HEISENBERG'S LASER
TEACHER NOTES

DESCRIPTION

The Heisenberg Uncertainty Principle is a long-established key component of quantum physics. It states that the minimum uncertainties in measurement of complementary variables (for example, momentum and position) are inversely proportional to each other. The more precisely we know one, the greater the uncertainty in the other. This is expressed mathematically as $\Delta p \, \Delta x \geq h/2\pi$ where p is momentum, x is position (distance from some point) and h is Planck's Constant, approximately 6.63×10^{-34} J s or 4.14×10^{14} eV s.

This activity is designed to enable students to empirically demonstrate the Uncertainty Principle by measuring the uncertainties in position and momentum of photons in a laser beam. We direct the laser through a small opening resulting in diffraction of the beam. The size of the opening reveals Δx while the width of the central maximum gives us Δp. Changing the size of the opening changes the width of the central maximum.

STANDARDS ADDRESSED

Next Generation Science Standards

Science and Engineering Practices

- 4. Analyzing and interpreting data
- 5. Using mathematics and analytical thinking
- 8. Obtaining, evaluating and communicating information

Common Core Literacy Standards

Reading

- 9-12.4 Determine the meaning of symbols, key terms . . .
- 9-12.7 Translate quantitative or technical information . . .

Common Core Mathematics Standards

- MP1. Make sense of problems and persevere in solving them.
- MP2. Reason abstractly and quantitatively.
- MP4. Model with mathematics.

AP and IB Standards

Check your current standards for AP Physics 1, AP Physics 2, International Baccalaureate.

ENDURING UNDERSTANDING

- Scientists can use data to develop models based on patterns in the data.

LEARNING OBJECTIVES

Students will know and be able to:

1. Follow laser safety rules.
2. Measure the central maximum for diffraction of light from a laser beam passing through a thin slit.
3. Show how changing the width of the slit changes the diffraction pattern.
4. Linearize a graph and determine the physical meaning of the slope.
5. Apply the deBroglie wavelength equation and experimental results to empirically demonstrate the relationship between uncertainty in position and uncertainty in momentum.

PRIOR KNOWLEDGE

Students must know and be able to:

- Plot and interpret a graph from data.
- Measure using a vernier caliper.
- Use the small angle approximation for sin θ and tan θ.

BACKGROUND MATERIAL

In the QuarkNet Data Activity *What Heisenberg Knew*, we use data from a beam of large molecules diffracting through a thin slit. These large molecules are still small enough to be quantum objects and so they create a wave-like diffraction pattern as they pass through the slit. The molecules traveling towards the slit can have any position in the x direction. When the molecules pass through the slit, they are confined to the width of the slit which gives us our Δx.

Now consider the diffraction pattern. Since there is a width to each maximum or minimum there must be an uncertainty in the DeBroglie wavelength $\Delta\lambda = h/\Delta p$, implying an uncertainty in momentum. Students receive the data from this experiment and can confirm empirically that $\Delta p = (\text{constant})/\Delta x$ or $\Delta p\Delta x = \text{constant}$, the mathematical expression of the Uncertainty Principle.

QuarkNet fellow Dr. Michael Wadness had an insight: what is physical for large molecules is physical for much smaller particles, like electrons. Electron diffraction experiments are well-known but beyond the capacities of high school physics laboratories to perform. Taking the logic a step further, however, perhaps the experiment can be performed with photons instead. This was successfully tested as a proof of concept at the University of Notre Dame with the assistance of undergraduate laboratory manager Dr. Thomas Loughran.

RESOURCES

- Video by Don Lincoln: https://www.youtube.com/watch?v=V5l_ehnM73w
- A review of linearization: https://quarknet.org/content/how-linearize-curved-data-plot
- Additional resources from QuarkNet Data Activities portfolio:
  What Heisenberg Knew
- Animation of particles forming a central maximum:
  https://www.glowscript.org/#/user/kcecire/folder/MyPrograms/program/movingsphere4-grok

MATERIALS

Method 1: Use the Data Images provided.

