.L., LL.D., F.R.S., M.R.I.* †.

Black-body spectrum

Michelson-Morley

Michelson-Morley experiment

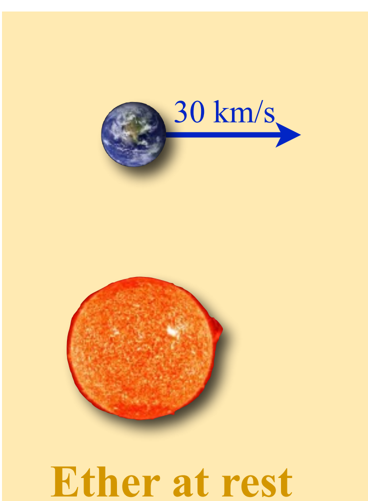
Maxwell equations predicted electromagnetic waves always traveling at speed $c = 1/\sqrt{\mu_0 \epsilon_0}$

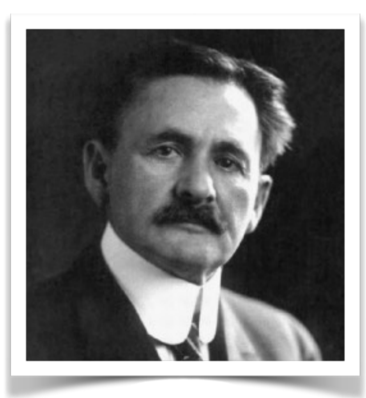
Speed c with respect to what???

$$\nabla^2 \boldsymbol{E} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\boldsymbol{E} = 0$$

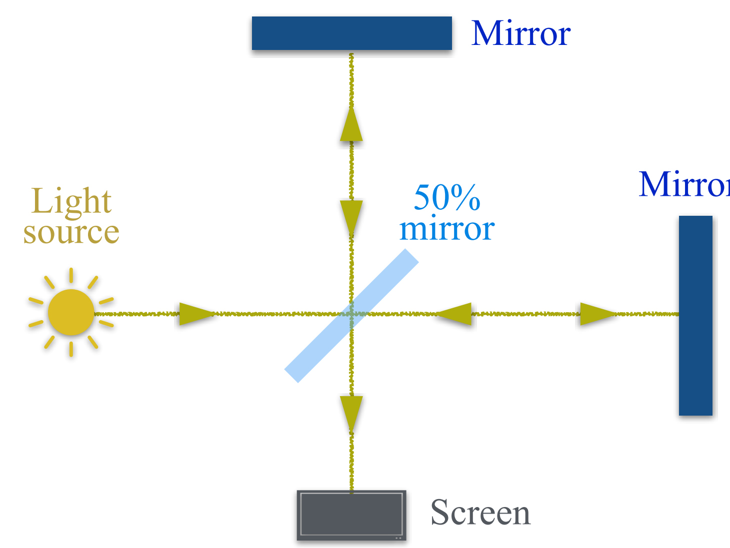
Hypothesis was that **Earth** was **moving at least at 30 km/s with respect to Ether**

If light moved at c with respect to Ether, **beam** would take **different times when aligned with Earth's speed versus transverse to it**





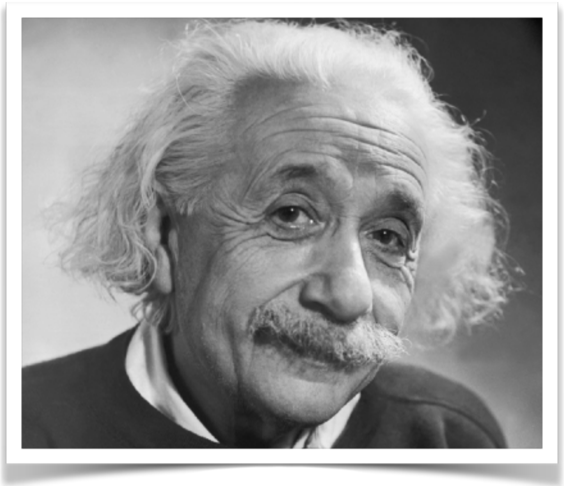
Michelson invented in 1881 (Naval academy, Annapolis) an **interferometer** that could measure a difference of one part in $c/v \approx 10^4$



No evidence for **light traveling** at **different speeds** in **different directions**

Albert Michelson would go onto win the **Nobel prize for Physics** in **1907,** the first American to do so!

Einstein's postulates



Relativity principle: all laws of Physics** have the **same form** in all **inertial frames of reference

*The **speed of light in vacuum** has the **same value c** in every direction in all **inertial frames of reference***

Zur Elektrodynamik bewegter Körper;
von A. Einstein.

dies für die Größen erster Ordnung bereits erwiesen ist. Wir wollen diese Vermutung (deren Inhalt im folgenden „Prinzip der Relativität" genannt werden wird) zur Voraussetzung erheben und außerdem die mit ihm nur scheinbar unverträgliche Voraussetzung einführen, daß sich das Licht im leeren Raume stets mit einer bestimmten, vom Bewegungszustande des emittierenden Körpers unabhängigen Geschwindigkeit V fortpflanze. Diese beiden Voraussetzungen genügen, um zu einer einfachen und widerspruchsfreien Elektrodynamik bewegter Körper zu gelangen unter Zugrundelegung der Maxwellschen Theorie für ruhende Körper. Die Einführung eines „Lichtäthers" wird sich

On the Electrodynamics of Moving Bodies;
A. Einstein

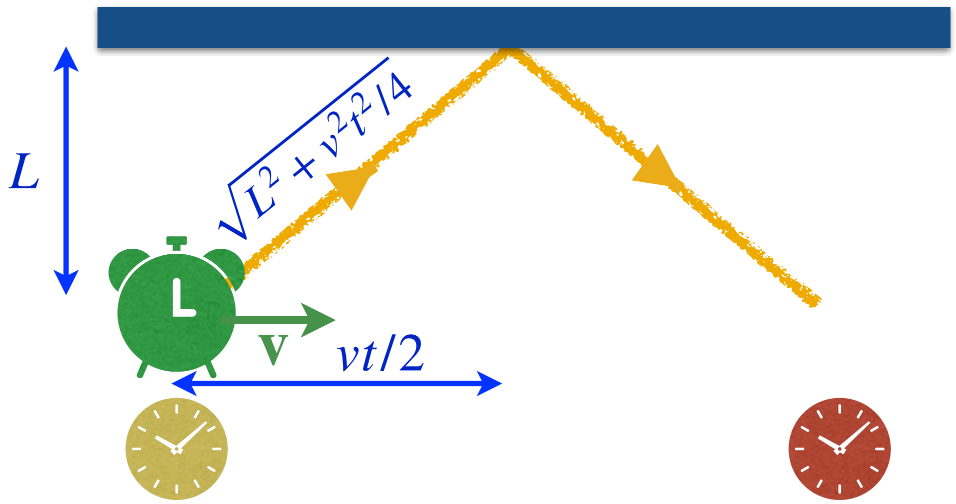
equations of mechanics hold good.[1] We will raise this conjecture (the purport of which will hereafter be called the "Principle of Relativity") to the status of a postulate, and also introduce another postulate, which is only apparently irreconcilable with the former, namely, that light is always propagated in empty space with a definite velocity c which is independent of the state of motion of the emitting body. These two postulates suffice for the attainment of a simple and consistent theory of the electrodynamics of moving bodies based on Maxwell's theory for stationary bodies. The introduction of a "luminiferous ether" will

Einstein's 1905 paper is considered the **first complete account of Special relativity**, but many elements had **also** been **derived** by **Hendrik Lorentz** and **Henri Poincaré**

Time dilation

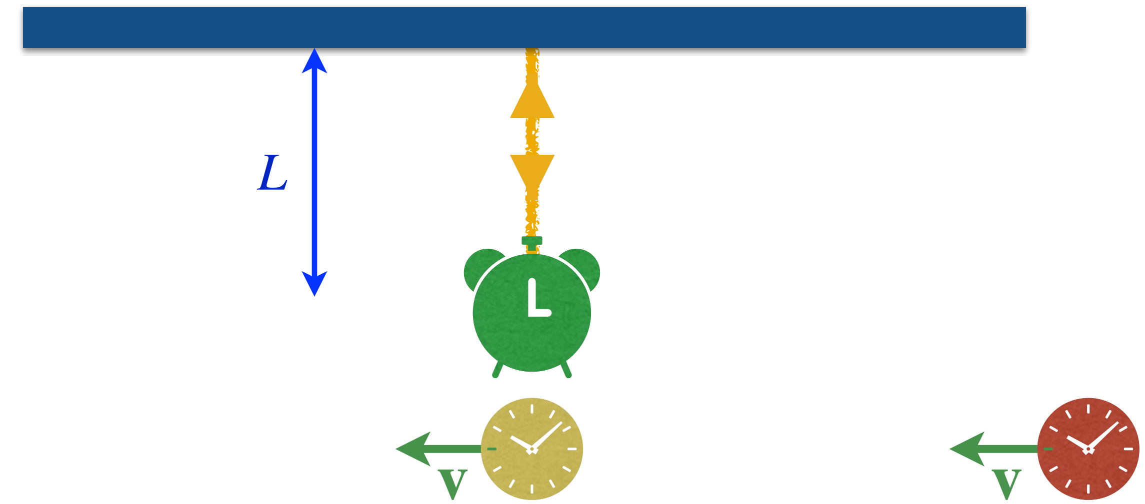
~ What's the **time** passed **between light is emitted** and **absorbed** by the **green clock**?

Rest frame



$$t = 2\frac{\sqrt{L^2 + v^2t^2/4}}{c} \Rightarrow t = \frac{2L}{c}\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Green clock frame



Total time between events is simply **total distance** covered by light **divided by speed**

$$t' = \frac{2L}{c}$$

We define a new quantity γ

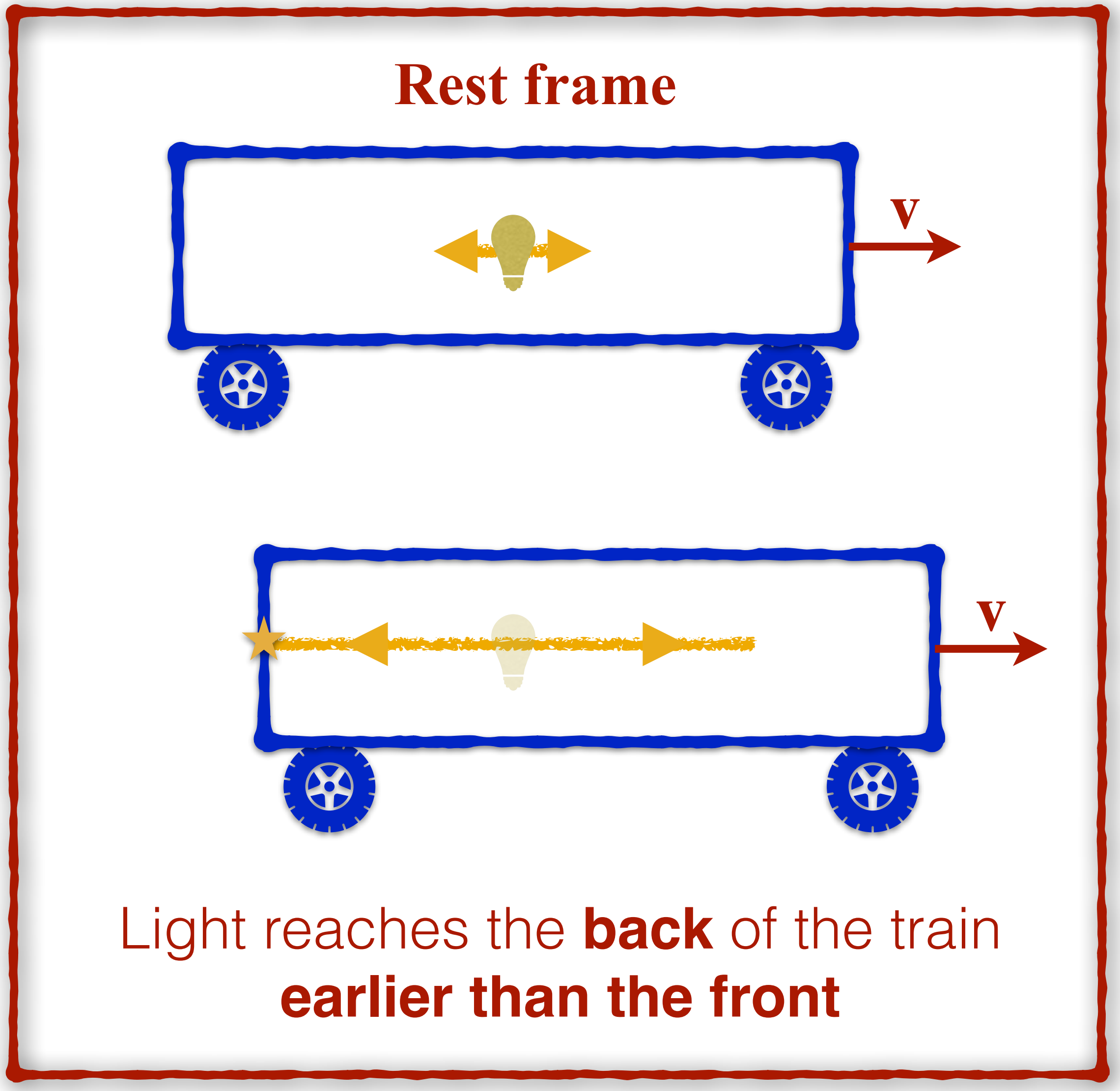
$$\gamma \equiv \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

