WHY RELATIVITY?
STUDENT GUIDE

For centuries Galileo had it right about relative velocity. Given reference frame A and reference frame B where frame A is at rest and frame B is moving at velocity v relative to frame A; Galileo posited that the distance between the frames at any time t is given by $x_B = x_A + vt$, with t measured to have the same value in both frames.

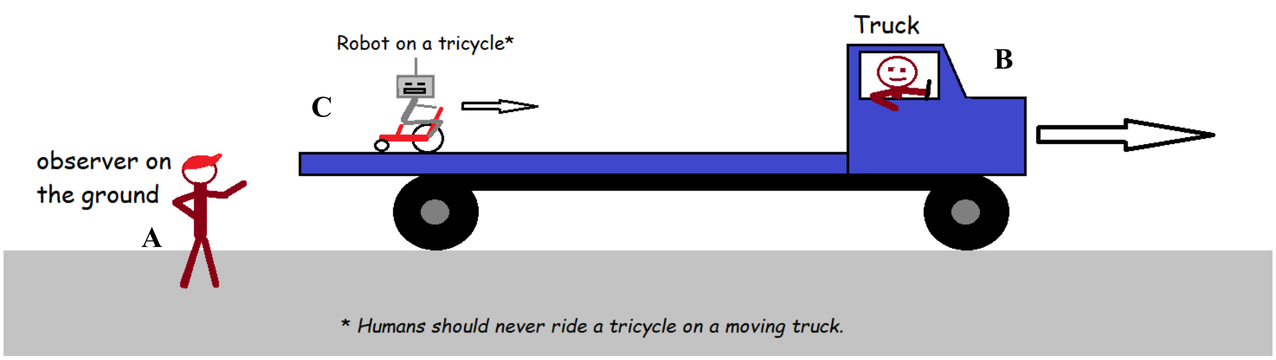
The velocity of an object can be found using the equation. $v = \frac{\Delta x}{\Delta t}$. We are going to test to see if Galileo got it right for cosmic ray muons. We will develop a predicted result using Galilean Relativity and compare that to an experimental result determined using muon detection rates at various elevations.

PART 1: UNDERSTANDING FRAMES OF REFERENCE

OBJECTIVES:
Students will know and be able to:
- Explain how two objects or people can have different velocities depending on which reference frames are used to describe the motion.

WHAT DO WE KNOW?
We are going to talk about the image below that shows three observers.
- Observer A is standing beside the road and is at rest with respect to the road.
- The truck travels to the right. The driver of the truck, Driver B, is at rest with respect to the truck and in motion with respect to the road.
- Robot C on the tricycle is traveling to the right on the bed of the truck. Robot C is at rest with respect to the tricycle but in motion with respect to Observer A and Driver B.

The road (Earth) is the reference frame for Person A. The truck is the reference frame for Driver B. The tricycle is the reference frame of Robot C.



The image shows:
- Observer A beside a road with velocity $v_A = 0$ m/s with respect to the road;
- Driver B in the truck traveling on the road with velocity $v_B = 30$ m/s with respect to the road;
- Robot C riding on a tricycle on the truck bed with velocity $v_C = 5$ m/s with respect to the truck.

Each observer can see and record the velocity of the other observers.

What do we do?

Complete the table:

Rest frame for each observer	Velocity of observer A	Velocity of driver B	Velocity of robot C
Person A/Road	0 m/s		
Driver B/Truck		0 m/s	
Robot C/Tricycle			0 m/s

Question:

Will all observers record the same velocity as the observers?

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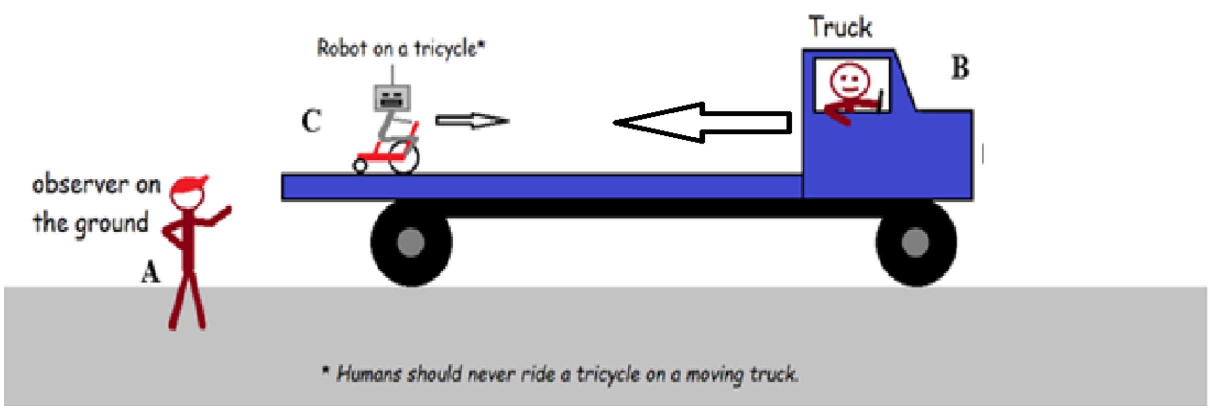
Extension:

Now assume that the robot is pedaling to the right at the same velocity with respect to the truck but **the truck is traveling to the left** at the same velocity with respect to the road.



