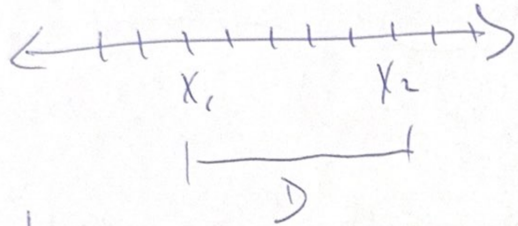
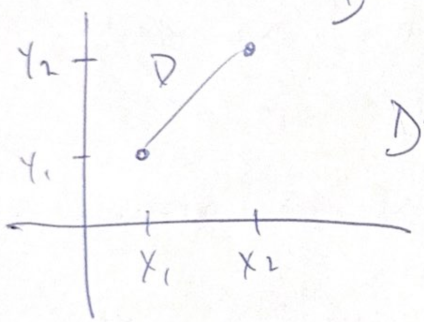
David Kaplan:
SR / GR / Cosmology

Recall:



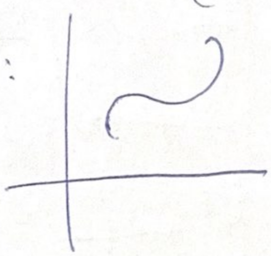
move x_1, x_2 by translation:
D is invariant under this



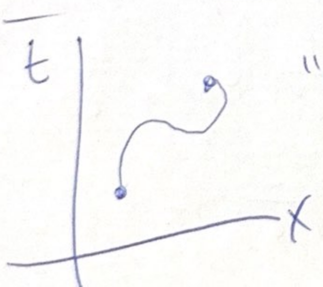
$D = \sqrt{\Delta x^2 + \Delta y^2}$ is also invariant in both rotation and translation (underlying symmetries?)

(same in 3D: $\sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}$)

Generalize:

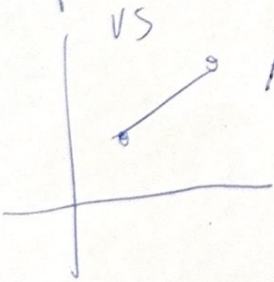


Recall: total length is just that Pythagorated $d\vec{l}$, integrated



"Events" happen
at time t
and coordinate x

Straight line version is constant velocity. If so, $\Sigma F = 0$



Avg velocity $= \frac{\Delta x}{\Delta t}$

slope $= \frac{\Delta t}{\Delta x}$

to at-rest-wrt-coordinate, $F = m\frac{dv}{dt}$

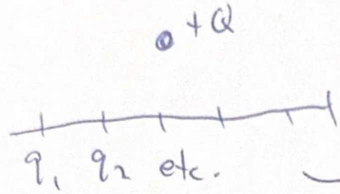
to driver, $F = m\frac{d(v-v')}{dt}$

$v \to v - v'$

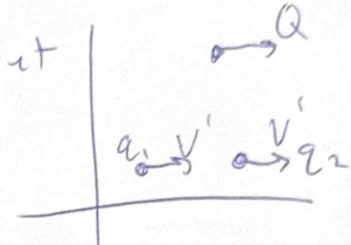
Galilean Invariance

SR: Recall origin in E&M:

Electrostatic case:



Felec points ↑ but

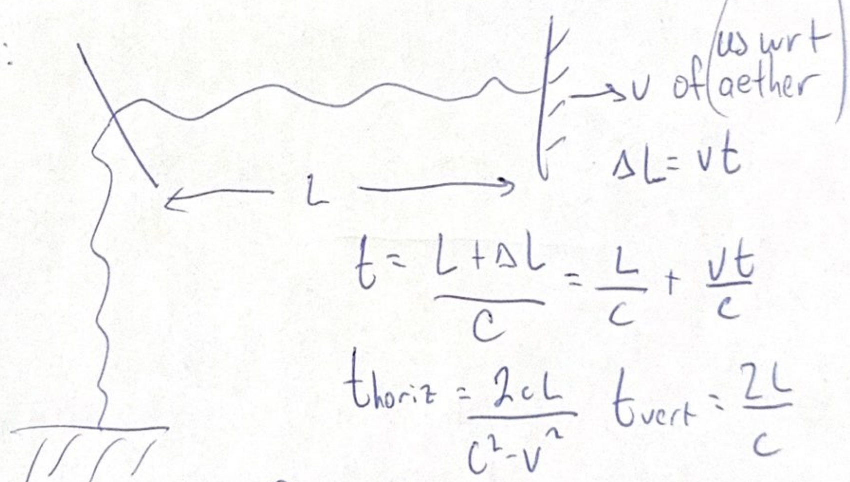


Fmag points down and Felec ↑ : why different??