Method 2: Run the experiment with students. The materials needed are:

- Two Vernier Calipers that can be adjusted and read to mm precision.
- Red laser, low intensity, with a known operating wavelength.
- Meter stick or similar length-measuring device.
- White or light-colored target to serve as the screen; e.g. poster board or wall.
- Stands and clamps to fix all items into place.
- Room with sufficient space which can be darkened.

IMPLEMENTATION

There are two different methods for obtaining the data. Method 1 relies on the provided images taken during the experiment at the University of Notre Dame. Method 2 has the class replicate the experiment and generate their own data.

Method 1

Use the data from the Data Images pages to calculate values of Δp and Δx, and then plot the results.

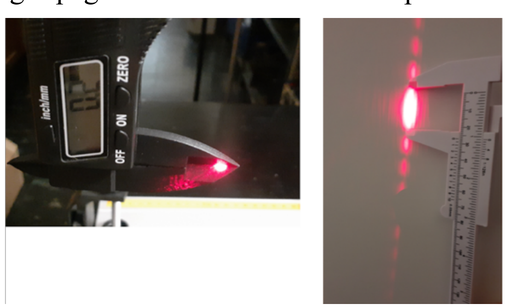


Figure 1. Data Images from Notre Dame experiment

Steps to gather data from the images:

- The image on the left in Figure 1 shows the slit width as the opening of the caliper: in this case, 0.2 mm.
- The image in Figure 2 shows that a second caliper, the measuring caliper, is used to measure the width of the central maximum. The width of the caliper opening is measured by finding where the zero on the left scale lines up with the central scale. In this case the central maximum width is 27 mm.

Figure 2

Method 2

Follow the instructions below to guide your students to collect the data to calculate values of Δp and Δx. ***Special care must be taken to train students in eye safety when using lasers.***

