WHY RELATIVITY?
TEACHER NOTES

DESCRIPTION
This activity focuses on using data collected by cosmic ray muon detectors to introduce the concept of reference frames and the need for special relativity when investigating very fast fundamental particles. We will develop a qualitative understanding of the need for length contraction and time dilation.

For centuries Galileo had it right about relative velocity. Given reference frame A and reference frame B where frame A is at rest and frame B is moving at velocity v relative to frame A; Galileo posited that the distance between the frames at any time t is given by $x_B = x_A + vt$, with t measured to have the same value in both frames.

The velocity of an object can be found using the equation. $v = \frac{\Delta x}{\Delta t}$. We are going to test to see if Galileo got it right for cosmic ray muons. We will develop a predicted result using Galilean Relativity and compare that to an experimental result determined using muon detection rates at various elevations.

STANDARDS ADDRESSED
Next Generation Science Standards

- Disciplinary Core Ideas
  - PS1.A: Structure and Properties of Matter
  - PS1.C: Nuclear Processes
  - PS2.B: Types of Interactions
  - PS2.C: Stability and Instability in Physical Systems
- Science and Engineering Practices
  - 4. Analyzing and interpreting data
  - 5. Using mathematics and analytical thinking
  - 6. Constructing explanations (for science) and designing solutions (for engineering)
  - 7. Engaging in argument from evidence
- Crosscutting Concepts
  - 1. Asking questions (for science) and defining problems (for engineering)
  - 4. Analyzing and interpreting data
  - 5. Using mathematics and computational thinking
  - 6. Constructing explanations (for science) and designing solutions (for engineering)
  - 7. Engaging in argument from evidence

IB Physics Standards

- 7.1 Discrete energy and radioactivity
- 7.3 The structure of matter
- A.2 Lorentz transformations

Common Core Literacy Standards

- Reading
  - 9-12.4 Determine the meaning of symbols, key terms . . .
  - 9-12.7 Translate quantitative or technical information . . .

Common Core Mathematics Standards

- MP2. Reason abstractly and quantitatively.

Enduring Understanding

Scientists make a claim based on data that comprise the evidence for the claim.

Background Material

Primary cosmic rays are particles such as protons and atomic nuclei that travel through space and on occasion interact with nuclei in the atmosphere of Earth. The primary cosmic ray collisions occur about 15 km above the earth's surface and cause showers of particles. These particles are secondary cosmic rays. The particles from these "air showers" which reach the surface of earth and are most easily detected are muons.

The type of detector used in QuarkNet to collect the data for this activity employs two or more counters made of scintillating material which will emit photons as a muon moves through the counter depositing some of its energy. The detector has an internal clock so rates of muon detection can be measured and recorded.

Based on muon lifetime distributions, muons have a rest frame mean lifetime of about 2.2 microseconds. For more information on muon lifetime refer to one or more of the three *Mean Lifetime* activities in the Data Activities Portfolio. The average muon speed is close to 3.0×10^8 m/s. For more on the speed of muons consider the *How Speedy are These Muons* activity in the Data Activities Portfolio.

To traverse the 15 km to Earth's surface before decaying would be rare were it not for special relativity. In the reference frame of the muon, the distance to the surface of Earth is contracted to much less than 15 km. On the other hand, in the reference frame of Earth, that distance is still 15 km but the time for the muon to traverse is dilated to a time much longer than the mean lifetime of the muon at rest.

The links below provide useful background material:

- Information about cosmic muons and showers:
  - Image of cosmic ray shower (Scroll down to see the images.)
  - Hyperphysics: Atmospheric Muons
- Information on Special Relativity:
  - Hyperphysics: Time Dilation and Length Contraction
  - Time Dilation: An Experiment with mu-mesons (1962) (This classic movie is 36 minutes long. It is historically accurate and easy to follow.)
  - Time Dilation Experiment (3 minutes)

Prior Knowledge

- Basic algebraic computational skills
- Familiarity with one-dimensional constant speed equation (unless using this activity as an introduction to one-dimensional motion)

Resources/Materials

Muon rate vs elevation data provided in the student pages with answers in the teacher notes.

Implementation

The activity consists of three parts which are outlined below. Included here are answers to the embedded questions found within. The students are given data tables to complete and spaces to write their answers to embedded questions. The answers appear in red italics.