→ Implies there must be a "preferred frame" where only they get the "right answer."

→ Stimulated aether experiments to find this frame

Michaelson-Morley:



$\Delta L = vt$

$$t = \frac{L + \Delta L}{c} = \frac{L}{c} + \frac{vt}{c}$$

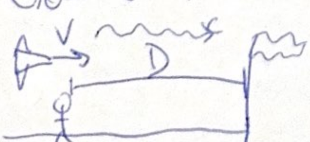
$$t_{horiz} = \frac{2cL}{c^2 - v^2} \qquad t_{vert} = \frac{2L}{c}$$

$$f_{vert} - f_{horiz} = \frac{2Lf}{c} \cdot \frac{v^2}{c^2}$$

For $\lambda \approx 500$ nm, $V_{sun} \approx 3\times10^4$ m/s, $L \approx 10$m, expect $\approx \frac{1}{2}$ bump diff.

But there was none! No aether!

Good ol' Rocket ship



$t_{gnd} = \frac{D}{c}$

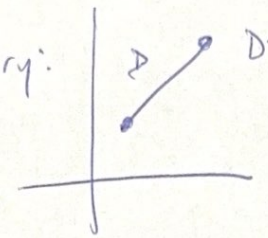
$t_{rocket} = \frac{D}{c+V}$

Everyone: ??

∴ - no simultaneity
- no agreement on time, distance
- new rule for V

E, p : - if you inelastically collide $\overset{m}{\circ}\xrightarrow{v} \xleftarrow{v}\overset{m}{\circ}$
two different frames disagree on whether $\vec{p}$ is conserved
- similarly, different frames disagree on Energy conservation

In normal geometry:

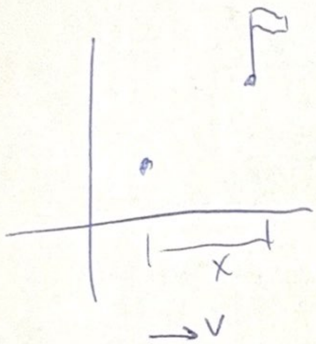


$D = \sqrt{\Delta x^2 + \Delta y^2}$

then rotate: $\Delta x' = \Delta x \cos\phi - \Delta y \sin\phi$

$\Delta y' = \Delta x \sin\phi + \Delta y \cos\phi$

D does not change
$(\sin^2 + \cos^2 = 1)$



$\frac{t}{x} = \frac{1}{c}$

$ct = x$

$0 = x^2 - c^2 t^2$

$I = \Delta x^2 - c^2 \Delta t^2 = \Delta x'^2 - c^2 \Delta t'^2$

invariant Δt

$(x, y, z)\begin{bmatrix} x \\ y \\ z \end{bmatrix} \Rightarrow I = $ Dot product or $\begin{bmatrix} x & y & z \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$

minus sign is a big deal! $\begin{bmatrix} t & x & y & z \end{bmatrix}\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} t \\ x \\ y \\ z \end{bmatrix}$

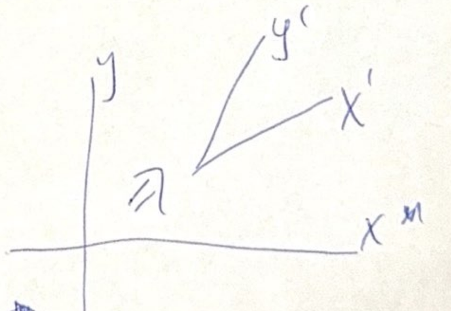
$-$ sign is only way to preserve the invariance

$\sin \rightarrow \sinh$

$\sin^2 + \cos^2 = 1$

$\Downarrow$

$\sinh^2 - \cosh^2 = 1$



Whoops! Diversion

Recall $i\frac{\partial}{\partial t}\psi = H\psi$ single derivative! What about Nima's Whole 2nd derivative thing? Where's the other $\frac{\partial}{\partial t}$?