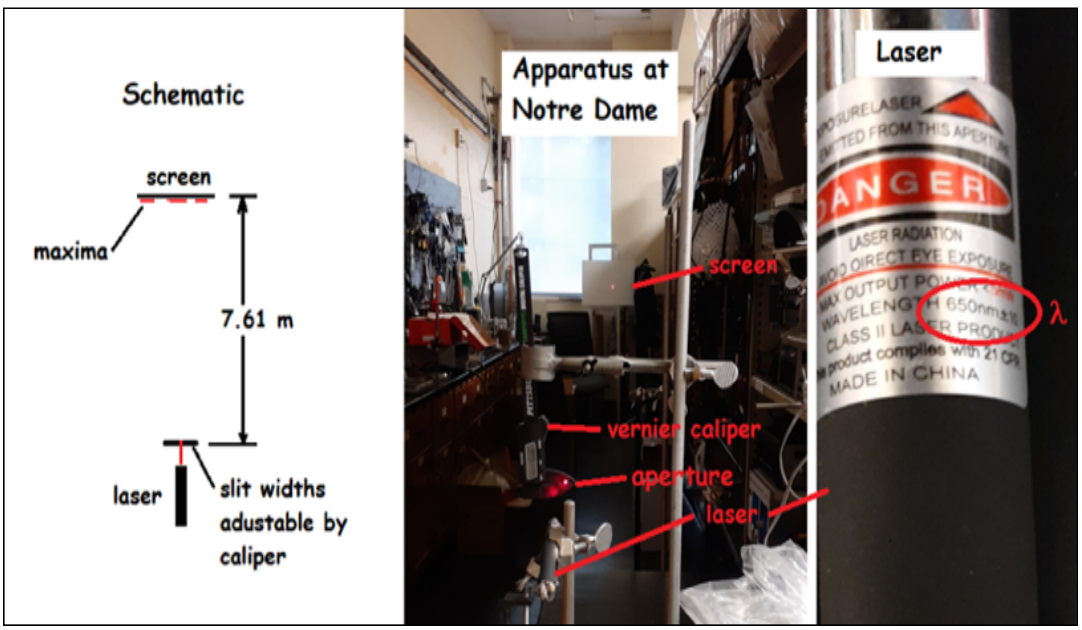


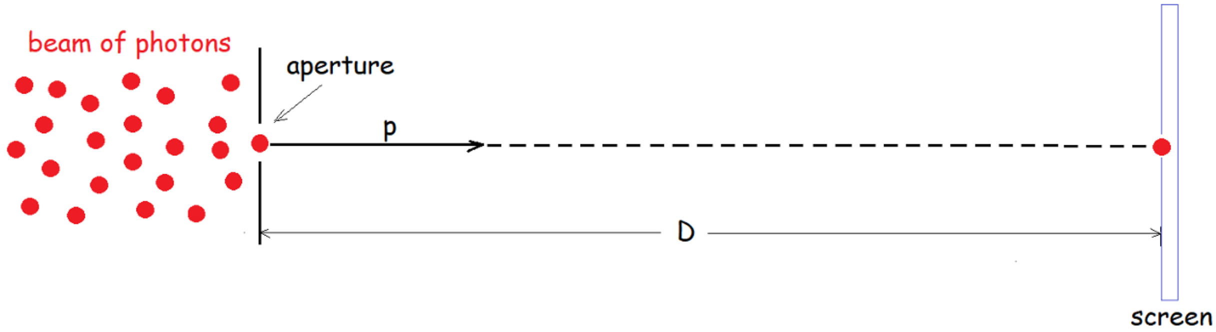
Figure 3. Experiment at Notre Dame.

1. Using stands and clamps, place the laser in a horizontal position to project the beam several meters towards a light-colored wall, poster paper or other target so that the beam end is clearly visible. See Figure 3.
2. Set up the slit caliper so that the plane of its slit opening is at right angles to the beam.
3. Adjust the slit opening to 0.1 mm.
4. Turn on the laser and adjust its position relative to the slit until you get a diffraction pattern on the screen. Note the wavelength of your laser, 650 nm for the pictured laser.
5. Using the measuring caliper, determine the width of the central maximum and record it.
6. Repeat steps 3-5 multiple times, using different slit opening widths up to 1.0 mm.

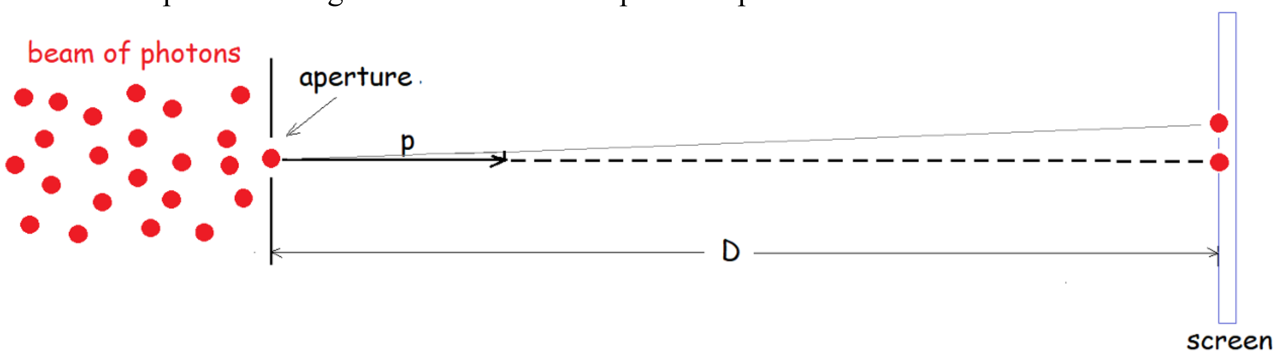
How to find Δp and Δx

Students must determine Δp from the width of the central maximum and Δx from the size of the slit opening for each data point. Use the series of images below or the animation listed in the resources for a more intuitive representation of the relationship between Δp and the width of the central maximum.