Part 1: Understanding Frames of Reference

Objectives:

Students will know and be able to:

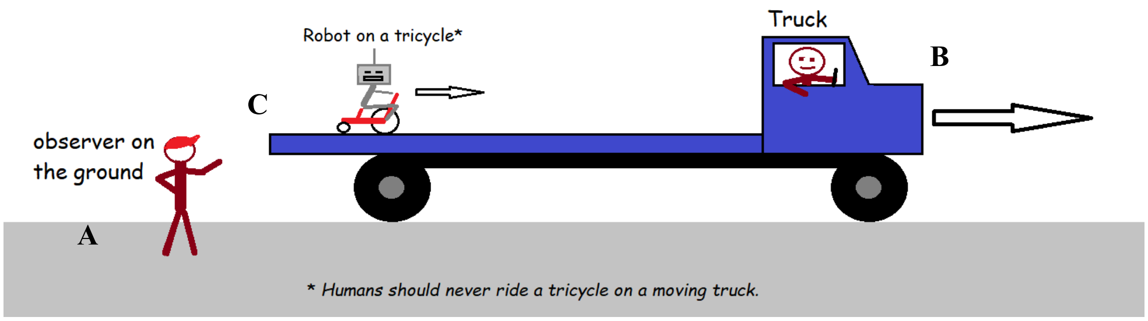
- Explain how two objects or people can have different velocities depending on which reference frames are used to describe the motion.

WHAT DO WE KNOW?

We are going to talk about the image below that shows three observers.

- Observer A is standing beside the road and is at rest with respect to the road.
- The truck travels to the right. The driver of the truck, Driver B, is at rest with respect to the truck and in motion with respect to the road.
- Robot C on the tricycle is traveling to the right on the bed of the truck. Robot C is at rest with respect to the tricycle but in motion with respect to Observer A and Driver B.

The road (Earth) is the reference frame for Person A. The truck is the reference frame for Driver B. The tricycle is the reference frame of Robot C.



The image shows:

- Observer A beside a road with velocity $v_A = 0$ m/s with respect to the road;
- Driver B in the truck traveling on the road with velocity $v_B = 30$ m/s with respect to the road;
- Robot C riding on a tricycle on the truck bed with velocity $v_C = 5$ m/s with respect to the truck.

Each observer can see and record the velocity of the other observers.

WHAT DO WE DO?

Complete the table:

Rest frame for each observer	Velocity of observer A	Velocity of driver B	Velocity of robot C
Person A/Road	0 m/s	*30 m/s*	*35 m/s*
Driver B/Truck	*-30 m/s*	0 km/hr	*5 m/s*
Robot C/Tricycle	*-35 m/s*	*-5 m/s*	0 m/s

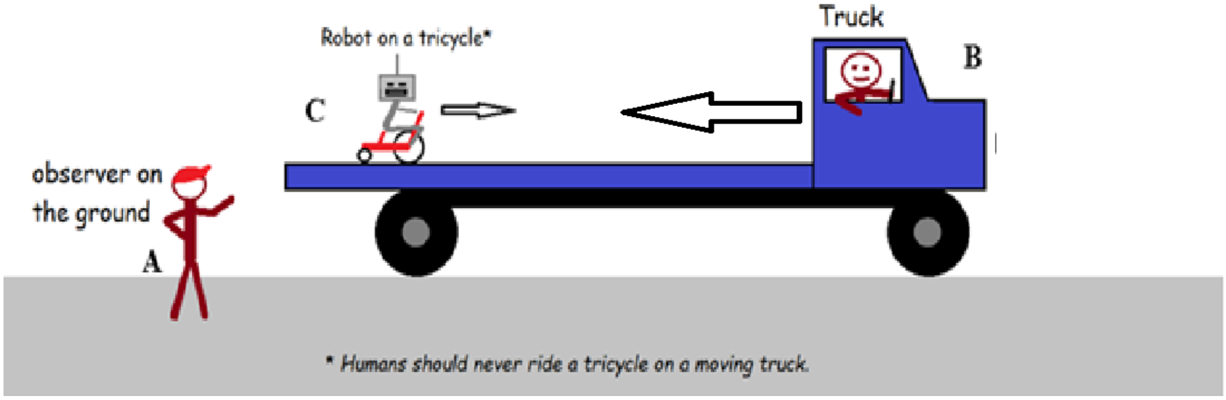
Question:

Will all observers record the same velocity as the other observers?

