α Radioactivity \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Name

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The natural radioactive decay sequence, given below, that starts with the abundant isotope U-238 (99.3% of all uranium found on Earth) was worked out in the early years of the 20th century by Marie and Pierre Curie, Ernest Rutherford, and several others.

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**⭨ α ⭨ β ⭨ β ⭨ α ⭨ α ⭨ α ⭨ α ⭨ α ⭨ β**

** stable**

**⭨ β ⭨ α ⭨ β ⭨ β ⭨ α**

Rutherford demonstrated that an α-particle is an He-4 nucleus. After Chadwick’s discovery of the neutron in 1932, He-4 was recognized as a combination of two protons and two neutrons. A β-particle is an electron emitted from an atomic nucleus. In radioactive decays α-particles are emitted with discrete kinetic energy values while   
β-particles emerge within a range of possible kinetic energies up to some maximum   
(end-point) value. Half-lives of nuclei in the U-238 decay chain range from billions of years (Ga for gigaannum) to microseconds (μs). The decay chain ends with the stable isotope of lead Pb-206.

The table below gives the isotope nuclear mass values, the decay process that occurs, the energy for most prevalent emitted α-particles or the end-point (highest observed) energy for emitted β-particles, and the half-life of the isotope. The nuclear isotope masses in the table below are adapted from Los Alamos National Laboratory website <http://t2.lanl.gov/nis/data/astro/molnix96/massd.html> .The decay energies and half-life values in the table are from the Brookhaven National Laboratory web site <http://www.nndc.bnl.gov/nudat2/reCenter.jsp?z=90&n=144> . The mass of an α-particle (He-4 nucleus), a β-particle (electron), and the unit conversions listed below are adapted from the National Institute of Standards and Technology website, <http://www.physics.nist.gov> .

*m*α = *m*He4 = 4.0015062 u , *m*β = *m*e = 5.4857991 x 10-4 u

*m*α = *m*He4 = 3727.3791 MeV/*c*2 , *m*β = *m*e = 0.51099891 MeV/*c*2

1 u = 1 atomic mass unit = 1.660538782 x 10-27 kg = 931.494028 MeV/*c*2 .

1 MeV = 1 megaelectronvolt = 106 eV = 1.6022 x 10-13 J.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Nuclear Mass  (u) | Nuclear  Isotope | Decay | α (discrete) or β (end point) Kinetic Energy  (MeV) | Half-life Time *t*1/2 | log10 *t*1/2  (*t*1/2 in s) |
| 238.001444 |  |  | 4.198 (79%) | 4.468 Ga |  |
| 233.995362 |  |  | 0.199 (78%) | 24.10 d |  |
| 233.994501 |  |  | 2.27 (98%) | 1.16 m |  |
| 233.991576 |  |  | 4.775 (71%) | 245.5 ka |  |
| 229.984860 |  |  | 4.687 (76%) | 75.4 ka |  |
| 225.978240 |  |  | 4.784 (94%) | 1.60 ka |  |
| 221.971510 |  |  | 5.489 (100%) | 3.824 d |  |
| 217.964006 |  |  | 6.002 (100%) | 3.098 m |  |
| 213.955939 |  |  | 1.02 (11%) | 26.8 m |  |
| 213.954279 |  |  | 3.27 (19%) | 19.9 m |  |
| 213.950203 |  |  | 7.687 (100%) | 164 μs |  |
| 209.940290 |  |  | 0.064 (16%) | 22.2 a |  |
| 209.939655 |  |  | 1.162 (100%) | 5.01 d |  |
| 209.937843 |  |  | 5.304 (100%) | 138.4 d |  |
| 205.930534 |  | STABLE |  | STABLE |  |

Note: 1 a = 1 year = 3.156 x 107 s

α-Decay Part 1

**Assume** a U-238 nucleus is at rest before it decays.   
**Calculate** **mass decrease** (Δ*m*) in atomic mass units (u) for U-238 decay to Th-234 + α.

**Calculate** **energy released** in that decay, Δ*E* = (Δ*m*)*c*2, in MeV units and in joules (J).

[In contrast, the energy per atom released in a chemical reaction is at most a few eV.]

Since the kinetic energies of the α-particles in the decay chain are much smaller than the energy associated with the mass of the α-particle, classical (Newtonian) expressions for particle kinetic energy and momentum can be applied with little error.