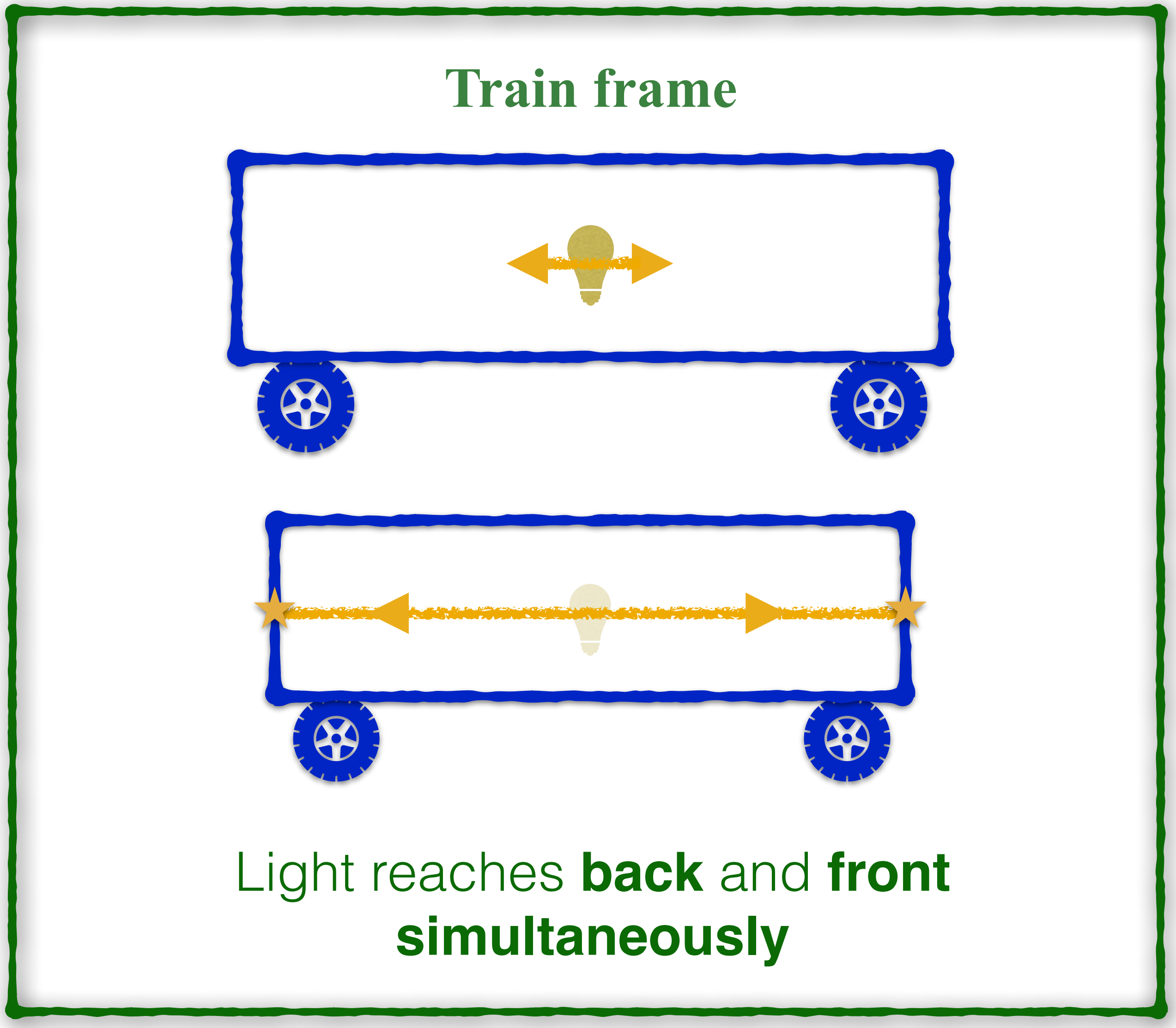
$$t = t'\gamma$$

Based on Einstein postulates alone, **time passes more slowly when moving**

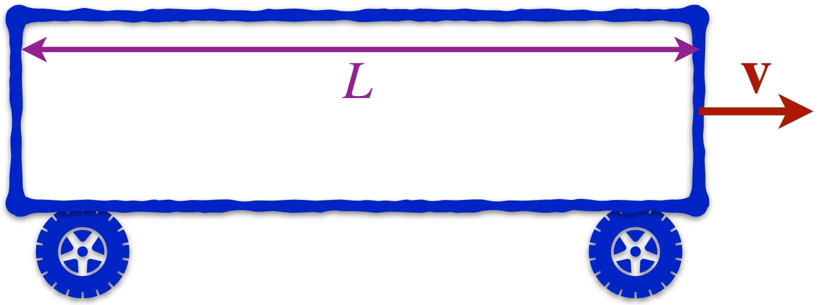
Simultaneity is frame dependent

- A light goes off in the middle of a moving train
  - ➡ Do the light rays **reach** the **front/back of the train at the same time?**

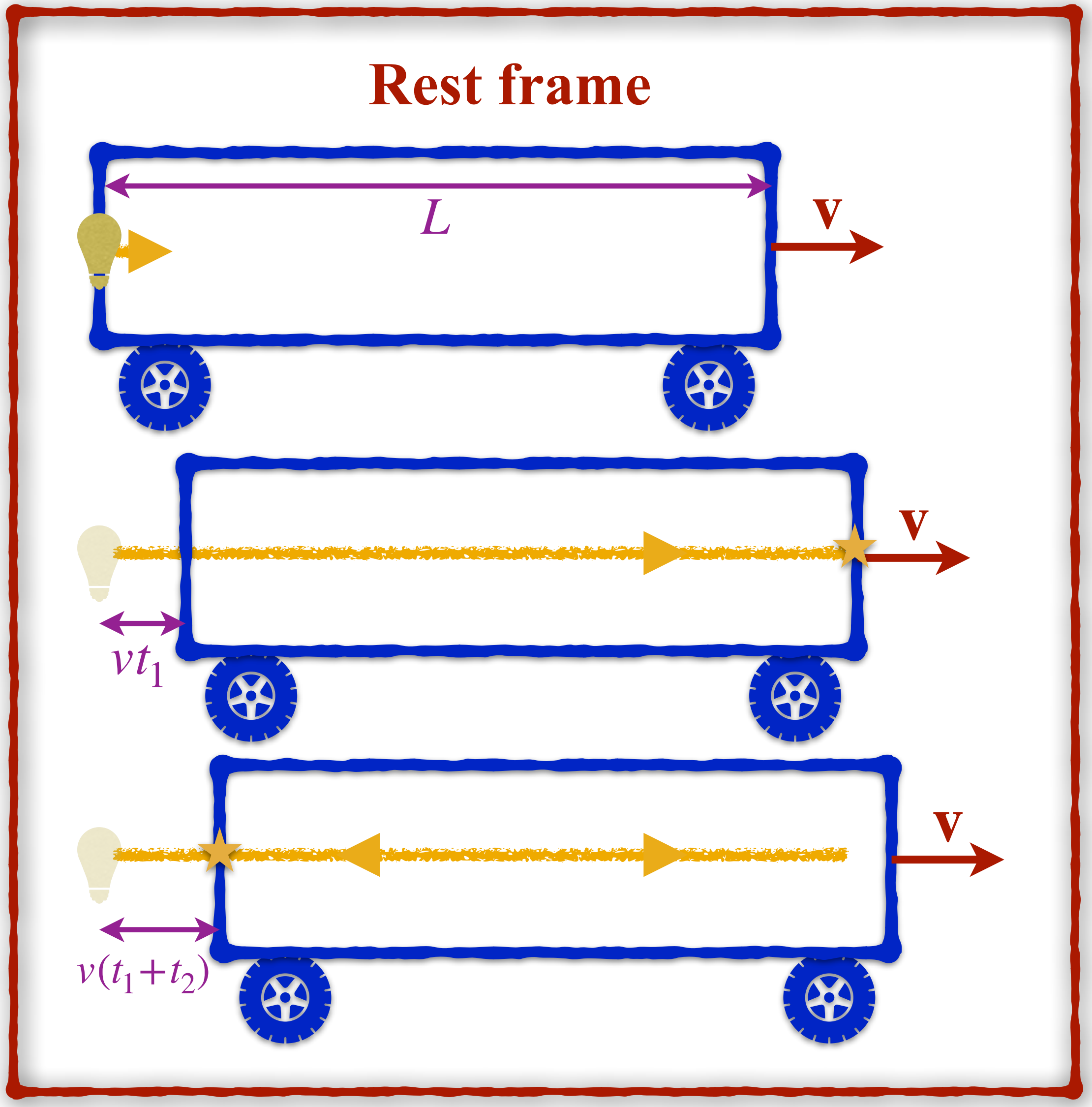


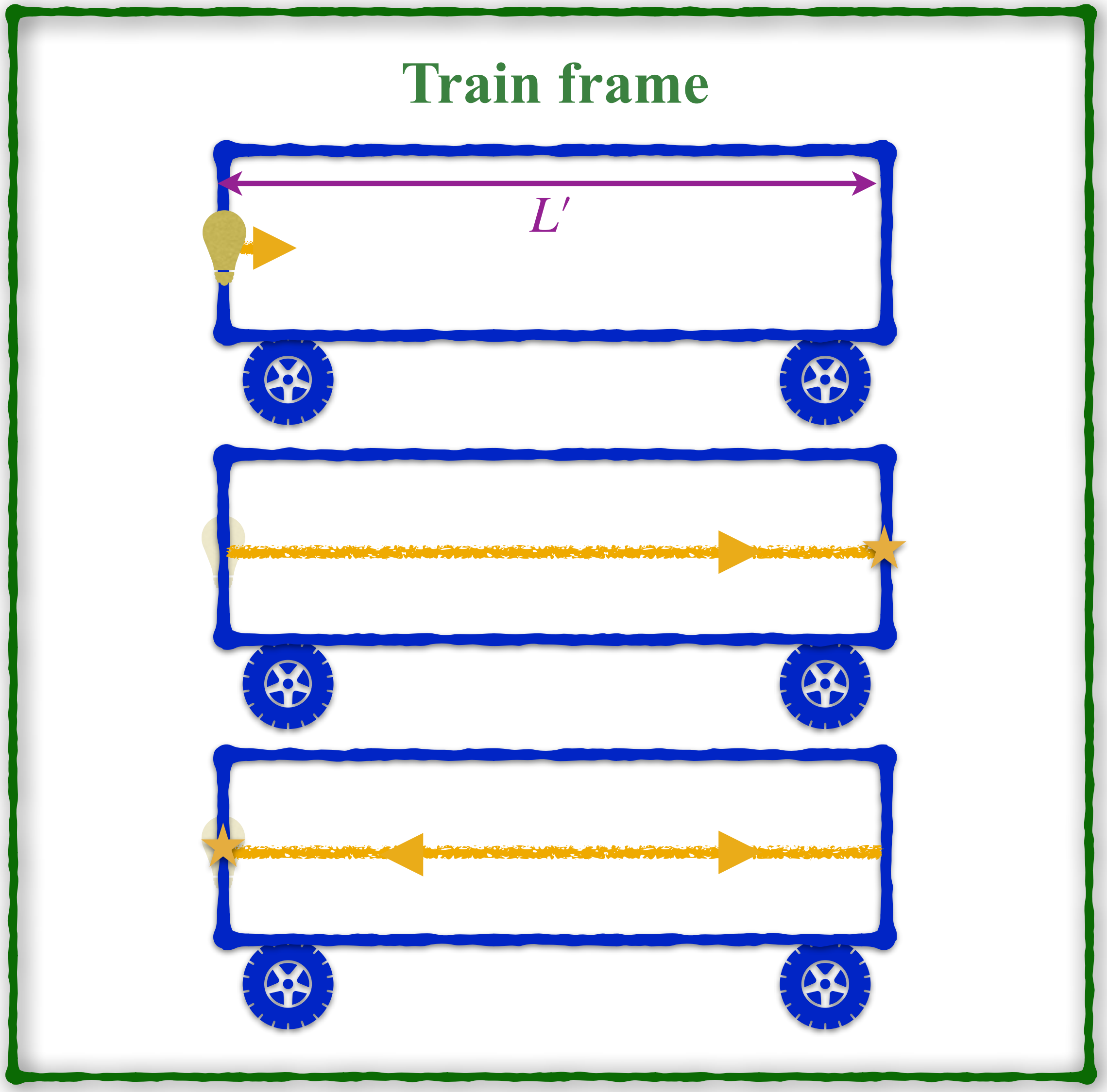


Whether two events that occurred far apart are simultaneous or not depends on the frame of reference



- Is the length of the train the same at rest and moving?





$$t_1 = (L + vt_1)/c \rightarrow t_1 = \frac{L}{c-v}$$

$$t_2 = (L - vt_2)/c \rightarrow t_2 = \frac{L}{c+v}$$

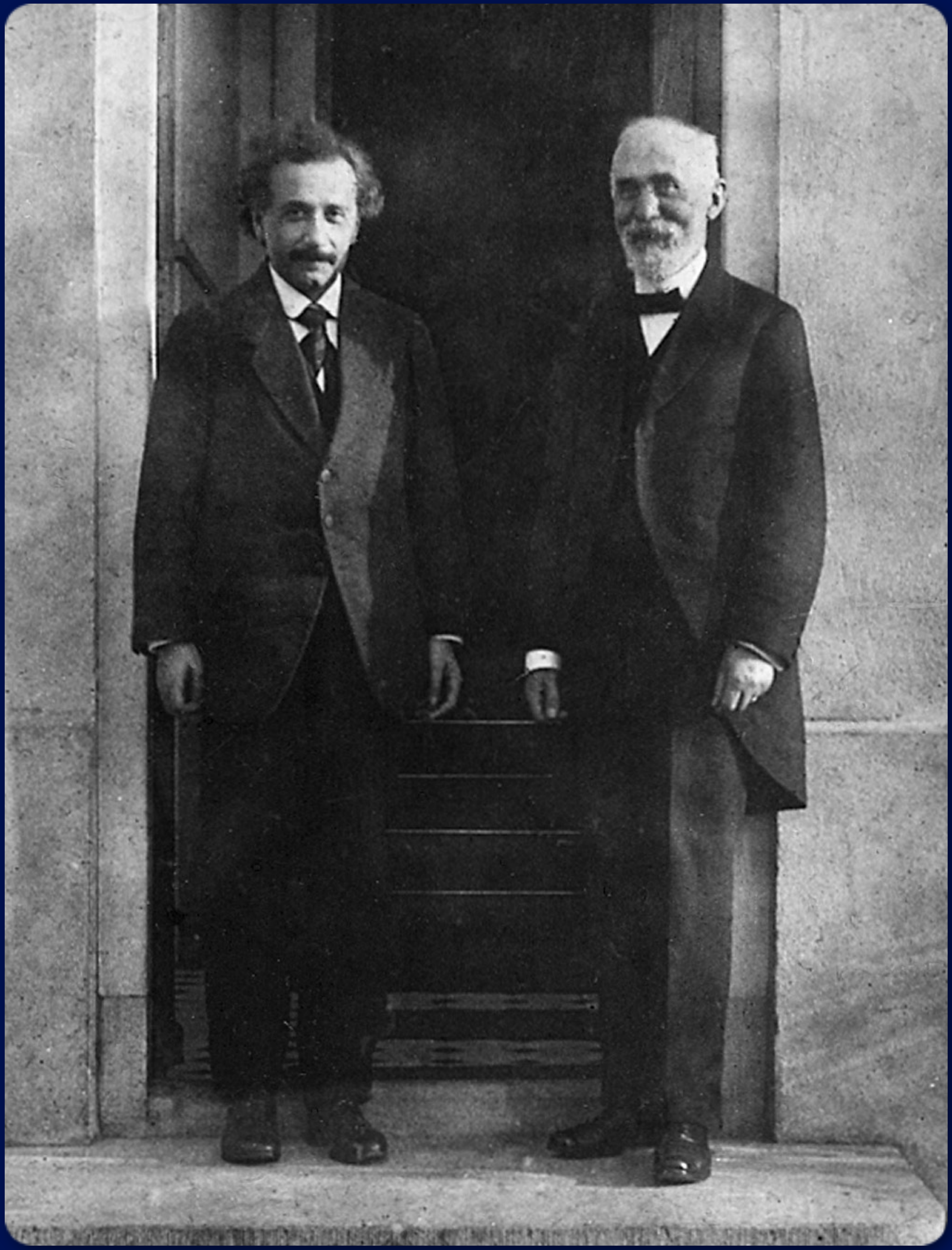
$$\Bigg\} \; t = t_1 + t_2 = \frac{2L}{c}\frac{1}{1 - v^2/c^2}$$

$$t' = \frac{2L'}{c}$$

Using time dilation $t = t'\gamma$

$$L = \frac{L'}{\gamma}$$

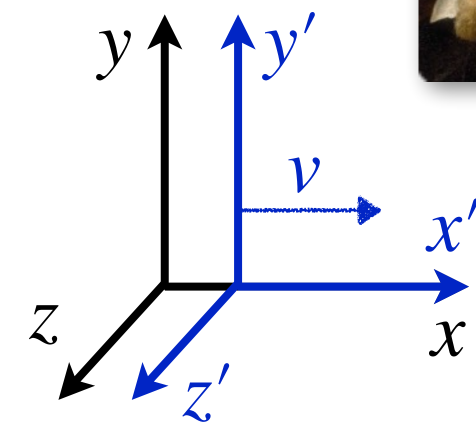
Moving objects are **contracted along the direction of movement**



