ENERGY, MOMENTUM, AND MASS
TEACHER NOTES

DESCRIPTION

Students have all heard of E = mc^2, and many can even say something about what it means. However, few are familiar with the more general form which includes momentum for objects not at rest in a given reference frame. In this activity, we can justify the more complete relationship,

$$E^2 = p^2c^2 + m^2c^4,$$

and show that when p = 0, the equation reverts to that equation that the whole world knows.

STANDARDS ADDRESSED

Next Generation Science Standards

Science and Engineering Practices

2. Developing and using models
4. Analyzing and interpreting data
5. Using mathematics and computational thinking
7. Engaging in argument from evidence
8. Obtaining, Evaluating, and Communicating Information

Disciplinary Core Ideas – Physical Science

PS1.A: Structure and Properties of Matter

Crosscutting Concepts

1. Patterns.
2. Cause and Effect: Mechanism and Explanation
4. Systems and System Models

Common Core Literacy Standards

Reading

9-12.4 Determine the meaning of symbols, key terms . . .

9-12.7 Translate quantitative or technical information . . .

Common Core Mathematics Standards

MP2. Reason abstractly and quantitatively.

IB Physics

Topic 7: Atomic and Nuclear Physics

7.2: Nuclear Reactions

7.3.1: Research that deals with the fundamental structure of matter . . .

ENDURING UNDERSTANDING

Particle physicists use conservation of energy and momentum to measure the mass of fundamental particles.

LEARNING OBJECTIVES

Students will know and be able to:

- Make plots of data showing energies and corresponding momenta of particles.
- Recognize when linearizing the graph of the data requires two steps.
- Interpret the physical meaning of the slope of the graph.
- Interpret the physical meaning of the intercept of the graph when the particle is at rest in the reference frame.
- Make a claim about the equation for the relationship between energy and momentum for particle moving in a reference frame. Support their claim with evidence and reasoning.

References

- A. H. Bucherer, *Ann. d. Phys.* **28**, 513, 1909.
- QuarkNet website, *Energy, Momentum, and Mass*, https://quarknet.org/page/energy-momentum-and-mass
- Shankland, Robert S., *Atomic and Nuclear Physics*, New York: MacMillan, 1955, pp. 33–36.
- Wikipedia, *Alfred Bucherer,* https://en.wikipedia.org/wiki/Alfred_Bucherer
- Wikipedia, *Walter Kaufmann*, https://en.wikipedia.org/wiki/Walter_Kaufmann_(physicist)
- W. Kaufmann, *Gottingen Nach*. **2**, 143, 1901.

Prior Knowledge

- Students must be able to make a graph from a table and manipulate data to linearize a graph.

Background Material

This activity enables students to develop an empirical understanding for the complete energy-momentum-mass relationship,

$$E^2 = p^2c^2 + m^2c^4.$$

Your students will use energy and momentum data for electrons from some rather old experiments by Walter Kaufmann and Alfred Bucherer in the early 20th century. In 1901, Kaufmann did an experiment to investigate the effect of changing energy on electrons in the 0.7 MeV to 1.6 MeV energy range. Later, Bucherer did a similar experiment which included lower energy electrons. Note that Kaufmann took his data *before* the introduction of the special theory of relativity in 1905. In both cases, the experimenters measured energy by deflecting an electron beam with an electric field while momentum was derived by deflecting the beam with a magnetic field, as seen in Figure 1 below.

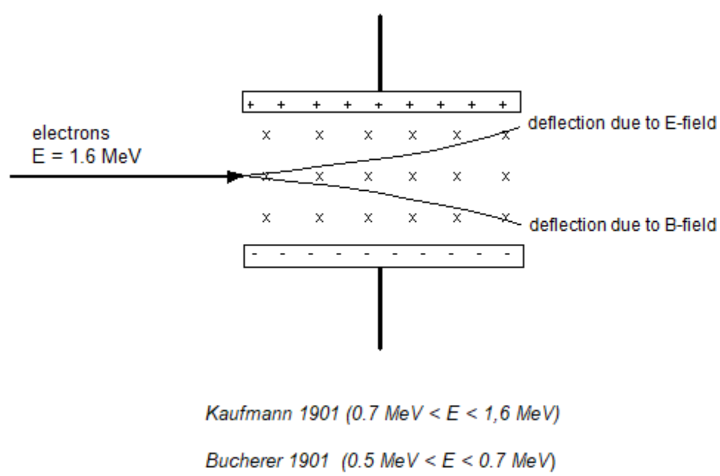


Figure 1: Diagram of the Kaufmann and Bucherer experiments.