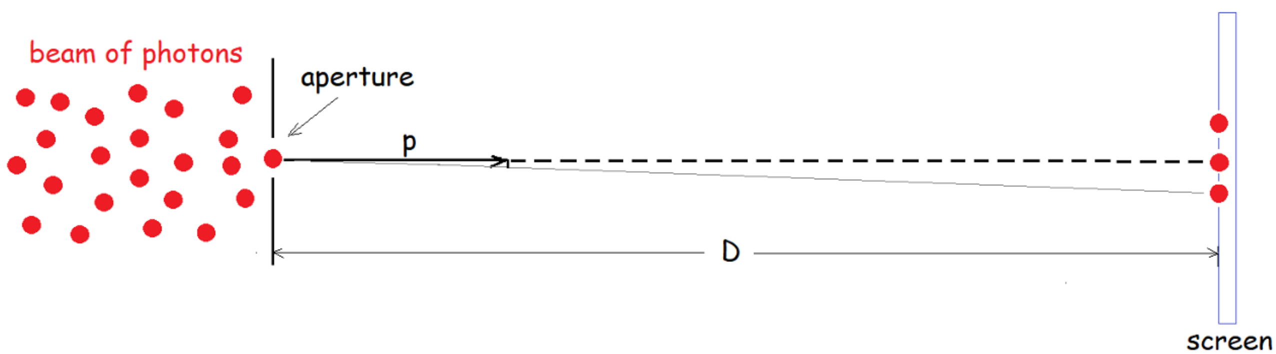
1. A photon gets through the aperture with zero transverse momentum and through to the screen.



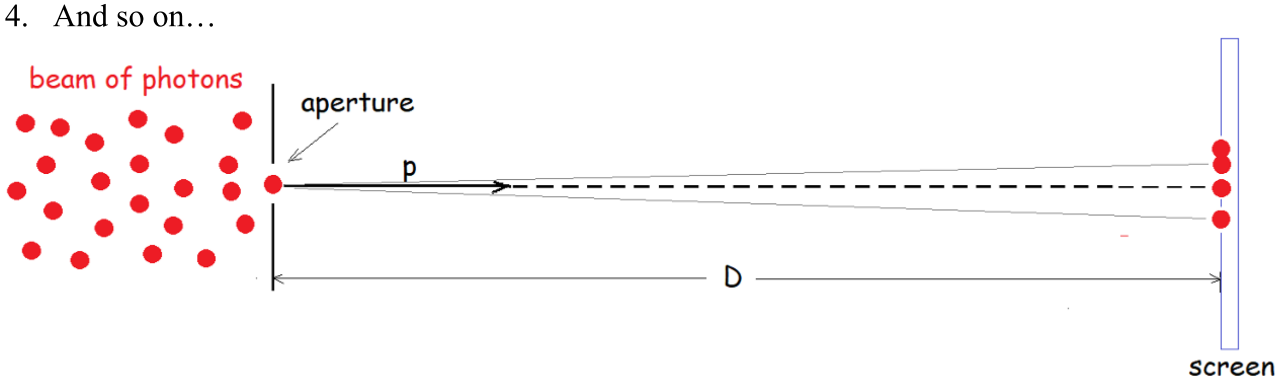
2. A small upward momentum – a result of the uncertainty in transverse momentum – results in the next photon hitting the screen above the previous photon.