Lorentz transformations and 4-vectors

Lorentz transformations

- **Galilean transformations** do **not account** for **time dilation** or **space contraction**



$$t' = t \qquad x' = x - vt \qquad y' = y \qquad z' = z$$

- **Lorentz transformations do**

Space and time are intimately intertwined

$$t' = \gamma t - \beta\gamma x/c \qquad y' = y$$

$$x' = \gamma x - \beta\gamma ct \qquad z' = z$$

➡ Where we defined

Normalized velocity $\beta = \frac{v}{c}$,

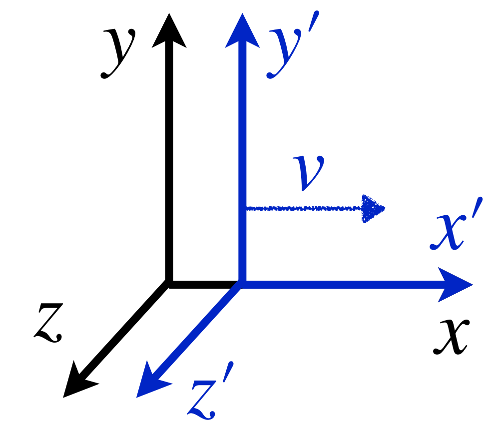
unitless number between 0 and 1

Lorentz factor $\gamma = \frac{1}{\sqrt{1-\beta^2}}$,

unitless number between 1 and ∞

- Instead of describing positions with 3D vectors, we use **4-vectors** that **combine time** and **space**

$$\begin{bmatrix} ct' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix}$$



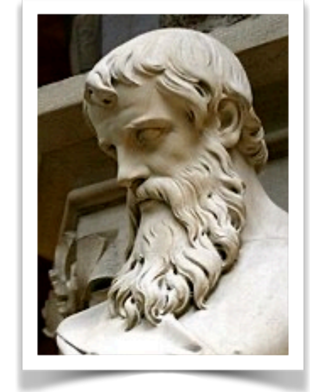
Lorentz transformation in matrix form

Normalized velocity $\beta = \frac{v}{c}$,
unitless number between 0 and 1

Lorentz factor $\gamma = \frac{1}{\sqrt{1-\beta^2}}$,
unitless number between 1 and ∞

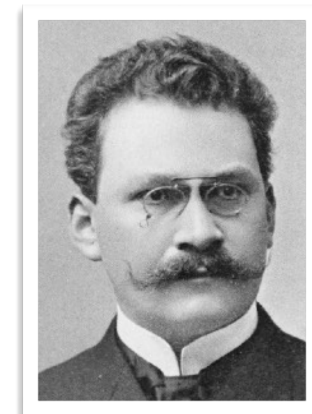
Norm of 4-vectors

- In **Euclidean space**, norm of vector found **adding up** the **squares** of **all the components**



$$|\vec{r}|^2 = x^2 + y^2 + z^2$$

- **4-vectors** live in **Minkowski space**, and the **norm** adds the **time** and **spatial** components with **different signs**
  - ➡ This **norm** is the **same in all frames** (invariant)

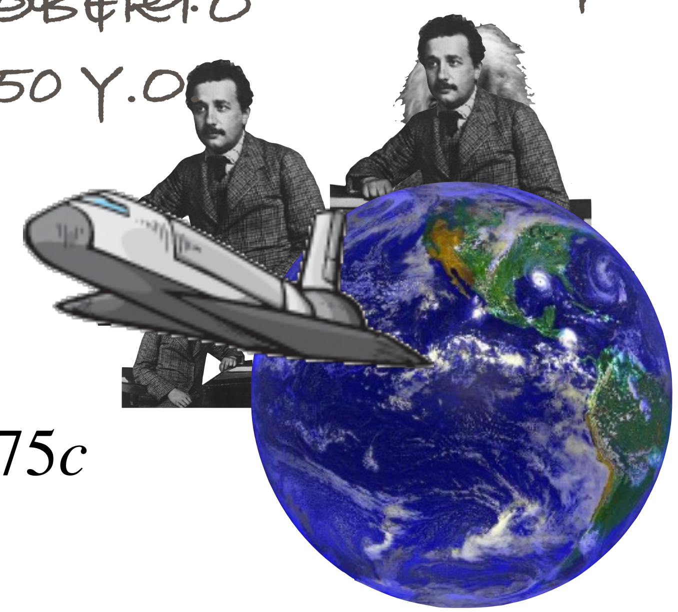


$$\sum r_\mu r^\mu = c^2t^2 - x^2 - y^2 - z^2 = \boldsymbol{c^2\tau^2}$$

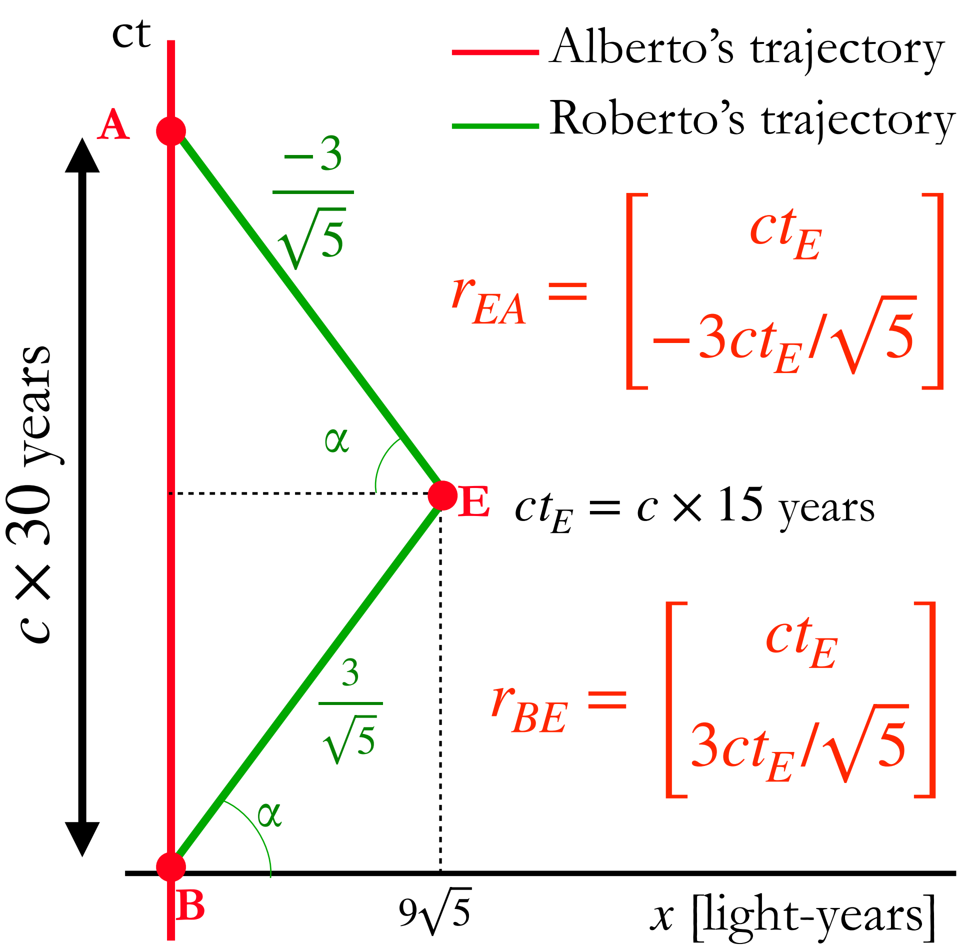
- For the **space-time 4-vector**, the **norm** is the **proper time τ** times c
  - ➡ The **proper time** is the **time** that **passed** for a **traveler** that **followed** the **space-time trajectory** given by $\boldsymbol{r^\mu}$

Twin paradox: what's Roberto's age?

$$v_R = \frac{-\sqrt{5}c}{3} \approx -0.75c$$



$$v_R = \frac{\sqrt{5}c}{3} \approx 0.75c$$



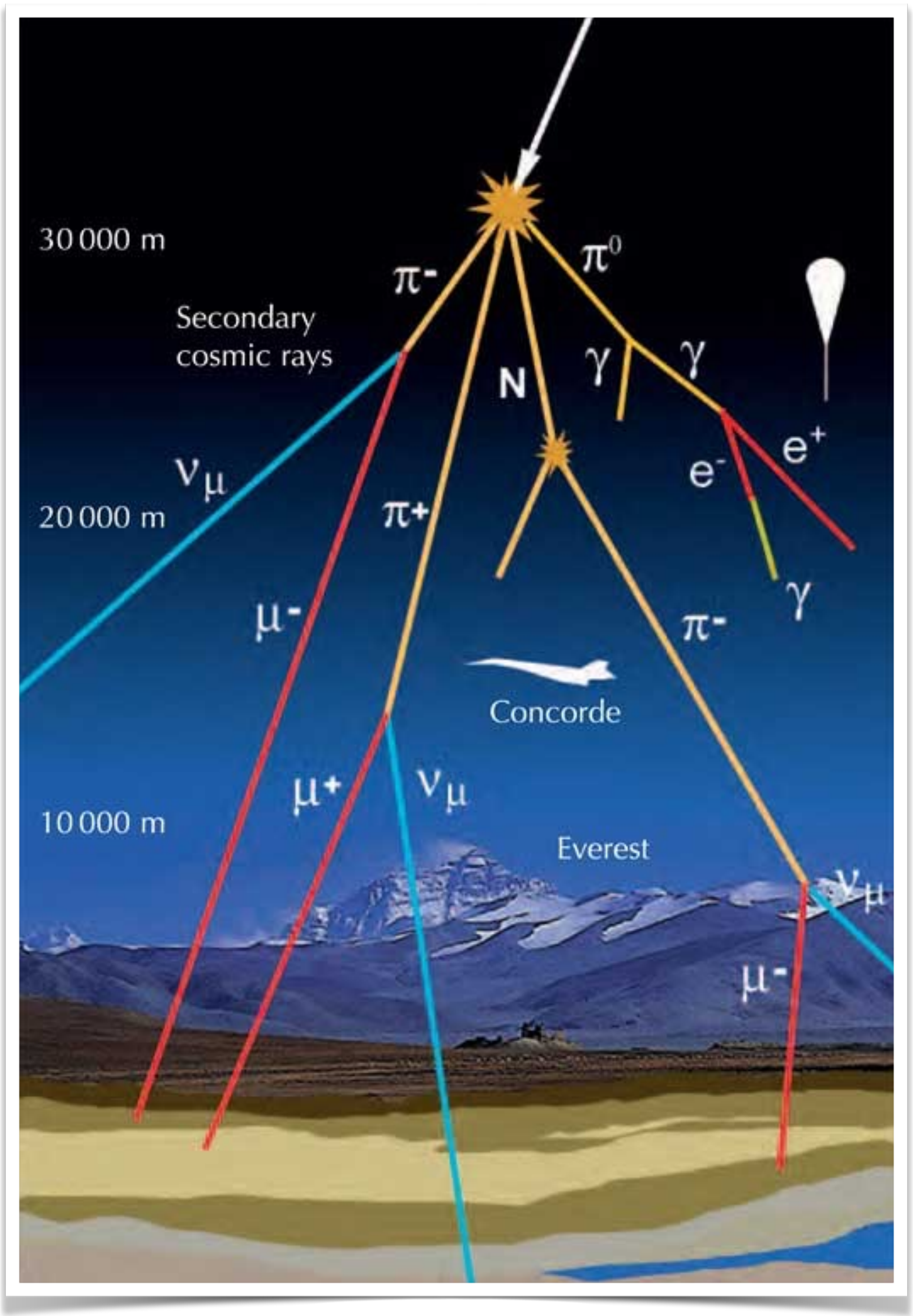
From B to E, the norm of the vector is

$$|r_{BE}|^2 = (ct_E)^2 - \left(\frac{3}{\sqrt{5}}ct_E\right)^2 = c^2\,(10 \text{ years})^2$$

$$|r_{EA}|^2 = |r_{BE}|^2 = c^2\,(10 \text{ years})^2$$

Roberto is 50 when he comes back!

Example: lifetime of the muon



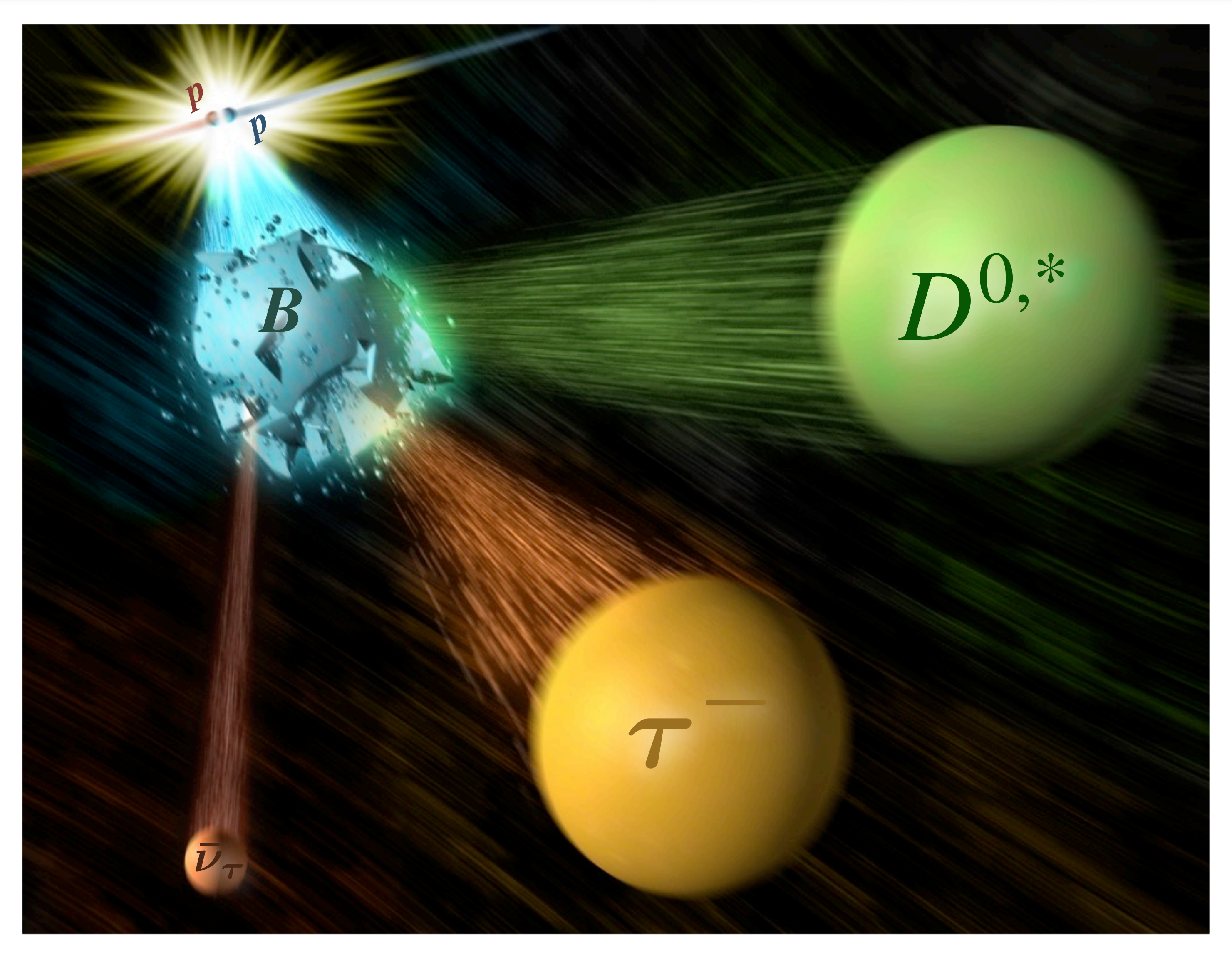
- Atmospheric muons typically come from $\pi^\pm \to \mu^\pm \nu_\mu$ decays
  - ➡ Then decay as $\mu^\pm \to e^\pm \nu_e \nu_\mu$
- What is the **lifetime** of a **muon**
  - ➡ **produced** at **25,000 m** altitude
  - ➡ **traveled vertically** towards the Earth
  - ➡ **decayed** at **5,000 m** after **66.7 μs**