Resources/Materials

- Handout on rules for linearizing graphs
- Provided data table
- Materials for making a graph or software for graphical analysis

Implementation

You can divide the class into groups of 2–3 students. Give each student a copy of the student page.

The data table contains some points generated using a simulation program, some points from the Bucherer experiment, and some points from the Kaufmann experiment.

Source of Data	Energy MeV	Momentum MeV/c
Simulated	0.515	0.053
	0.52	0.08
	0.53	0.132
	0.54	0.158
Bucherer 1909	0.542	0.172
	0.552	0.209
	0.567	0.243
	0.593	0.305
	0.700	0.481
Kaufmann 1901	0.767	0.603
	0.848	0.700
	1.022	0.882
	1.237	1.121
	1.584	1.494

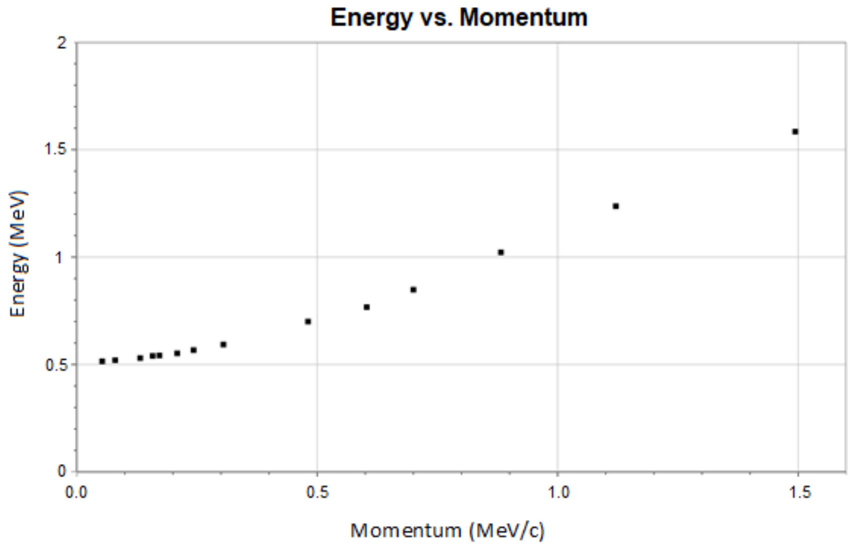
Student Instructions:

1. **Plot** energy (E) as a function of momentum (p).
2. **Describe** the relationship between energy and momentum.
3. **Use** the shape of the graph to determine how to modify the data to make a linear graph.
4. **Plot** the modified data.
5. If this graph is not linear, **use** the shape of this graph to determine how to modify the data to make a linear graph.
6. **Write** the equation of the linear graph using the slope value and intercept value. Remember units on values!
7. **Describe** the physical meaning of the slope of the linearized graph. Use the units as a clue.
8. **Describe** the physical meaning of the intercept of the linearized graph. Remember that when momentum equals zero, the electron is at rest in the reference frame. Use the units as a clue.
9. **Make** a claim about the relationship between m, p, and E. Support your claim with evidence and reasoning.