- *The table shows that the measured velocities vary from observer to observer.*

Extension:

Now assume that the robot is pedaling to the right at the same velocity with respect to the truck but **the truck is traveling to the left** at the same velocity with respect to the road.



Complete the table:

Rest frame for each observer	Velocity of observer A	Velocity of driver B	Velocity of robot C
Person A/Road	0 m/s	*-30m/s*	*-25 m/s*
Driver B/Truck	*30 m/s*	0 km/hr	*5 m/s*
Robot C/Tricycle	*25 m/s*	*-5 m/s*	0 m/s

Question:

Will all observers record the same velocity as the observers?

- *The table shows that the measured velocities vary from observer to observer.*

Discussion Question:

As you can see, distance and time appear to be the same for all observers. Is this also true for velocity? Give evidence from your tables to support your claim.

PART 2: Expected values for muon rate vs elevation

OBJECTIVES:

Students will know and be able to:

- Construct a graph of muon rate vs elevation using Galilean assumptions.

WHAT DO WE KNOW?

These data are generated from predictions in the reference frame of the earth for muons traveling towards the earth's surface from the upper atmosphere.

Note the symbol for muon is μ. This symbol is also used in scientific notation for the prefix micro ($1x10^{-6}$) as in 2.2 μs referenced below.

WHAT DO WE NEED?

The definition of half-life is the time it takes for a sample of particles to decay to ½ of the initial number of particles. Similarly, lifetime is the time for the sample of particles to decay to 1/*e* of the initial number of particles. Mathematically, *e* is a constant of nature equal to approximately 2.718.

The mean lifetime (τ) of a muon is approximately 2.2 μs and the speed of a muon (v_μ) is established to be slightly less than 3.0 x 10^8 m/s based on previous experimentation.

Question 1:

How far (Δy) can a muon travel in one lifetime in the reference frame of the muon?
This can be found by multiplying the mean lifetime by the speed of the muon. $\Delta y = \tau * v_\mu$

$\Delta y = 2.2x10^{-6}s * 3.0x10^{8}m/s$ = 660m

Question 2:

How does the rate of muons counted by the detector change during the first lifetime?
Given an initial rate of 17 counts/minute, determine the new rate after one lifetime.
(17 counts/minute)/e = 6.25 counts/minute

What do we do?

Question 3:

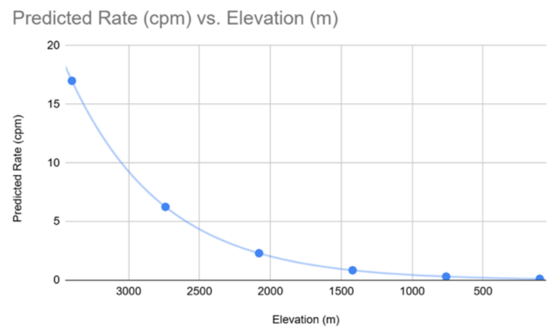
What are the expected elevation and rate for each lifetime of the descending muon?
To help us answer Question 3, use the muon speed and the lifetime value to find the expected elevation and the rate after each lifetime, assuming that Galilean physics applies.

Use the table below to guide your calculations.

Number of lifetimes τ	Elevation y (m)	Expected rates using Galilean physics (counts/minute)
0	3400	17
1	3400 m - 660 m = 2740 m	17/e = 6.25
2	2740 m - 660 m = 2080 m	6.25/e = 2.30
3	*1420 m*	*0.85*
4	*760 m*	*0.31*
5	*100 m*	*0.11*

Use the data from the table to make a plot of expected data rate (cpm) vs elevation.

The expected graph is shown below:



Part 3:

Objectives:

Students will know and be able to:

- Explain that discrepancies between the results of the lifetime and velocity experiments can be resolved using special relativity.

What do we know?

These data were collected and reported using the rest frame of the earth. The data in the measured rate column in the table below were obtained using small cosmic ray detectors during an ascent up the South Baldy mountain in New Mexico.

What do we need?