$$c^2\tau^2 = c^2t^2 - x^2 - y^2 - z^2$$

$$\tau = \frac{\sqrt{c^2t^2 - x^2}}{c}$$

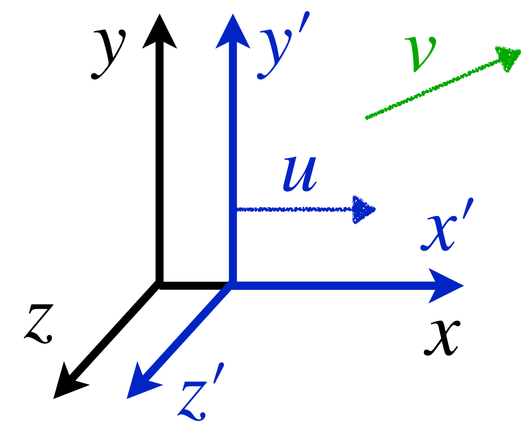
$$= \frac{\sqrt{c^2(66.7 \times 10^{-6})^2 - 20{,}000^2}}{c} = 2.2\ \mu\text{s}$$

Relativistic kinematics: conservation of 4-momentum



The 4-velocity

- **4-velocity** defined as $\eta^\mu \equiv \frac{dx^\mu}{d\tau} = \gamma \frac{dx^\mu}{dt} = \gamma(c, v_x, v_y, v_z)$
  - ➡ Greek indices like μ refer to the 0, 1, 2, 3 components
    - ✦ $x^0 = ct,\ x^1 = x,\ x^2 = y,\ x^3 = z$
- Like **all 4-vectors**
  - ➡ **Lorentz transforms** into **other reference frames**



$$\eta'^\mu = \begin{bmatrix} \gamma(v')c \\ \gamma(v')v'_x \\ \gamma(v')v'_y \\ \gamma(v')v'_z \end{bmatrix} = \begin{bmatrix} \gamma(u) & -\beta(u)\gamma(u) & 0 & 0 \\ -\beta(u)\gamma(u) & \gamma(u) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \gamma(v)c \\ \gamma(v)v_x \\ \gamma(v)v_y \\ \gamma(v)v_z \end{bmatrix}$$

$$\gamma(v) = \frac{1}{\sqrt{1 - v^2/c^2}}$$

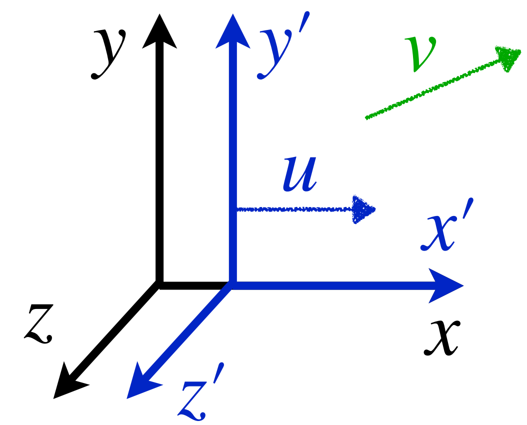
  - ➡ Its **norm** is **frame invariant**

$$\sum \eta_\mu \eta^\mu = (\gamma c)^2 - (\gamma v_x)^2 - \left(\gamma v_y\right)^2 - (\gamma v_z)^2 = \gamma^2\left(c^2 - v^2\right) = c^2$$

4-momentum

- **4-momentum** defined as $p^\mu = m\eta^\mu = \left(m\gamma c, m\gamma\vec{v}\right)$
  - This is for a particle of mass m
  - Also defined for a **set of particles** as the **sum of individual 4-momenta**
- Like **all 4-vectors**
  - **Lorentz transforms** into **other reference frames**

$$p'^\mu = \begin{bmatrix} m\gamma(v')c \\ m\gamma(v')v'_x \\ m\gamma(v')v'_y \\ m\gamma(v')v'_z \end{bmatrix} = \begin{bmatrix} E'/c \\ p'_x \\ p'_y \\ p'_z \end{bmatrix} = \begin{bmatrix} \gamma(u) & -\beta(u)\gamma(u) & 0 & 0 \\ -\beta(u)\gamma(u) & \gamma(u) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} E/c \\ p_x \\ p_y \\ p_z \end{bmatrix}$$



  - Its **norm** is **frame invariant**

$$\sum p_\mu p^\mu = E^2/c^2 - p_x^2 - p_y^2 - p_z^2 = m^2c^2 \;\rightarrow E = \sqrt{m^2c^4 - p^2c^2}$$

$$p^2 = p_x^2 + p_y^2 + p_z^2$$

4-momentum for $v \ll c$

- **Spatial part** looks like **standard momentum**

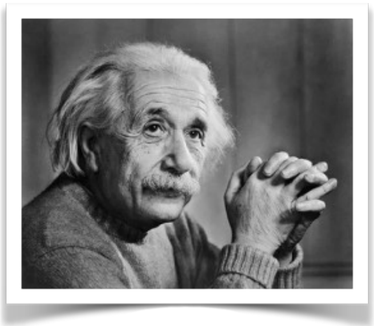
$$p^\mu = \begin{bmatrix} m\gamma c \\ m\gamma v_x \\ m\gamma v_y \\ m\gamma v_z \end{bmatrix} = \begin{bmatrix} E/c \\ p_x \\ p_y \\ p_z \end{bmatrix}$$

$$m\gamma\vec{v} = \frac{m\vec{v}}{\sqrt{1-\frac{v^2}{c^2}}} = m\vec{v} + m\vec{v}\frac{v^2}{2c^2} + \mathcal{O}\left(\frac{v^4}{c^4}\right)$$

- **Time part** is **mc^2 constant** plus **kinetic energy**

$$\frac{E}{c} = m\gamma c = \frac{mc}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{c}\left[mc^2 + m\frac{v^2}{2} + \mathcal{O}\left(\frac{v^4}{c^2}\right)\right]$$

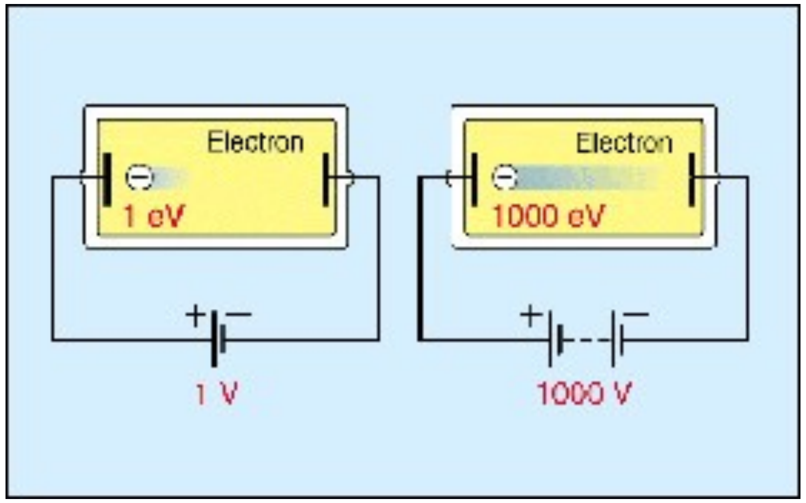
Einsteins brilliant insight
$E = mc^2$ (at rest)



When not at rest,
$$E = \sqrt{m^2c^4 - p^2c^2}$$

Units in particle physics

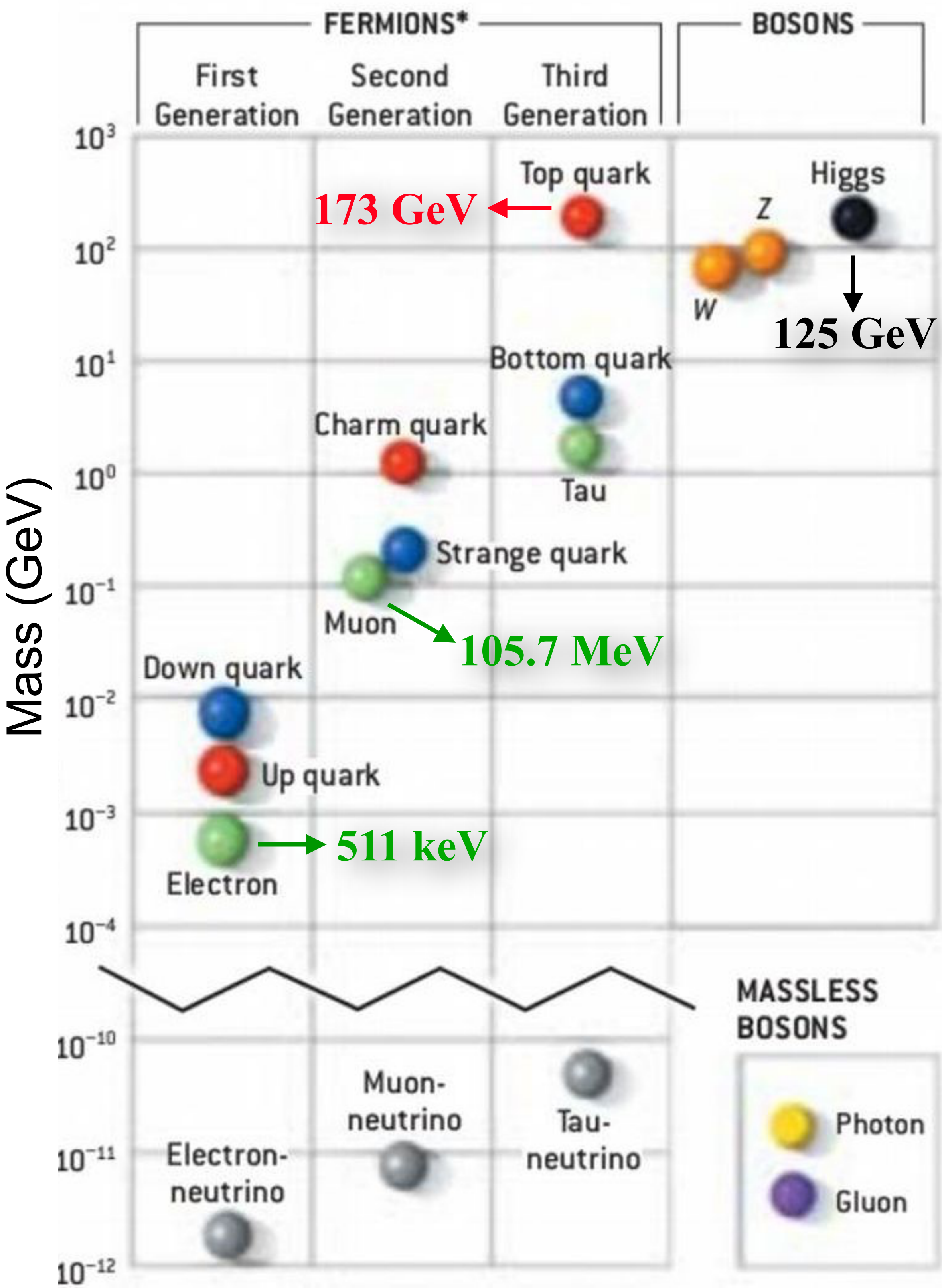
- Typically use ***electronvolts*** **(eV)** to **measure energy**
  - ➡ Energy of an electron accelerated by a 1 V potential
  - ➡ Very useful in the original experiments
  - ➡ 1 eV = 1.602×10^{-19} J



- Use **eV/c** for **momentum**
- Use **eV/c²** for **mass**
- What is the energy of an **electron** ($\mathbf{m_e}$ **= 511 keV/c²**) that has a **momentum of 1 MeV/c**?

$$E_e = \sqrt{p_e^2 c^2 + m_e^2 c^4} = \sqrt{(1\ \mathrm{MeV}/c)^2 c^2 + \left(0.511\ \mathrm{MeV}/\mathrm{c}^2\right)^2 c^4} = 1.12\ \mathrm{MeV}$$

- Very often, we just set **c = 1**, so ***energy, mass,*** *and* ***momentum*** *are* ***measured in eV***



- Elementary particles masses cover a large range
  - Photon/gluon are massless
  - Neutrinos ~ meV
  - 1st generation fermions ~1 MeV
  - 2nd generation fermions ~ 0.1 - 1 GeV
  - 3rd generation fermions ~ 1 - 173 GeV
  - W, Z bosons ~ 90 GeV
  - Higgs boson ~ 125 GeV
- **Masses** of **non-elementary particles** heavily determined by **binding energy**
  - eg, m(u) = 2.3 MeV, m(d) = 4.8 MeV
    - **Mass of proton (uud) is 938 MeV**

4-momentum conservation

- In SR, it is **4-momentum** that is **conserved**
  - ➡ Generalization of energy and 3-momentum conservation
- For a **closed system** of $\boldsymbol{n}$ **particles**

$$\sum_{i}^{n} p_\mu^{i,ini} = \sum_{i}^{n} p_\mu^{i,fin} \quad \text{for each coordinate } \mu$$

$$\begin{bmatrix} \frac{E^{1,ini}}{c} \\ p_x^{1,ini} \\ p_y^{1,ini} \\ p_z^{1,ini} \end{bmatrix} + \begin{bmatrix} \frac{E^{2,ini}}{c} \\ p_x^{2,ini} \\ p_y^{2,ini} \\ p_z^{2,ini} \end{bmatrix} + \cdots + \begin{bmatrix} \frac{E^{n,ini}}{c} \\ p_x^{n,ini} \\ p_y^{n,ini} \\ p_z^{n,ini} \end{bmatrix} = \begin{bmatrix} \frac{E^{1,fin}}{c} \\ p_x^{1,fin} \\ p_y^{1,fin} \\ p_z^{1,fin} \end{bmatrix} + \begin{bmatrix} \frac{E^{2,fin}}{c} \\ p_x^{2,fin} \\ p_y^{2,fin} \\ p_z^{2,fin} \end{bmatrix} + \cdots + \begin{bmatrix} \frac{E^{n,fin}}{c} \\ p_x^{n,fin} \\ p_y^{n,fin} \\ p_z^{n,fin} \end{bmatrix}$$