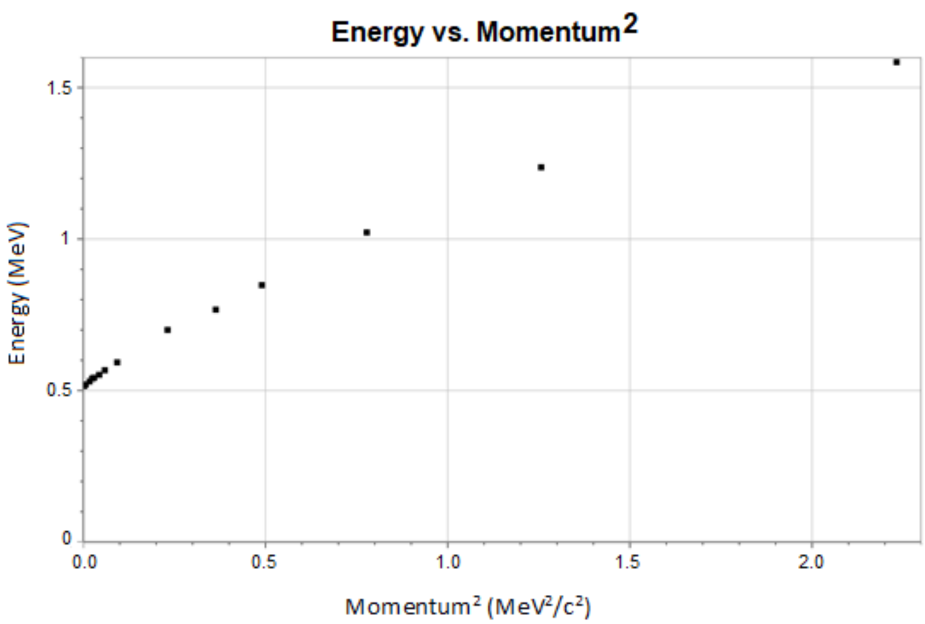
ASSESSMENT

If you are using this activity as an exercise in linearization, the activity can serve as a summative assessment. If the focus of the activity is an introduction to special relativity relationships, the activity is best done using formative assessments such as group discussions, whiteboarding, or a gallery walk.

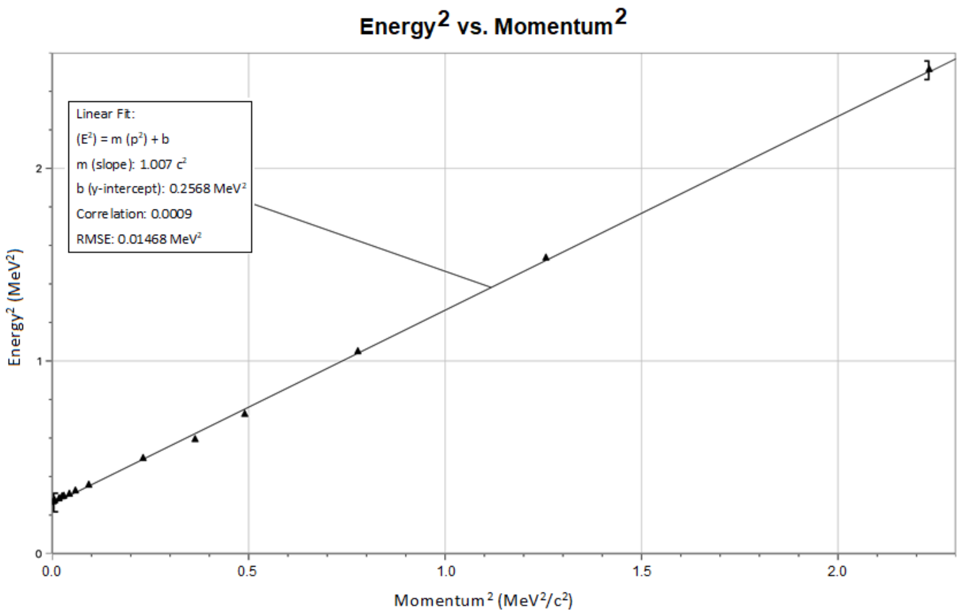
1. **Plot** energy (E) as a function of momentum (p).



2. **Describe** the relationship between energy and momentum.
   - *The graph looks like an over-achiever or parabola.*
3. **Use** the shape of the graph to determine how to modify the data to make a linear graph.
   The momentum data should be squared and energy vs. momentum^2 will be the next graph.
4. **Plot** the modified data.



5. If this graph is not linear, **use** the shape of this graph to determine how to modify the data to make a linear graph.
   This graph looks like an under-achiever or a square root graph. Therefore, the energy data should be squared and energy^2 vs. momentum^2 will be the next graph.



6. **Write** the equation of the linear graph using the slope value and intercept value. Remember units on values!
    - $E^2 = 1.00c^2p^2 + 0.257\ MeV^2$
7. **Describe** the physical meaning of the slope of the linearized graph. Use the units as a clue.
    - *Since the slope is equal to one, E^2 is directly proportional to p^2c^2.*
8. **Describe** the physical meaning of the intercept of the linearized graph. Remember that when momentum equals zero, the electron is at rest in the reference frame. Use the units as a clue.
    - *When p equals 0, E^2 equals 0.257 MeV^2. The square root of 0.257 MeV^2 equals 0.507 MeV. The invariant mass of an electron is 0.511 MeV/c^2 which is found using the famous equation 0.511 $MeV/c^2 = mc^2$.*
9. **Make** a claim about the relationship between m, p, and E. Support your claim with evidence and reasoning.
    - *The resulting physical equation for these data is:*

$$E^2 = p^2c^2 + m^2c^4.$$