Complete the table:

Rest frame for each observer	Velocity of observer A	Velocity of driver B	Velocity of robot C
Person A/Road	0 m/s		
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Robot C/Tricycle			0 m/s

Question:

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Discussion Question:

As you can see, distance and time appear to be the same for all observers. Is this also true for velocity? Give evidence from your tables to support your claim.

Part 2: Expected values for muon rate vs elevation

Objectives:

Students will know and be able to:

- Construct a graph of muon rate vs elevation using Galilean assumptions.

What do we know?

These data are generated from predictions in the reference frame of the earth for muons traveling towards the earth's surface from the upper atmosphere.

Note the symbol for muon is μ. This symbol is also used in scientific notation for the prefix micro ($1x10^{-6}$) as in 2.2 μs referenced below.

What do we need?

The definition of half-life is the time it takes for a sample of particles to decay to ½ of the initial number of particles. Similarly, lifetime is the time for the sample of particles to decay to $1/e$ of the initial number of particles. Mathematically, e is a constant of nature equal to approximately 2.718.

The mean lifetime (τ) of a muon is approximately 2.2 μs and the speed of a muon (v_μ) is established to be slightly less than 3.0×10^8 m/s based on previous experimentation.

Question 1:
How far (Δy) can a muon travel in one lifetime in the reference frame of the muon?
This can be found by multiplying the mean lifetime by the speed of the muon. $\Delta y = \tau * v_\mu$

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Question 2:
How does the rate of muons counted by the detector change during the first lifetime?
Given an initial rate of 17 counts/minute, determine the new rate after one lifetime.

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What do we do?

Question 3:
What are the expected elevation and rate for each lifetime of the descending muon?
To help us answer Question 3, use the muon speed and the lifetime value to find the expected elevation and the rate after each lifetime, assuming that Galilean physics applies.

Use the table below to guide your calculations.

Number of lifetimes τ	Elevation y (m)	Expected rates using Galilean physics (counts/minute)
0	3400	17
1	3400 m – 660 m = 2740 m	17/e = 6.25
2	2740 m – 660 m	6.25/e
3		
4		
5		

Part 3:

Objectives:

Students will know and be able to:

- Explain that discrepancies between the results of the lifetime and velocity experiments can be resolved using special relativity.

WHAT DO WE KNOW?

These data were collected and reported using the rest frame of the earth. The data in the observed rate column in the table below were obtained using small cosmic ray detectors during an ascent up the South Baldy mountain in New Mexico.

WHAT DO WE NEED?

Copy the elevation and expected rates values from the table above:

Number of lifetimes τ	Elevation y (m)	Expected rates (counts/minute)	Observed rate (counts/minute)
0	3400	17	17
1	2740 m	6.25	13
2			10
3			8
4			6*
5			5*

Note*: The Observed rates for 4 lifetimes and 5 lifetimes are estimated.

WHAT DO WE DO?

- Plot the expected rates vs elevation.
- On the same graph plot the observed rate vs elevation.

DISCUSSION QUESTIONS: TIME DILATION

1. Of the two curves in the plot, which represents our calculations of what we expect the rates to be? Which represents the actual measured rates?

2. Are the observed rates higher or lower than our expectations?

3. The rate is the number of muons per minute. Does the higher rate mean more or fewer muons have decayed in each minute? Remember that the detectors count muons, not their decay products.

4. Does the answer to question 3 imply a longer or a shorter lifetime?

5. The lifetime of a muon is 2.2 microseconds in its own reference frame. The muons move close to the speed of light relative to Earth. What does this imply about the lifetime - the average time to decay - in our reference frame?

6. What does this imply about time in different reference frames?

7. And why did Galileo never notice this?

8. Who did?

DISCUSSION QUESTIONS 2: LENGTH CONTRACTION

1. Of the two curves in the plot, which represents our calculations of what we expect the rates to be? Which represents the actual measured rates?

2. Are the observed rates higher or lower than our expectations?

3. The rate is the number of muons per minute. Does the higher rate mean more or fewer muons remain at the altitude they are detected? Remember that the detectors count muons, not their decay products.

4. If muons decay at a steady rate, does the fact that we measure more muons than expected mean that the muon traveled a shorter or longer distance relative to Earth?

5. What does this imply about distance in different reference frames?

6. And why did Galileo never notice this?

7. Who did?