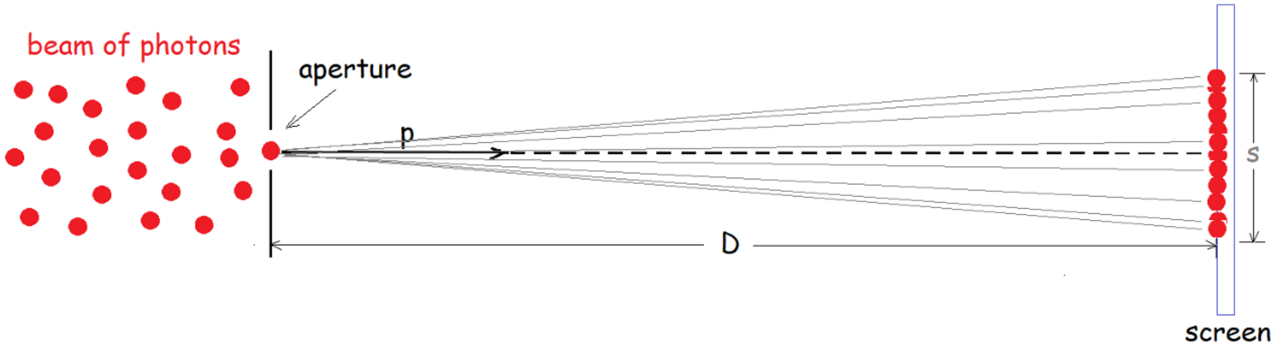
3. The next photon has a small downward momentum due to the uncertainty.



4. And so on…



5. And so on as we build up a central maximum of width s



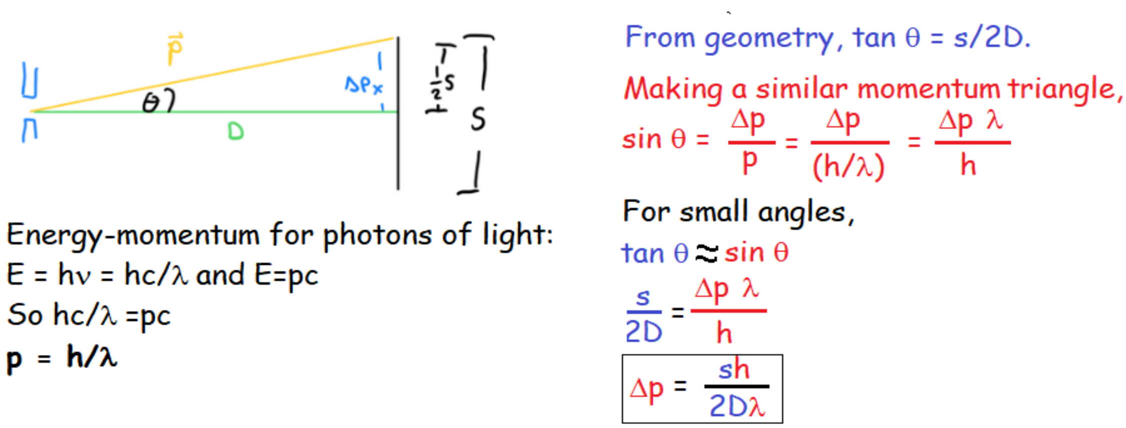
It is not hard to get Δx: it is just the width of the aperture for the laser beam made by the slit caliper at the source. Finding Δp is a little more involved.

The equation to calculate the uncertainty in momentum Δp is:

$$\Delta p = \frac{hs}{2D\lambda}$$

Where h is Planck's Constant (6.63 x 10^{-34} kg-m^2/s), s is the width of the central maximum, D is the distance from the slit opening to the screen, and λ is the wavelength of the laser light.

See the derivation below using similar values of sin θ and tan θ for very small angles:



Sample calculation to find Δp

Let's make a sample calculation for an aperture 0.2 mm. In that case, the width of central maximum s = 27 mm as shown in Figure 1. The wavelength of the laser light is 650 nm and the distance from the aperture to the screen is 7.61 m. Then we can calculate Δp in this instance to be:

$$\Delta p = \frac{hs}{2D\lambda}$$

$$\Delta p = \frac{6.63x10^{-34}\,kg\frac{m^2}{s} * 27x10^{-3}m}{2 * 7.61m * 650x10^{-9}m}$$