Copy the elevation and expected rates values from the table above:

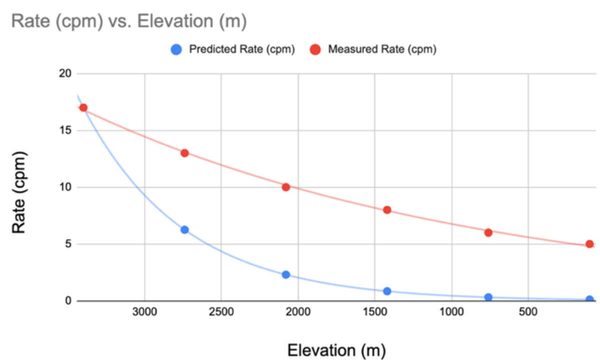
Number of lifetimes τ	Elevation y (m)	Expected rates (counts/minute)	Measured rate (counts/minute)
0	3400	17	17

1	2740 m	6.25	13
2	2740 m - 660 m = 2080 m	6.25/e = 2.30	10
3	*1420 m*	*0.85*	8
4	*760 m*	*0.31*	6*
5	*100 m*	*0.11*	5*

Note*: the measured rates for 4 lifetimes and 5 lifetimes are estimated.

What do we do?

- Plot the measured rates vs elevation on the same graph as the expected rate vs elevation.



This graph is in the earth reference frame. Notice that the predicted values are not in agreement with the measured values. This is evidence that the Galilean transformations do not correctly predict the behaviors of cosmic muons moving close to the speed of light. In this case a new model is needed.

While it is important to be able to make the graphs, **the significance of this activity comes from drawing meaning from the graphs.**

Two sets of discussion questions follow. Set 1 focuses on time dilation and set 2 focuses on length contraction. Select either set or both sets if you have time.

Discussion Questions 1: Time Dilation

1. Of the two curves in the plot, which represents our calculations of what we expect the rates to be? Which represents the actual measured rates?
   - *Expected rates (calculations) is the lower curve; measured rates is the upper curve.*
2. Are the observed rates higher or lower than our expectations?
   - *Higher.*
3. The rate is the number of muons per minute. Does the higher rate mean more or fewer muons have decayed in each minute? Remember that the detectors count muons, not their decay products.
   - *You are seeing more muons because fewer have decayed.*
4. Does the answer to question 3 imply a longer or a shorter lifetime?
   - *Longer lifetime means we see more muons and fewer have decayed*.
5. The lifetime of a muon is 2.2 microseconds in its own reference frame. The muons move close to the speed of light relative to Earth. What does this imply about the lifetime - the average time to decay - in our reference frame?
   - *The lifetime in the earth's reference frame is longer than the lifetime in the muon's reference frame.*
6. What does this imply about time in different reference frames?
   - *Elapsed time does not appear to be the same for all observers.*

7. And why did Galileo never notice this?
    - *Nothing in Galileo's observations included objects moving at speeds near the speed of light.*
8. Who did?
    - *Albert Einstein introduced the concept of time dilation in his famous 1905 paper on special relativity. This was developed using his realization that the speed of light has the same value in all reference frames.*

DISCUSSION QUESTIONS 2: LENGTH CONTRACTION

1. Of the two curves in the plot, which represents our calculations of what we expect the rates to be? Which represents the actual measured rates?
    - *Expected rates (calculations) is the lower curve; measured rates is the upper curve.*
2. Are the observed rates higher or lower than our expectations?
    - *Higher.*
3. The rate is the number of muons per minute. Does the higher rate mean more or fewer muons remain at the altitude they are detected? Remember that the detectors count muons, not their decay products.
    - *You are seeing more muons because fewer have decayed.*
4. If muons decay at a steady rate, does the fact that we measure more muons than expected mean that the muon traveled a shorter or longer distance relative to Earth?
    - *If fewer muons are left undecayed, they have traveled farther.*
    - *Since more muons are left undecayed, the distance traveled is shorter in the muon reference frame.*
5. What does this imply about distance in different reference frames?
    - *Distance traveled does not appear to be the same for all observers.*
6. And why did Galileo never notice this?
    - *Nothing in Galileo's observations included objects moving at speeds near the speed of light.*
7. Who did?
    - *Albert Einstein introduced the concept of length contraction in his famous 1905 paper on special relativity. This was developed using his realization that the speed of light has the same value in all reference frames.*