- A particle α decays into two other particles λ and ω: **α → λ ω**

General conservation of 4-momentum

$$\begin{bmatrix} \sqrt{p_\alpha^2 + m_\alpha^2 c^2} \\ p_{\alpha,x} \\ p_{\alpha,y} \\ p_{\alpha,z} \end{bmatrix} = \begin{bmatrix} \sqrt{p_\lambda^2 + m_\lambda^2 c^2} \\ p_{\lambda,x} \\ p_{\lambda,y} \\ p_{\lambda,z} \end{bmatrix} + \begin{bmatrix} \sqrt{p_\omega^2 + m_\omega^2 c^2} \\ p_{\omega,x} \\ p_{\omega,y} \\ p_{\omega,z} \end{bmatrix}$$

Conservation of 4-momentum in two-body decay

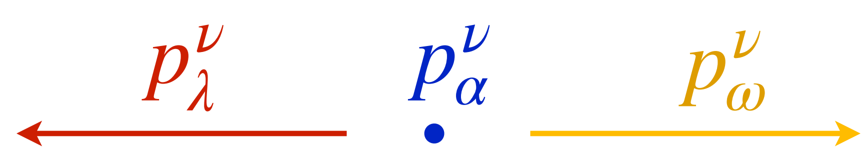
$$\begin{bmatrix} m_\alpha c \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \sqrt{p_\lambda^2 + m_\lambda^2 c^2} \\ p_\lambda \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \sqrt{p_\omega^2 + m_\omega^2 c^2} \\ p_\omega \\ 0 \\ 0 \end{bmatrix}$$

$$p_\alpha = \sqrt{p_{\alpha,x}^2 + p_{\alpha,y}^2 + p_{\alpha,x}^z}$$

$$p_\lambda = \sqrt{p_{\lambda,x}^2 + p_{\lambda,y}^2 + p_{\lambda,x}^z}$$

$$p_\omega = \sqrt{p_{\omega,x}^2 + p_{\omega,y}^2 + p_{\omega,x}^z}$$

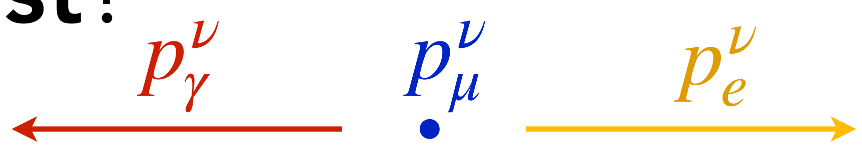
	Degrees of freedom				Equations
	α	λ	ω	Total	(constraints)
Original	4	4	4	**12**	**4**
Masses	3	3	3	**9**	**4**
α at rest	0	3	3	**6**	**4**
Symmetry	0	1	1	**2**	**2**



Cannot know λ and ω direction (isotropic decay), but we know that they will **both travel along the same line** in **opposite directions** by **conservation of momentum**
→ Choose x axis along the direction of the decay

(Forbidden) decay of a muon

- What is the **energy** of the (**massless**) photon $\boldsymbol{\gamma}$ emitted in the decay $\boldsymbol{\mu^-} \rightarrow \mathbf{e^-} + \boldsymbol{\gamma}$ assuming the **muon is initially at rest**?



Conservation of 4-momentum

$$\begin{bmatrix} \sqrt{p_\mu^2 + m_\mu^2 c^2} \\ p_\mu \end{bmatrix} = \begin{bmatrix} \sqrt{p_e^2 + m_e^2 c^2} \\ p_e \end{bmatrix} + \begin{bmatrix} \sqrt{p_\gamma^2 + m_\gamma^2 c^2} \\ p_\gamma \end{bmatrix}$$

Cannot know γ and e⁻ direction (isotropic decay), but we know that they will **both travel along the same line** in **opposite directions** by **conservation of momentum**

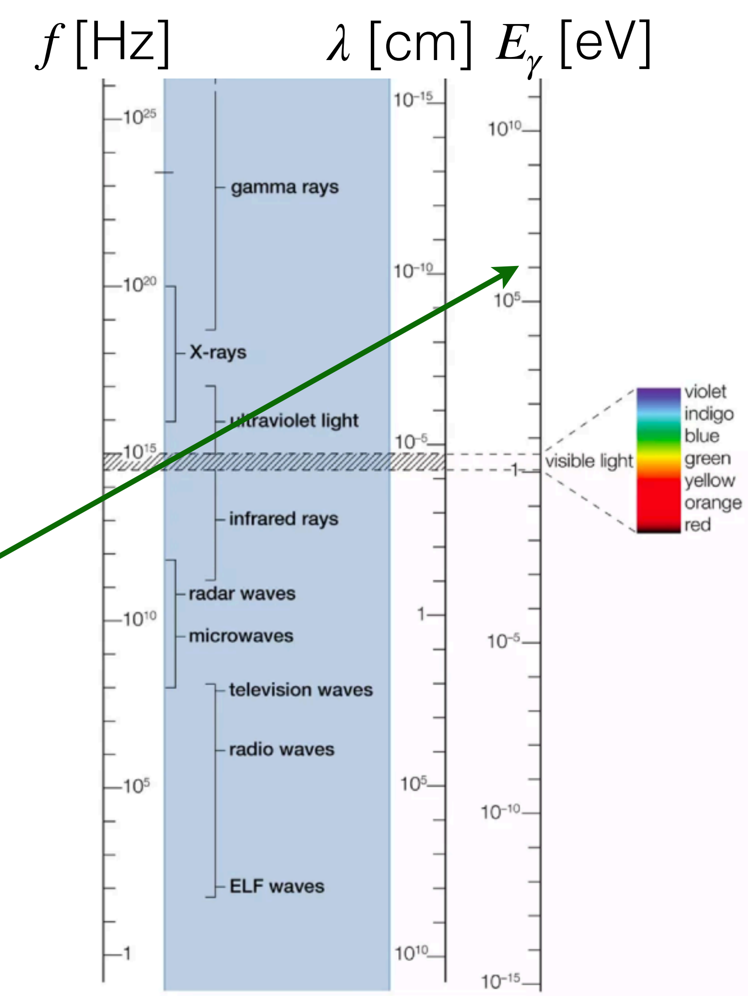
Since the muon is at rest, $\mathbf{p_\mu = 0}$, the photon is massless, $\mathbf{m_\gamma = 0}$

$$\begin{bmatrix} m_\mu c \\ 0 \end{bmatrix} = \begin{bmatrix} \sqrt{p_e^2 + m_e^2 c^2} \\ p_e \end{bmatrix} + \begin{bmatrix} |p_\gamma| \\ p_\gamma \end{bmatrix} \Rightarrow \begin{array}{c} \sqrt{p_e^2 + m_e^2 c^2} = m_\mu c - |p_\gamma| \\ 0 = p_e + p_\gamma \end{array}$$

Solve for positive $\mathbf{p_\gamma}$

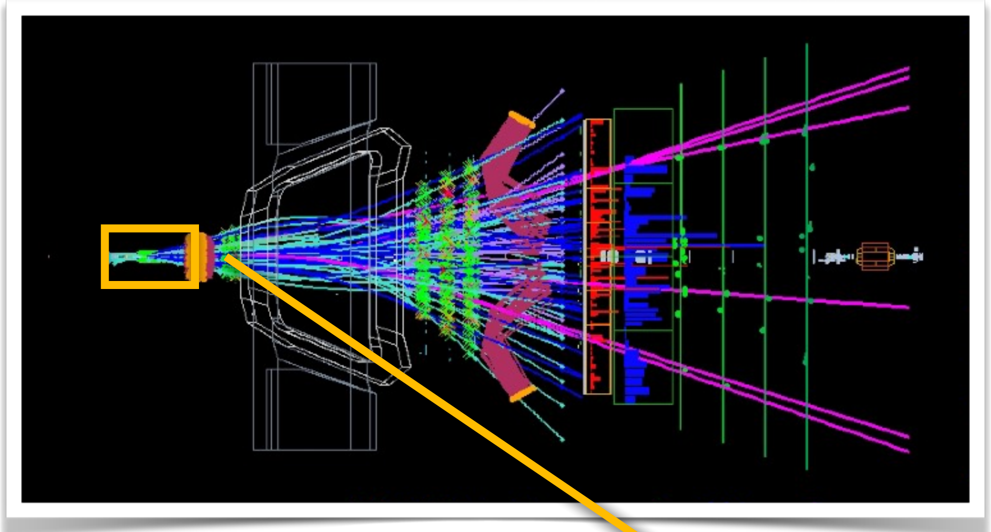
$$p_\gamma^2 + m_e^2 c^2 = m_\mu^2 c^2 - 2 m_\mu c p_\gamma + p_\gamma^2 \Rightarrow p_\gamma = \frac{m_\mu^2 c - m_e^2 c}{2 m_\mu}$$

$$E_\gamma = p_\gamma c = \frac{m_\mu^2 c^2 - m_e^2 c^2}{2 m_\mu} = 52.83 \text{ MeV}$$



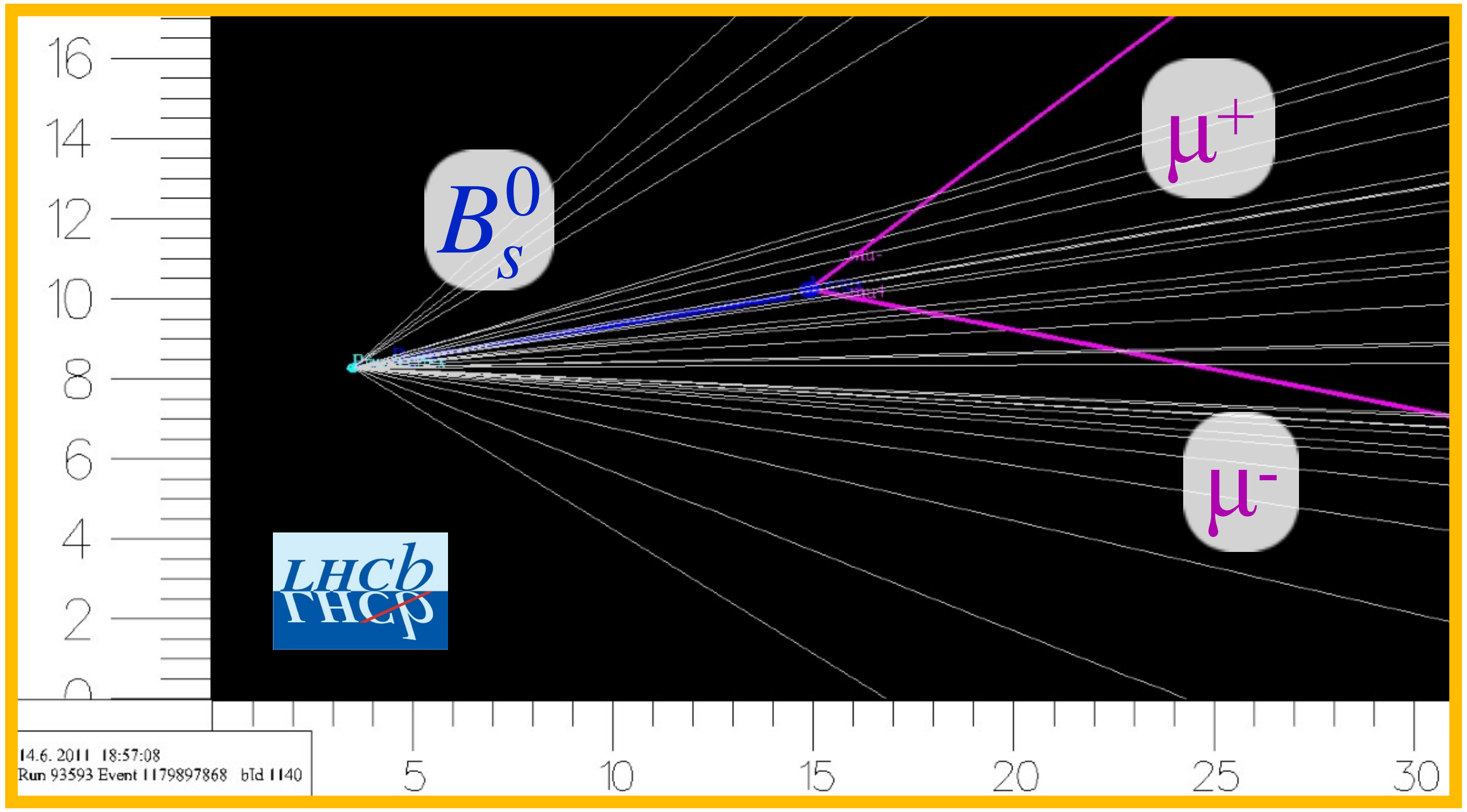
Reconstructing a particle

- How do you discover a new particle?
  - ➡ Look for particles decaying from a common vertex