$$\Delta p = 1.8x10^{-30}kg\frac{m}{s}$$

Data Collection

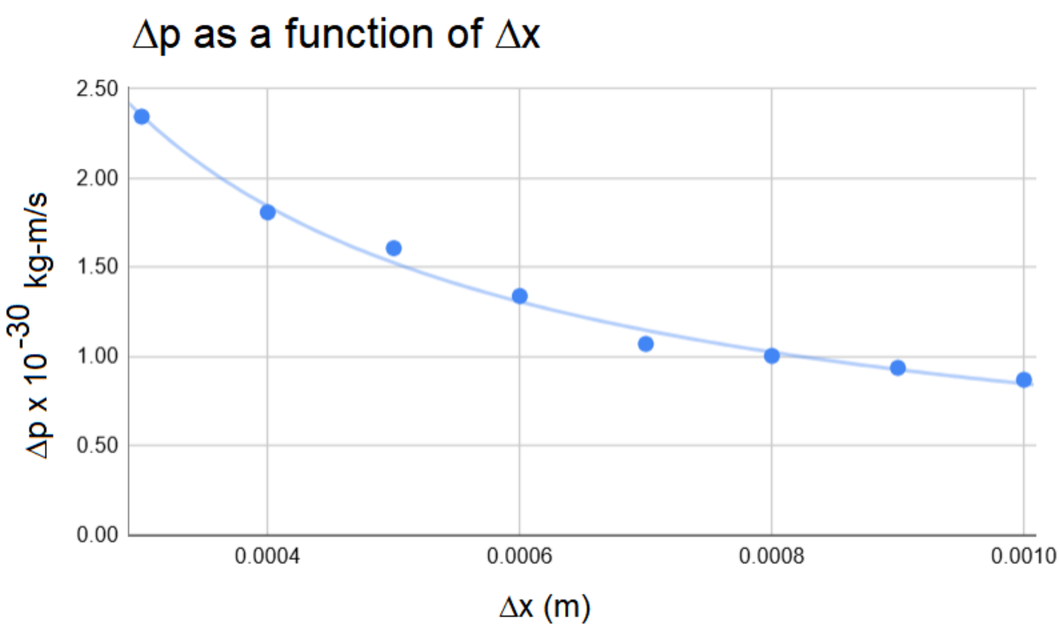
Divide your class into groups of two or three students. Have your students build a table like the one below to organize their data. Students must calculate Δp and 1/Δx to complete the last two columns.

Event	App (mm)	Width s (mm)	Width s (m)	Δ x (m)	Δ p $x10^{-30}$ $(kg\frac{m}{s})$	$\frac{1}{\Delta x}(m^{-1})$

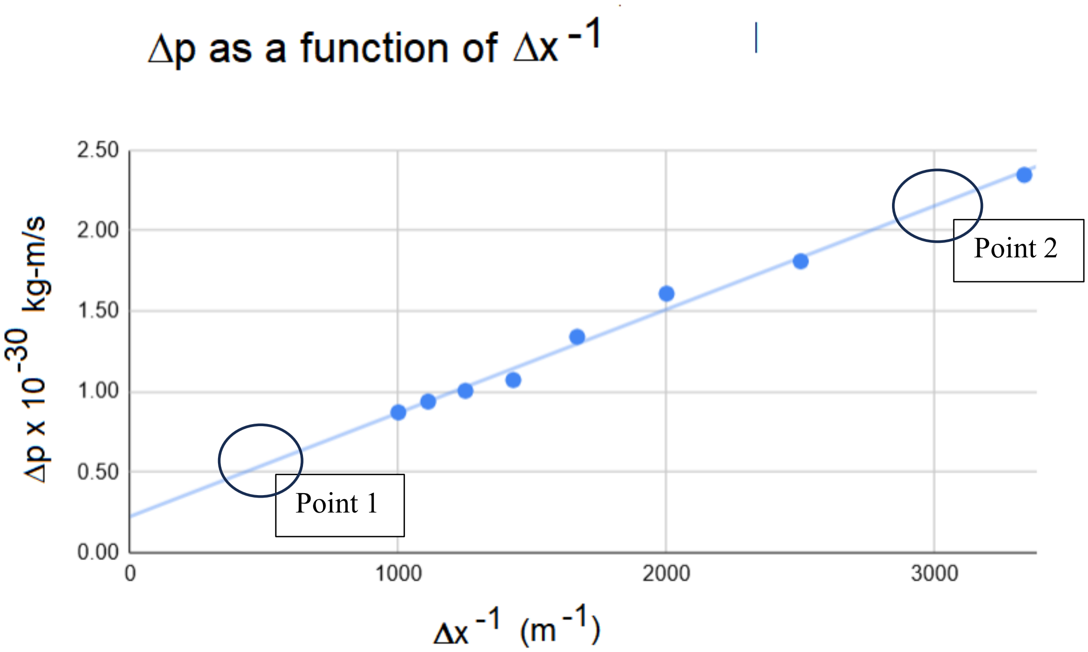
Expected results using supplied data
The table below shows the expected results for data gathered from the University of Notre Dame images.

