

# DICE, HISTOGRAMS & PROBABILITY

## TEACHER NOTES

### DESCRIPTION

Students roll provided dice (> 1 die per roll) and record the resulting individual values as well as the sum of the values. They create histograms of the data. Students develop insight into the concept of “degrees of freedom” by rolling differing numbers of dice and noting how the histograms change. Students can extend the activity by using a spreadsheet to simulate the pseudo-random numbers they would obtain with dice rolls. You can extend the activity further by providing loaded dice; sufficiently large numbers of rolls with these can create a “bump” in the histogram.

### STANDARDS ADDRESSED

#### *Next Generation Science Standards*

##### Science Practices

1. Asking questions
2. Developing and using models
3. Planning and carrying out investigations
4. Analyzing and interpreting data
5. Using mathematics and computational thinking
6. Constructing explanations
7. Engaging in argument from evidence
8. Obtaining, evaluating, and communicating information

##### Disciplinary Core Ideas – Physical Science

PS1.A: Structure and Properties of Matter

PS2.C: Stability and Instability in Physical Systems

##### Crosscutting Concepts

1. Patterns.
2. Cause and effect: Mechanism and explanation.
3. Scale, proportion, and quantity.
4. Systems and system models.

#### *Common Core Literacy Standards*

##### Reading

9-12.7 Translate quantitative or technical information . . .

#### *Common Core Mathematics Standards*

MP5. Use appropriate tools strategically.

MP6. Attend to precision.

#### *IB Physics Standards*

##### Topic 1: Measurement and Uncertainty

1.2.6 Describe and give examples of random and systematic errors.

1.2.8 Explain how the effects of random errors may be reduced.

1.2.11 Determine the uncertainties in results.

### ENDURING UNDERSTANDING

- Scientists can analyze data more effectively when they are properly organized; charts and histograms provide methods of finding patterns in large data sets.

### LEARNING OBJECTIVES

Students will know and be able to:

- Precisely record simple observations.
- Create and interpret a histogram.
- Describe the degrees of freedom (variability) present in the system under study

### **PRIOR KNOWLEDGE**

Students must be able to keep careful records of observations and sum integers.

### **BACKGROUND MATERIAL**

A fair die has an equal probability of producing any of the available numbers. Histograms of the individual rolls shouldn't show any features at all—they should be flat.

Degrees of freedom can be thought of as the number of independent pieces of information needed to find the answer minus the number of calculations needed to arrive at the answer. In this case, the number of dice minus one since the interesting representation is the sum of all of the rolls for that student. This is a definition used in statistics. For biologists and chemists, molecular bonds have three degrees of freedom to undergo thermal vibrations: they can stretch, they can twist and they can flex. Each of those vibrational modes corresponds to a different energy, which means when you expose these molecules to infrared light, they will absorb light at three distinct wavelengths per bond; furthermore, a C=O bond would have different resonances from a C-H or N-H bond. Thus infrared spectroscopy becomes an important tool for chemists trying to determine the structure of a molecule. For physicists, degrees of freedom is an important topic in quantum mechanics and specifically in particle physics when discussing the many pathways for particle decay.

Students will see a peak in the histogram of sums of the rolled dice, (e.g., 3 d6s (die 6 sided). The location of the peak depends on two things: how many rolls they combine, and the number of sides on the rolled dice. There are three ways to roll a 10 when rolling two d6: 6+4, 4+6, and 5+5. There are 27 ways to roll a 10 when rolling *three* d6! *The peak of the histogram appears at the sum that can be most easily created.*

If only six sided dice are used, the degrees of freedom are number of dice minus one (N-1). If the dice used have different numbers of sides, then the degrees of freedom are number of dice minus 2 (N-2)

### **IMPLEMENTATION**

We do not provide a student handout with this activity.

- Provide students with sets of similar dice. All groups should have the same number and type of dice if you wish to pool individual or group data into a larger, shared set.
- Ask the students to record their dice rolls and draw histograms of their data. Each group will make two histograms:
  - A single histogram of all of the single roll values for their own data. The group will also make a histogram of the total of dice thrown for each roll.
  - In addition to a histogram for a single die, students should generate a histogram for pairs of thrown dice; this means the number of bins in the histogram changes from six to eleven.
- Students should compare and contrast the appearance of these histograms and propose explanations for any observed similarities and differences using the Claim/Evidence/Reasoning framework.
- You can use a spreadsheet to combine the class data and generate one histogram with a large data set.

**Extensions:**

1. Many spreadsheets have a facility for creating random numbers (e.g. the “randbetween()” command). You might consider asking your students to do this assignment using a spreadsheet instead of dice. This allows students to easily adjust both the number of dice used per throw, and the number of sides on each die; in fact, this allows for dice that wouldn't be possible otherwise (e.g. a seven-sided die).
2. Provide the students with a mixture of die types and repeat the experiment.

**ASSESSMENT**

For a formative assessment, the students display their group histograms around the room. Ask them to note differences in the histograms. The students then confer in their group, present their answers to the questions listed below on a white board and share their ideas and answers with the class.

You might ask students questions such as:

- What conclusion can you draw about the histogram of how many times your number was rolled?
- What number appeared most often in the rolls? Discuss the reasons for this.
- What would happen to the histogram if there were twice as many dice rolls recorded on it?
- If you suspected that there was a “trick die” hidden in your materials, would you be able to infer this from your data? Why or why not?
- Make a spreadsheet of the combined class data to form a new histogram to test the prediction.
- Write a paragraph length conclusion based on the results. Be sure to include a discussion of the effects of ‘degrees of freedom’ or ‘variability’ in the explanation.

**EXTENSION QUESTIONS:**

- Predict the effect of using a mixture of dice types. What will the new total histogram look like?
- Predict the effect of increasing the number of thrown dice from two to three, or of increasing the number of sides on each die.
- Test the prediction.
- Write a conclusion. Be sure to include your claim, the evidence from data and the reasoning explaining why the evidence supports the claim.