The B_s^0 is invisible, but we see the muons and reconstruct its momentum

$$B_s^0 \rightarrow \mu^+\mu^-$$



Reconstruct invariant mass

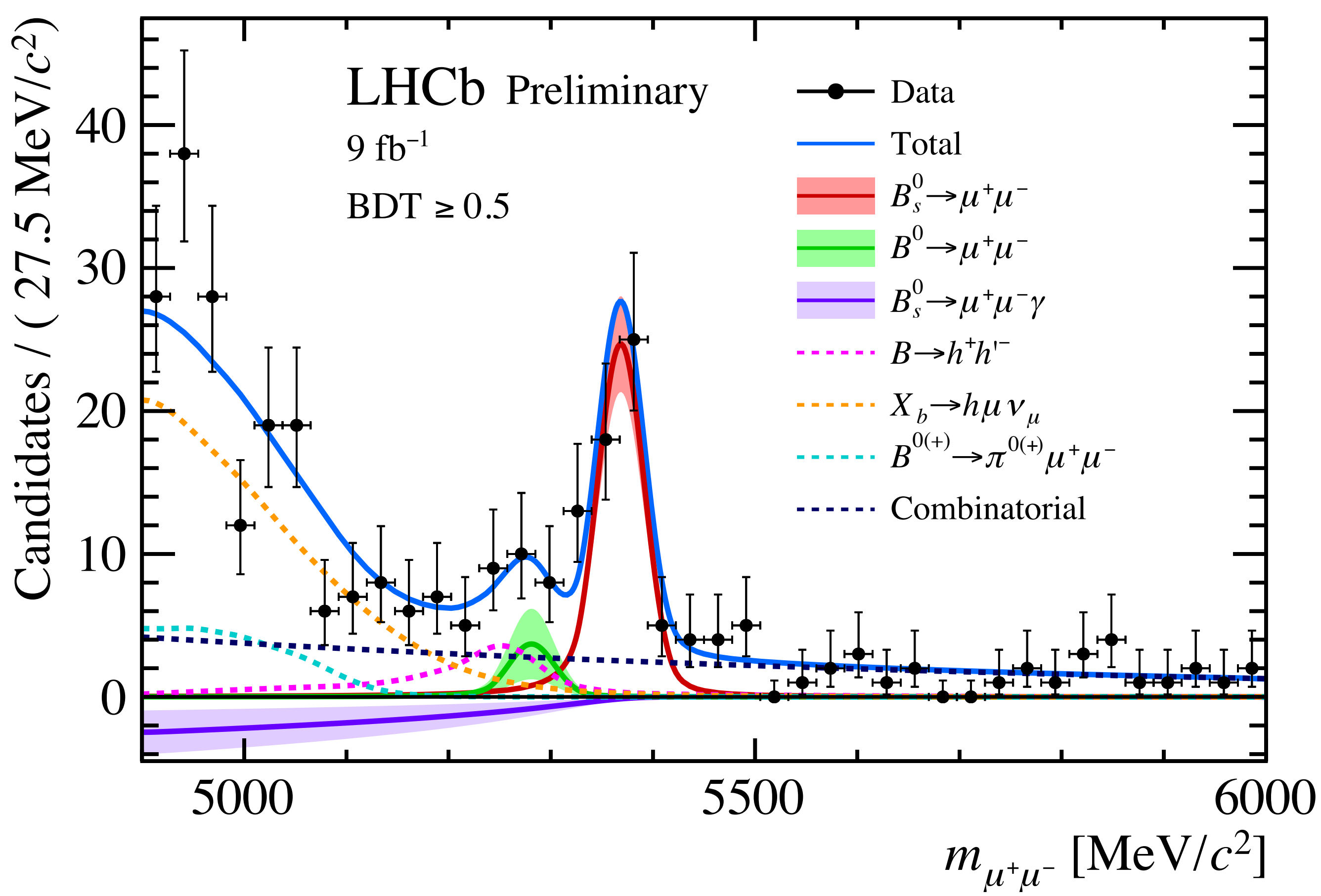
- How do you discover a new particle?
  - Look for particles decaying from a common vertex
  - Add up the momenta of the daughter particles

$$\begin{bmatrix} \sqrt{p_1^2 + m_\mu^2 c^2} \\ p_1 \end{bmatrix} + \begin{bmatrix} \sqrt{p_2^2 + m_\mu^2 c^2} \\ p_2 \end{bmatrix} = \begin{bmatrix} E_B \\ p_B \end{bmatrix}$$

  - Calculate invariant mass

$$m_B = \sqrt{E_B^2 - p_B^2}$$

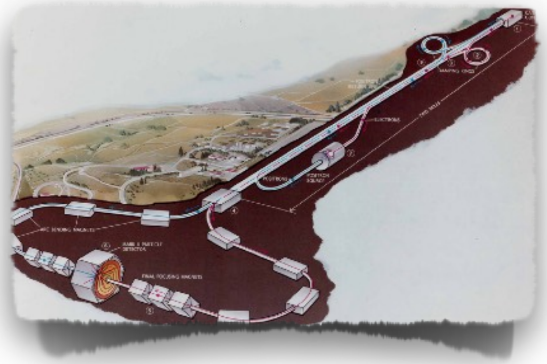
  - Plot histogram and find peak!



Particle accelerator types

Geometry

Linear collider
Stanford Linear Collider (SLC)



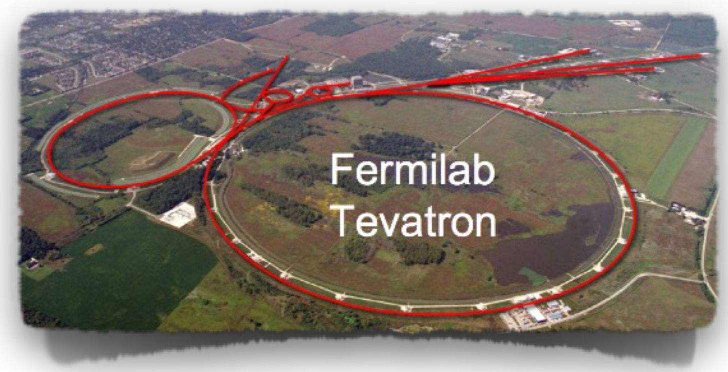
vs

Circular collider
Large Electron-Positron collider (LEP)

Type of collision

Fixed target
NA62

vs



Collider
Tevatron

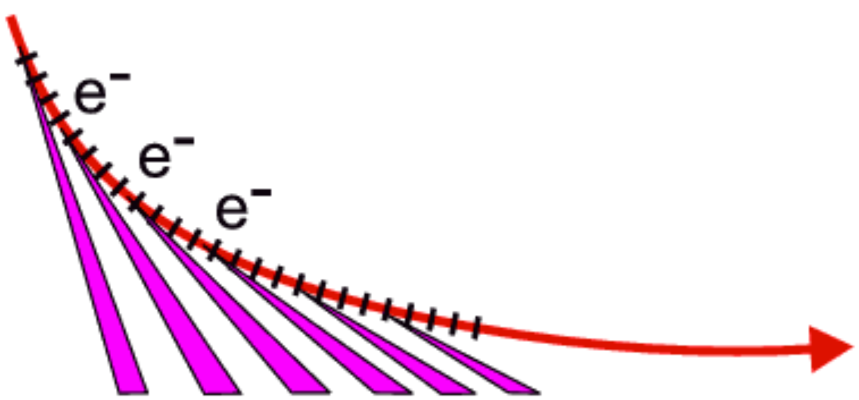
Type of particle

Lepton collider
Positron-Electron Project II (PEP-II)

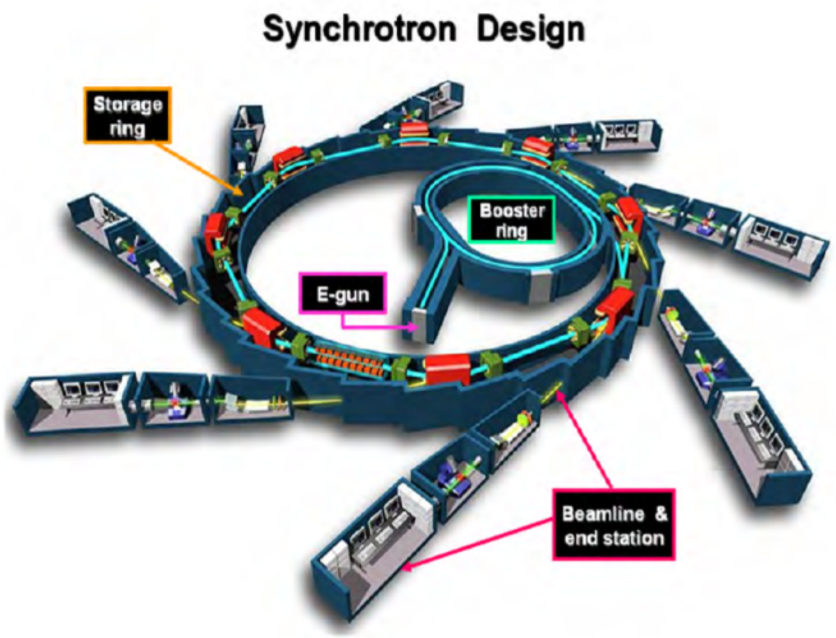
vs

Hadron collider
Large Hadron Collider (LHC)

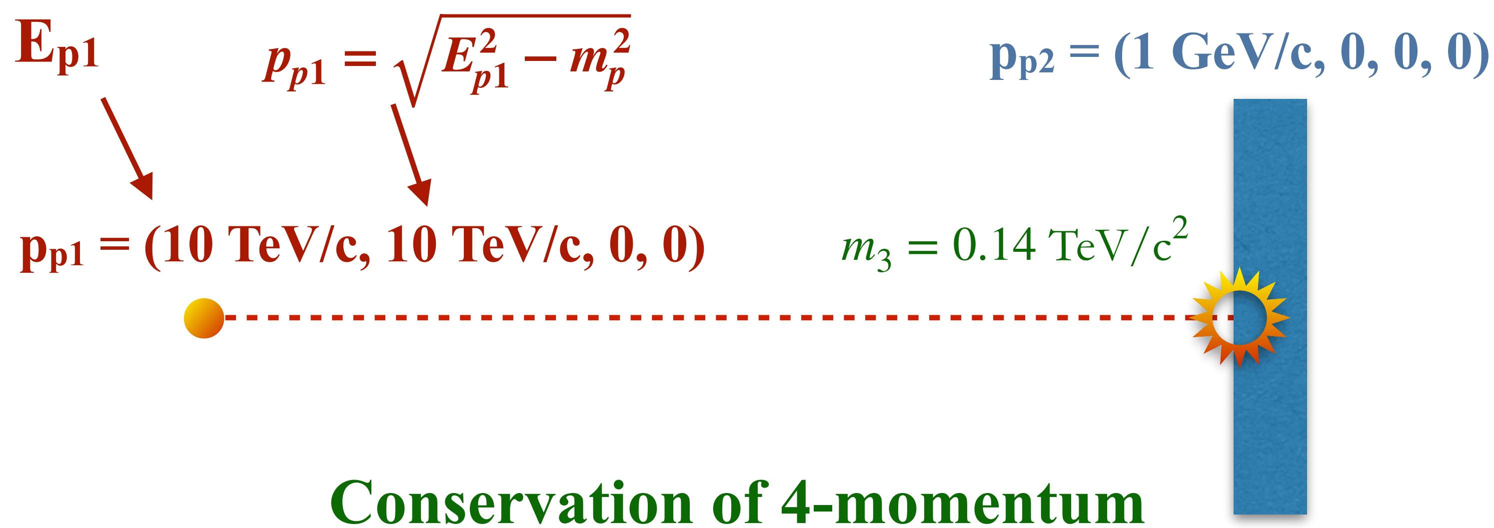
- **Circular** geometry has **key advantages** as particles circle around
  - Can have **higher energies** as particles are accelerated at each cycle
  - Can have **higher collision rates** as particle bunches have more opportunities to collide
- However, charged particles emit **synchrotron radiation when changing direction**
  - **Power loss** grows very **quickly with energy**
  - Circular geometry **not feasible for light particles** (eg, electrons) for $\sqrt{s} \geq \sim 250$ GeV
    - Use protons ($m_p \sim 2000 m_e$) for higher energies



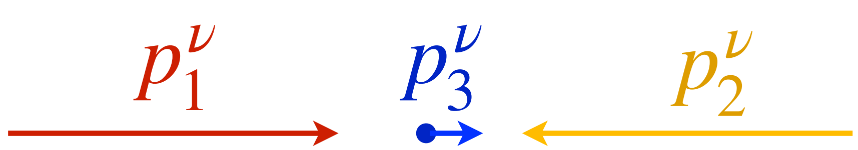
$$P_{\text{sync}}^{\text{loss}} \propto \frac{E^4}{m^4 R^2}$$



- In **Fixed target experiments** a **beam of high-energy particles** collides against a **large** and **stationary target**



Conservation of 4-momentum



$$\begin{bmatrix} \sqrt{p_1^2 + m_1^2 c^2} \\ p_1 \end{bmatrix} + \begin{bmatrix} \sqrt{p_2^2 + m_2^2 c^2} \\ p_2 \end{bmatrix} = \begin{bmatrix} \sqrt{p_3^2 + m_3^2 c^2} \\ p_3 \end{bmatrix}$$

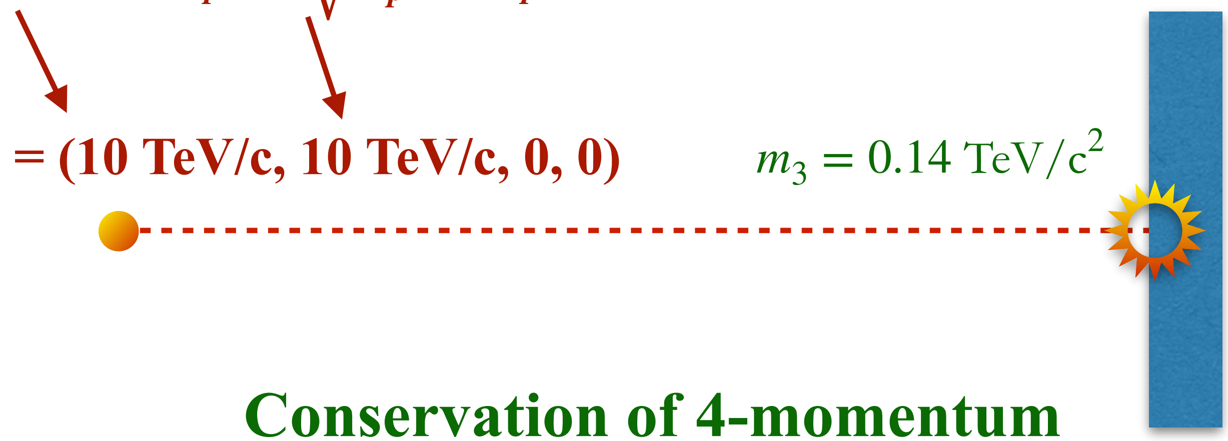
$$\begin{bmatrix} p_1 \\ p_1 \end{bmatrix} + \begin{bmatrix} m_p \\ 0 \end{bmatrix} = \begin{bmatrix} \sqrt{p_3^2 + m_3^2 c^2} \\ p_3 \end{bmatrix}$$

$$p_3 = p_1$$

$$m_3^2 c^2 = \left(p_1 + m_p c\right)^2 - p_1^2 = 2 p_1 m_p c \quad \rightarrow \quad m_3 = 0.14 \text{ TeV/c}^2$$

Fixed target vs Colliders

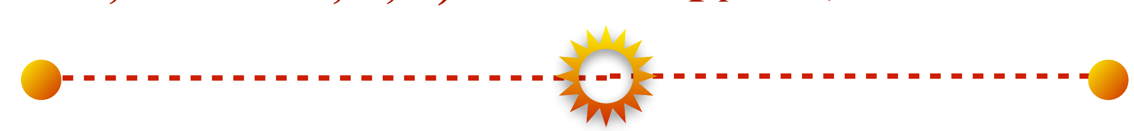
- In **Fixed target experiments** a **beam of high-energy particles** collides against a **large** and **stationary target**
  - ➡ Easy, cheap
  - ➡ Inefficient → difficult to reach high energies



Conservation of 4-momentum

$$\begin{bmatrix} \sqrt{p_1^2 + m_1^2 c^2} \\ p_1 \end{bmatrix} + \begin{bmatrix} \sqrt{p_2^2 + m_2^2 c^2} \\ p_2 \end{bmatrix} = \begin{bmatrix} \sqrt{p_3^2 + m_3^2 c^2} \\ p_3 \end{bmatrix}$$

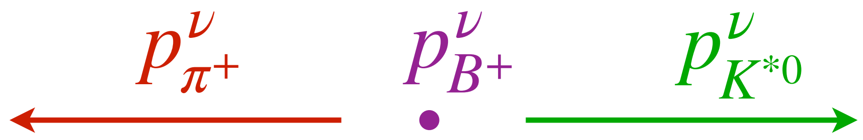
- **Colliders** crash **two narrow beams of particles**
  - ➡ Difficult but efficient



$$m_3 = 20\ \mathrm{TeV/c^2}$$

- **Conservation of 4-momentum** allows us to **solve particle physics problems**