Event	Apperture (mm)	Width s (mm)	Width s (m)	Δx (m)	Δp $x10^{-30}$ $(kg\frac{m}{s})$	$\frac{1}{\Delta x}(m^{-1})$
1	0.3	35	0.035	0.0003	2.35	3333
2	0.4	27	0.027	0.0004	1.81	2500
3	0.5	24	0.024	0.0005	1.61	2000
4	0.6	20	0.020	0.0006	1.34	1667
5	0.7	16	0.016	0.0007	1.07	1429
6	0.8	15	0.015	0.0008	1.01	1250
7	0.9	14	0.014	0.0009	0.94	1111
8	1.0	13	0.013	0.001	0.87	1000

Data Analysis
Plotting Δp as a function of Δx, we get:



This is close to the expected curve – if Δp Δx ~ h/4π, then as Δx increases, Δp decreases. You can linearize the plot to gain better insight into the relationship between Δp and Δx as well as determining the physical meaning of the slope of the linearized graph.



Point 1: $(500m^{-1}, 0.5 * 10^{-30}\frac{kg*m}{s})$ Point 2: $(3000m^{-1}, 2.2 * 10^{-30}\frac{kg*m}{s})$

Finding the slope:

$$Slope = \frac{(\Delta p_2 - \Delta p_1)}{(\frac{1}{\Delta x_2} - \frac{1}{\Delta x_1})}$$

Upon substitution

$$Slope = \frac{(2.2 - 0.5) * 10^{-30}}{(3000 - 500)} \frac{kg * m/_s}{\frac{1}{m}}$$

$$= \frac{1.7 * 10^{-30}}{2500} \frac{kg * m/_s}{\frac{1}{m}}$$

$$= 6.8 * 10^{-34}\, kg * m^2/_s$$

Therefore; the equation of this graph is:

$$\Delta p\, \Delta x = 6.8 * 10^{-34}\, kg * m^2/_s$$

The equation developed by Werner Heisenberg is:

$$\Delta p\, \Delta x \geq h/_{4\pi}$$

$$\Delta p\, \Delta x \geq 0.53 * 10^{-34}\, kg * m^2/_s$$

Our result of $6.8 * 10^{-34}\frac{kg*m^2}{s}$ is greater than the Heisenberg values of $0.53 * 10^{-34}\frac{kg*m^2}{s}$. These results indicate that the optical interference pattern formed by photons passing through a slit is consistent with Heisenberg's Uncertainty Principle.

Assessment

Suggestions for assessment include:

Students may submit:

1. One sample calculation for Δp.
2. The graph of Δp vs. Δx.
3. The linearized graph and the calculation of the slope of the linearized plot.
4. A written paragraph about how the data supports or fails to support the claim that photons passing through a slit can be examples of Heisenberg Uncertainty Principle.

Acknowledgement

This activity is based on an idea proposed by Michael Wadness, a QuarkNet LHC fellow in the Boston Center and physics teacher at Medford High School. Thomas Loughran of the Department of Physics and Astronomy at the University of Notre Dame helped develop the experimentation.