**Assume** that all the energy released in the decay goes to the kinetic energy of the   
Th-234 nucleus and the α-particle. Use Newtonian conservation laws and the Einstein expression *E* = *mc*2 to **show** that   
 Δ*E*/*c*2 = (½)*m*α(*v*α/*c*)2(1+*m*α/*m*Th).

**Calculate** *v*α/*c* and *v*Th/*c* . Are the values much less than 1, as required for our Newtonian assumption to be valid?

**Calculate** the classical values of KEα and KETh in MeV units. **Compare** KEα to the measured α energy value for this decay in the table. **Compare** (KEα + KETh) to (Δ*m*)*c*2 in MeV units.

**Repeat** the calculations and comparison above for another α-decay in the U-238 decay sequence.

α-Decay Part 2:   
Rutherford scattering indicates that the radius (*R*) of an atomic nucleus can be given by  , where *r*0 = 1.2 fm = 1.2 x 10-15 m and   
*A* = the number of protons and neutrons in the nucleus.  
**Explain** what *r*0 represents for *A* = 1.

*r0 represents the apparent radius of a proton (or neutron).*

**What model** of the nucleus is suggested by the proportionality between *R* and A1/3 ? **Explain**. [Note: In contrast, atoms have diameter about 10-10 m independent of mass.]

**Calculate** the radius values of the Th-234 nucleus and the α-particle, and **show** that the sum of the two values is slightly less than 10 fm.

Within the U-238 nucleus the residual strong force of attraction between adjacent nucleons is stronger than the electrical repulsion force between adjacent protons. However, the residual strong force diminishes much more quickly with separation distance than the electric force. As a result, large nuclei, like U-238, are not able to hold themselves together forever. They undergo radioactive decay. The diagram below represents the potential energy of an α-particle as a function of distance (*r*) from the center of the Th-234 nucleus. The deep potential well represents the effect of the residual strong force attraction overpowering the electrical repulsion for *r* < 10 fm. Beyond 10 fm the strong force potential is negligible and only the 1/*r* electrical repulsion potential is effective. [Note: The -16 MeV potential energy for *r* < 9 fm in the graph is arbitrary.]



Suppose an α-particle exists momentarily as a unit within the U-238 nucleus. Use the Heisenberg relation between the uncertainty in momentum and uncertainty in position measurements for a particle, Δ*p*xΔ*x* ≥ *h*/2π , to **calculate** a minimum α-particle momentum (J/(m/s)) in a U-238 nucleus with a diameter of 14.9 fm.

**Convert** your momentum value to MeV/*c* units.

Note: 1 kg⋅m/s =  = 1.875 x 1021 MeV/*c*

**Show** that the minimum α-particle kinetic energy within the U-238 is about 0.024 MeV.

**Show** that the repulsive electric potential energy between the Th-234 nucleus and the   
α-particle just outside the nucleus at a separation of 10 fm would be about 26 MeV.

The energy available from converting nuclear mass to kinetic energy in the U-238   
α-particle decay is only 4.3 MeV. Energy conservation prevents the α-particle escaping from the nucleus at *r* = 10 fm since it would carry away 26 MeV at that distance.

**Show** that the repulsive electric potential energy between the Th-234 nucleus and the   
α-particle is equal the 4.3 MeV energy available for the decay if *r* = 60 fm.

An α-particle can escape from the nucleus if it can get to *r* = 60 fm. However, an α-particle in the nucleus does not have enough energy to surmount the potential barrier at 10 fm to escape from the nucleus. **Speculate** how the decay could happen in spite of this energy barrier.

In 1928, using the wave mechanics of the then new quantum theory, George Gamow, Edward Condon, and Ronald Gurney proposed a mechanism for α-decay. According to their quantum mechanical model of the nucleus an α-particle has a small, but finite, probability of appearing outside the nucleus, in spite of the energy barrier. The phenomenon is called tunneling, because the α-particle behaves as if it could tunnel through the potential energy “hill” and appear some distance away from the nucleus consistent with energy conservation.

**Calculate** the approximate distance that the α-particle has to “tunnel” to escape from the U-238 nucleus.

According to quantum theory, the probability of tunneling decreases exponentially with the required tunneling distance. Lower tunneling probability means a longer half-life for the nucleus. **What does this imply** for the relation between α-particle energy and   
half-life time?

**Calculate** the log10 of the α-decay half-life times for the U-238 sequence and **enter** the values in the table. **Plot** log10 *t*½ (vertical axis) *vs*. α-decay energy (horizontal axis) for the α decays in the U-238 sequence.   
Is the relation you expect between half-life and decay energy apparent in the plot?