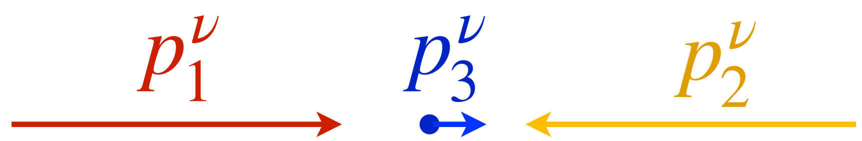
Conservation of 4-momentum in a particle decay



$$\begin{bmatrix} cm_{B^+} \\ 0 \end{bmatrix} = \begin{bmatrix} \sqrt{p^2_{K^{*0}} + c^2 m^2_{K^{*0}}} \\ p_{K^{*0}} \end{bmatrix} + \begin{bmatrix} \sqrt{p^2_{\pi^+} + c^2 m^2_{\pi^+}} \\ p_{\pi^+} \end{bmatrix}$$

2-body decays are always collinear

Conservation of 4-momentum in a head on particle collision

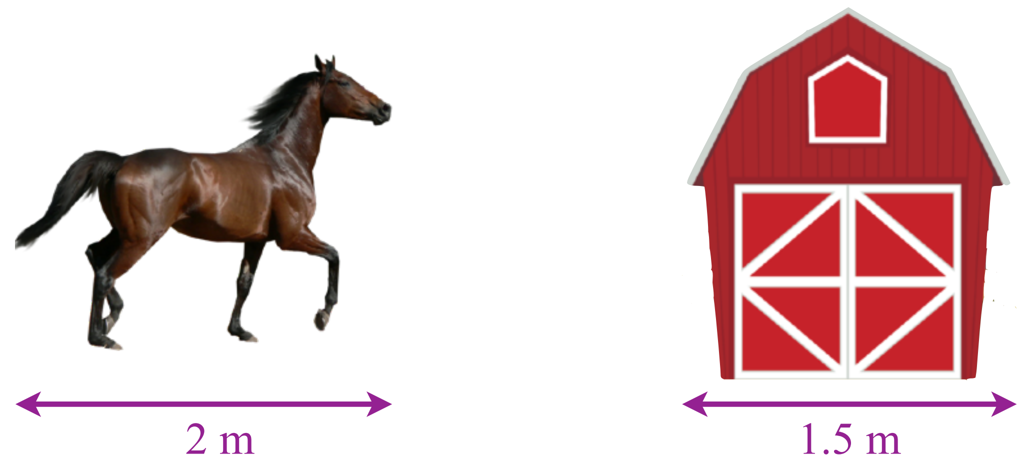


$$\begin{bmatrix} \sqrt{p_1^2 + m_1^2 c^2} \\ p_1 \end{bmatrix} + \begin{bmatrix} \sqrt{p_2^2 + m_2^2 c^2} \\ p_2 \end{bmatrix} = \begin{bmatrix} \sqrt{p_3^2 + m_3^2 c^2} \\ p_3 \end{bmatrix}$$

Backup

Horse-barn paradox

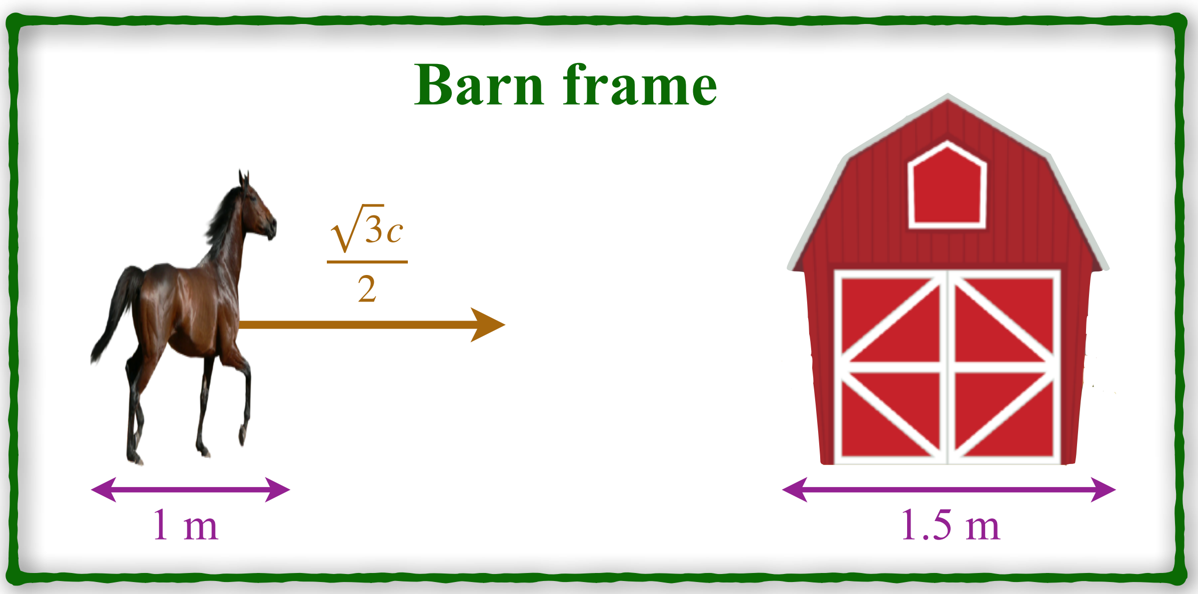
Could we **fit** a **2 m horse** into a **1.5 m barn** using relativity?

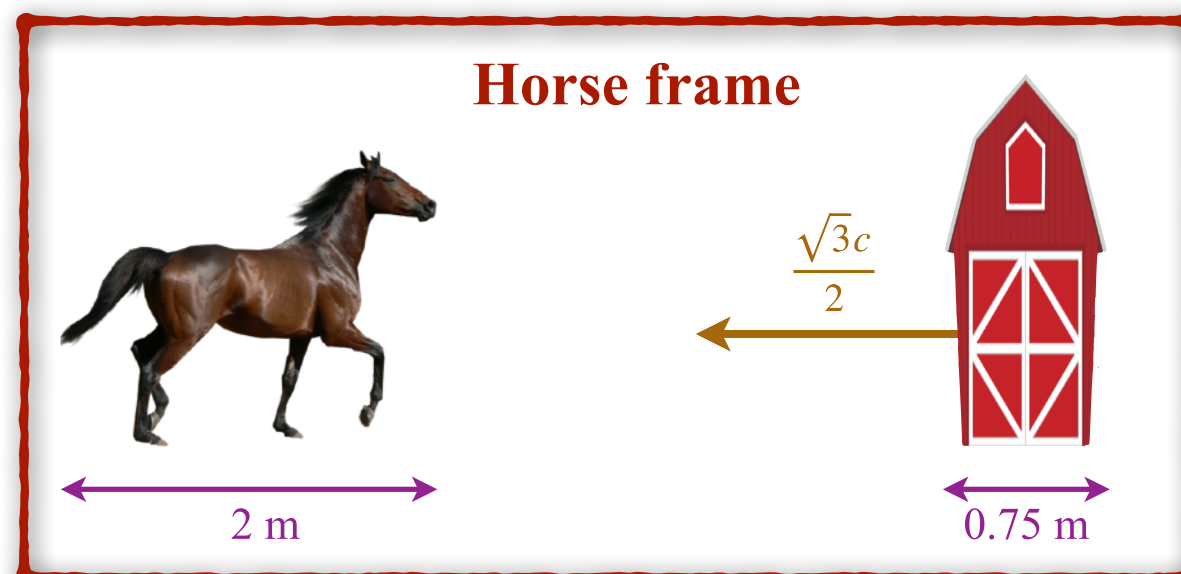


Yes! Accelerate the horse to

$$v_{\text{horse}} = \frac{\sqrt{3}c}{2} \rightarrow \gamma = \frac{1}{\sqrt{1 - \frac{v^2_{\text{horse}}}{c^2}}} = 2$$

How can the 2 m horse **both fit in the 1.5 m barn** in the rest frame and **not fit in the barn** in its own frame?

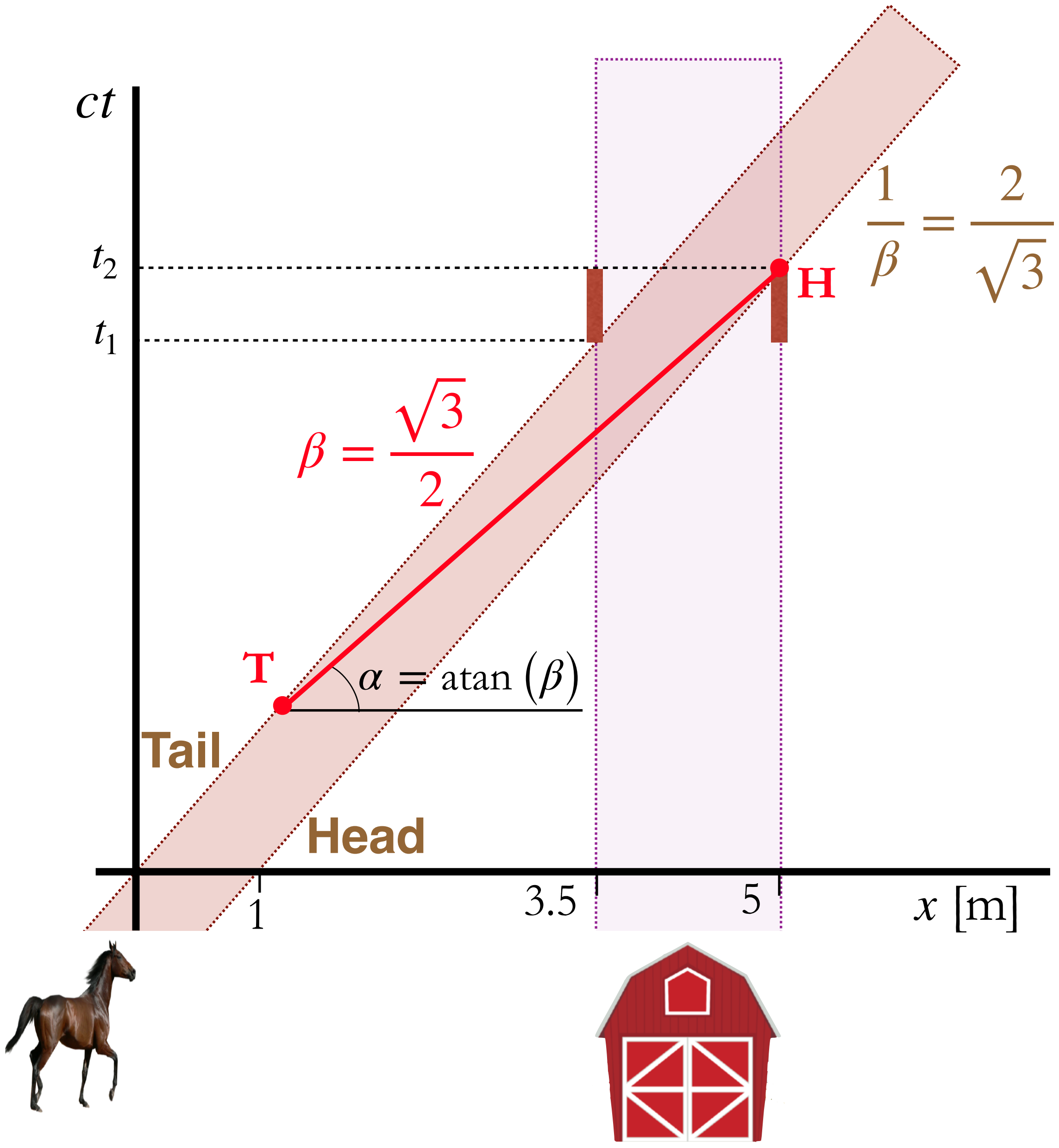




Lack of simultaneity is what leads to many of SR's paradoxes

Horse-barn paradox resolution

- It is simply a problem of simultaneity
  - The tail and head of the horse being inside the barn at the same time depends on the speed that you are going



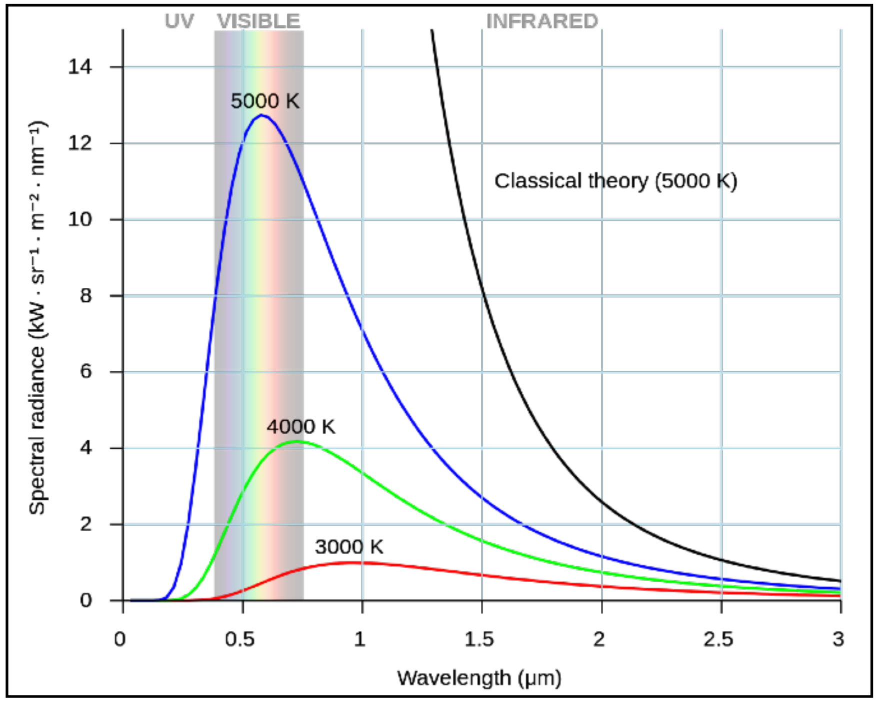
In the barn rest frame, the horse is (2m)/γ = 1m long, and the slopes of the head and tail of the horse are $\frac{1}{\beta} \approx 1.15$ Since the barn is at rest in this frame, its doors have infinite slope.

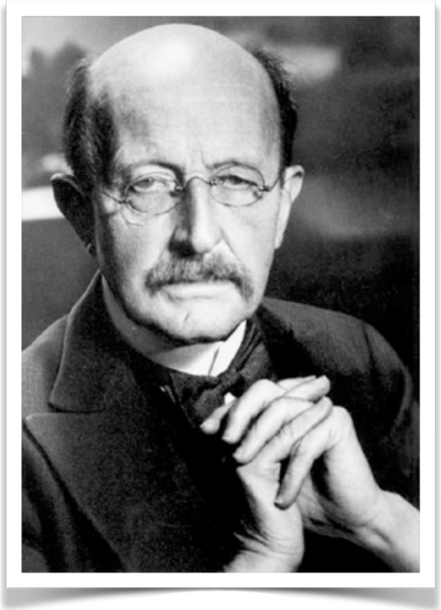
Between t_1 and t_2 the **horse** is **fully enclosed inside the barn** because both its **tail** and **head are inside**.

Black body radiation
Ultraviolet catastrophe

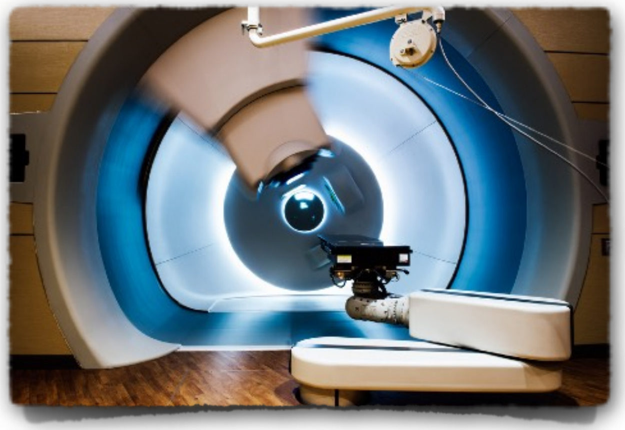
→ **Energy quantization** →

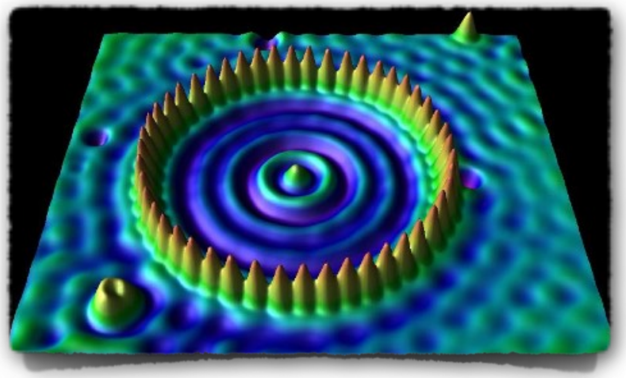
Deeper understanding of the Universe's fundamental constituents





$$E = h\nu$$



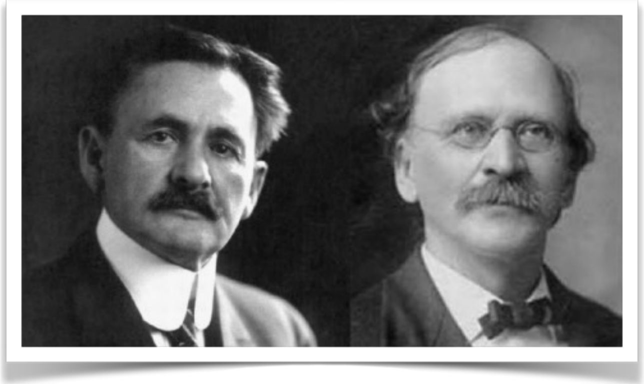


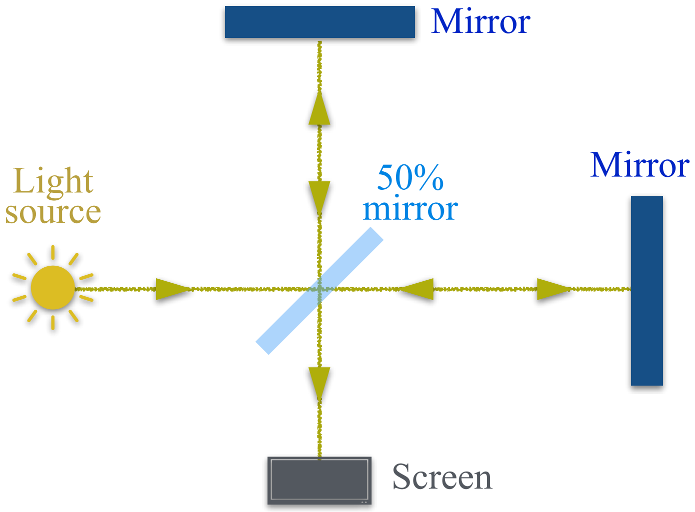
+ many other experiments (photoelectric effect…)

+ work from many others (Einstein, Schrödinger, Heisenberg…)

Relativity in a nutshell

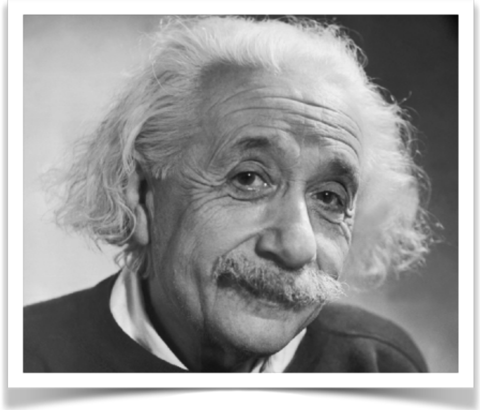
Michelson-Morley experiment
Could not find ether, the medium light travels with respect to





+ experiments from many others (ether dragging…)

→ **Einstein's postulates** →



The laws of Physics have the same form in all inertial frames of reference

The speed of light in vacuum has the same value c in every direction in all inertial frames of reference

+ work from many others (Poincaré, Lorentz…)

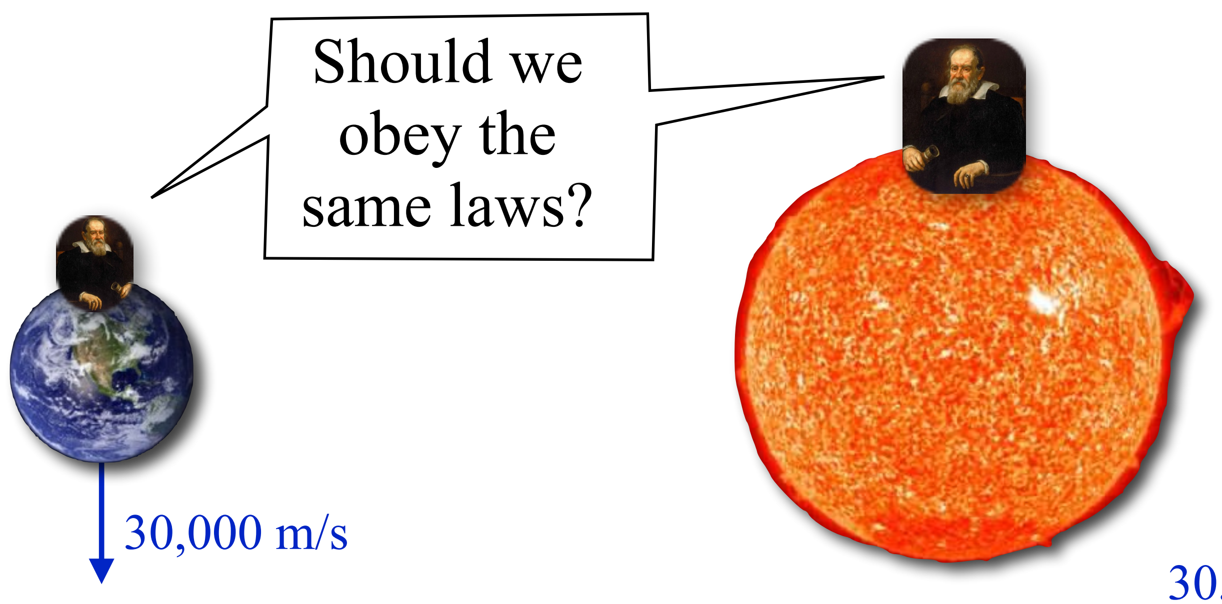
Deeper understanding of the Universe's spacetime, mass, and energy

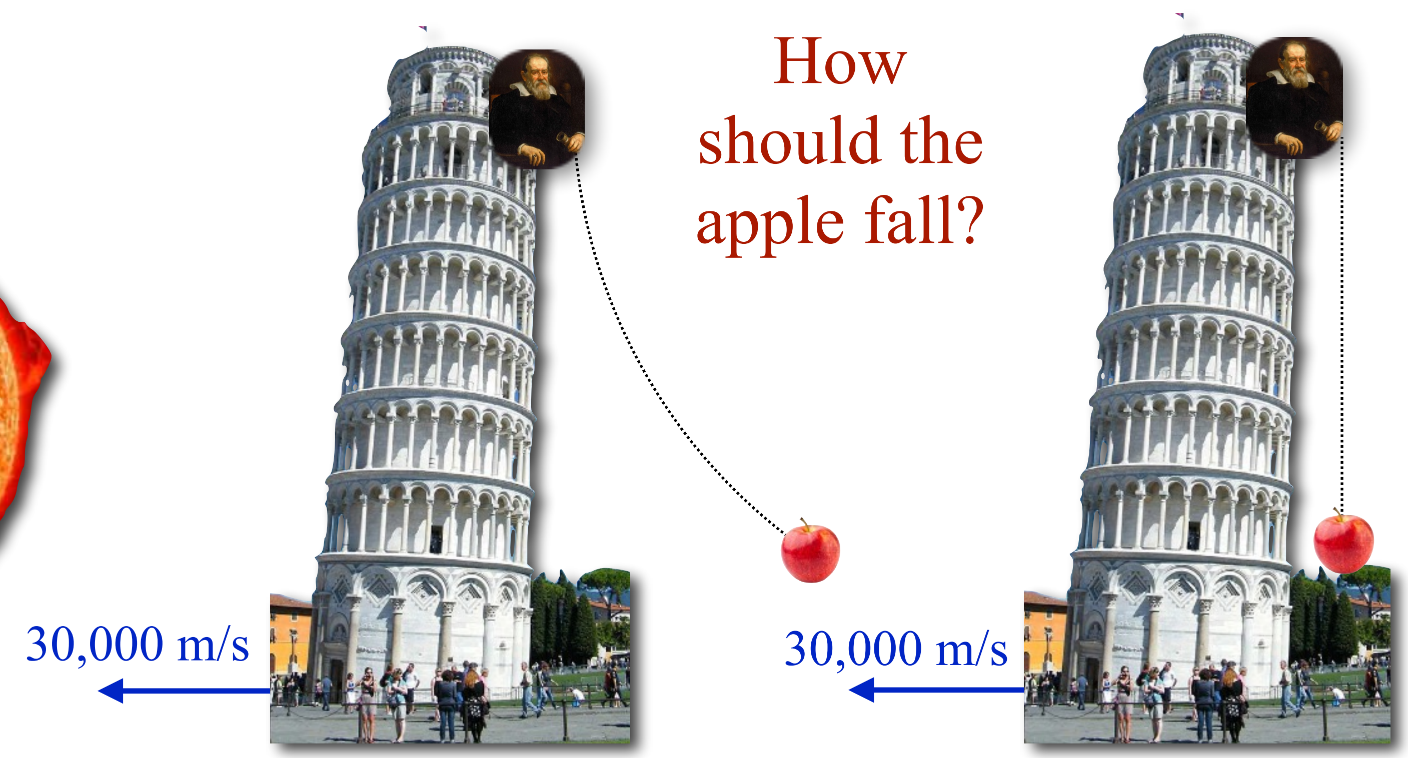
$$E = mc^2$$

$$\Delta t' = \frac{\Delta t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Galilean relativity

- Not obvious whether Earth's laws of physics should apply elsewhere





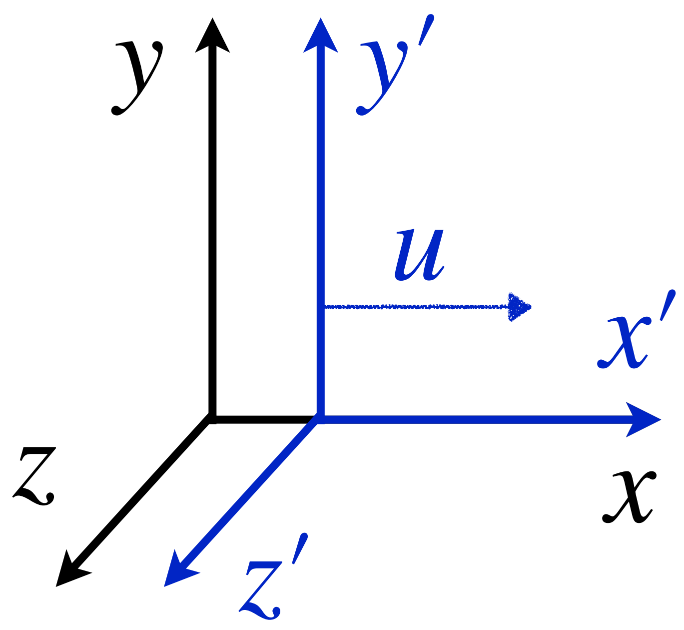
Postulate from Galileo Galilei (1564 - 1642)

The laws of Motion have the same form in all inertial frames of reference

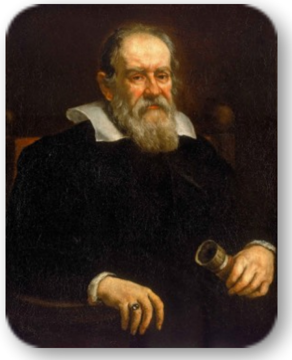
Inertial frames of reference are those that move at **constant relative velocities**

Galilean transformations

- The Galilean transformations rely on two axioms
  - ➡ There exists an absolute space
  - ➡ All inertial frames share a universal time
- Thus, the **coordinate transformation between two inertial frames** where the **origins are the same at $t = 0$** and the **velocity u is aligned with the x axis**



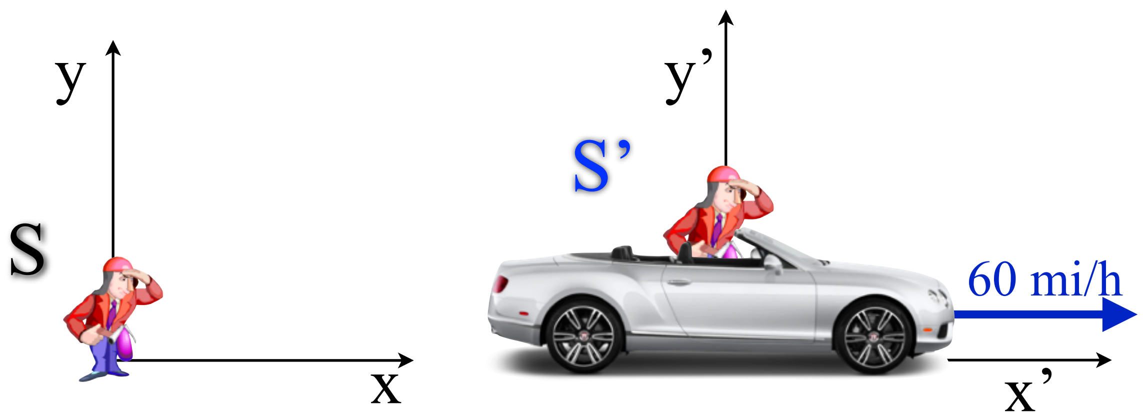
$$t' = t \qquad x' = x - ut$$
$$y' = y \qquad z' = z$$



Galilean example

- For two frames of references S and S′ with the same origin at t = 0 and aligned

$t' = t$	$x' = x - (60\,\text{mi/h})\,t$
$y' = y$	$z' = z$



$$x_{\text{train}} = 10 + (200\,\text{mi/h})t - (10\,\text{mi/h}^2)t^2$$

$$L_{\text{train}} = 20\,\text{m}$$

$$v_{\text{train}} =$$

$$a_{\text{train}} =$$

$$x'_{\text{train}} =$$

$$L'_{\text{train}} =$$

$$v'_{\text{train}} =$$

$$a'_{\text{train}} =$$