$$H(x,p) = \frac{p^2}{2m} + V(x)$$

↳ second derivative hiding in p

You can derive $\cancel{\vec{\nabla}\times\vec{E} = \nabla\times}$ $\frac{dE}{dt} = \vec{\nabla}\times\vec{B} + \vec{J}$

but you have to impose $\vec{\nabla}\cdot\vec{E} = \rho$ and if you do, you need a background + $\boxed{\text{const}}$ term in Gauss

To gravity: $ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$ if $c=1$

$$= \begin{bmatrix} dt & dx & dy & dz \end{bmatrix} \begin{bmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} dt \\ dx \\ dy \\ dz \end{bmatrix}$$

↓

$$\begin{bmatrix} 16 \\ \text{terms!} \end{bmatrix}$$ the metric tensor $g_{\mu\nu}$

Invariant adds a function a^2 to the space part and redefines length

Monkey-gun $\bigcap = \bigwedge$

Euclid required parallel lines remain so. Remove this, you have curved space - meridians converge

$$G^{\mu\nu} = 8\pi G_N T^{\mu\nu}$$

mass-energy tensor

(which includes $g_{\mu\nu}$)

Isotropic matter

$$ds^2 = \underbrace{-g_{00}(t)\,dt^2}_{dT^2} + g_{xx}(t)\left(dx^2 + \ldots\right)$$

$$= -dT^2 + g_{xx}(T)\,dx^2$$

$$D_{12} = \sqrt{g(t)\left(x_2^2 - x_1^2\right)} \text{ etc.} = \sqrt{g(T)}\, D_{12}^{\text{normal}}$$

call this $a(T)$:)

define $\dot{a} = \frac{da}{dt}$, $\ddot{a} = \frac{d^2a}{dt^2}$

$$G^{\mu\nu} = 8\pi G_N T^{\mu\nu} \Rightarrow G^{tt}: \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G_N \rho$$

energy density

pressure

$$G^{xx} = G^{yy} = G^{zz}: \frac{\ddot{a}}{a} + \frac{1}{2}\left(\frac{\dot{a}}{a}\right)^2 = -4\pi G_N P$$

important

4-vector: $p^\mu = [E, p_x, p_y, p_z]$

in $v \ll c$ realm, cold gas, $v \Rightarrow 0$

$p^\mu \Rightarrow [mc^2, 0, 0, 0]$

so $T \Rightarrow \begin{bmatrix} mn & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ pressure

here, $P = \frac{1}{3}\rho$

"matter" = stuff w/ no pressure

In SR, density contracts ?!

\# of particles $N = (n, f_x, f_y, f_z)$

flux of particles

$$T^{\mu\nu} = p^\mu N^\nu$$

if $P = 0$, $\dfrac{\ddot{a}}{a} = \dfrac{-1}{2}\left(\dfrac{\dot{a}}{a}\right)^2$ implies negative pressure

and if there is pressure, $\dfrac{\ddot{a}}{a} = \dfrac{-1}{2}\left(\dfrac{\dot{a}}{a}\right)^2 - 4\pi G_N P$

still is shrinking universe

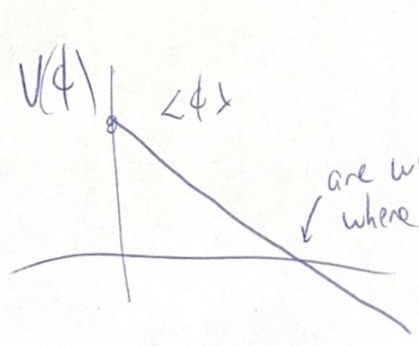
we see that $\dfrac{\ddot{a}}{a} > 0$

so there must be a cosmological constant Λ

so $G^{\mu\nu} = 8\pi G_N \left(T^{\mu\nu} + g^{\mu\nu}\Lambda\right)$

Λ background energy density (can come from lots of places)

$\Lambda \sim \langle \text{stuff} \rangle \sim \infty$ bad :(



are we here where Λ is small?

could it change?

$\sim M_{planck}^4$ (estimate)

$\sim (\text{TeV})^4$ exp. est.

but LHC says $\sim (10^{-3}\,\text{eV})^4$

10^{-124} discrepancy :|

$$i\frac{d}{dt}\psi = H(\vec{A}, \vec{E})\psi$$, leads to $\dot{E} = \vec{\nabla} \times \vec{B} - \vec{J}$

impose $\vec{\nabla} \cdot \vec{E} = \rho_e + \text{const}$

$$\frac{d}{dt} \vec{\nabla} \cdot \vec{E} = -\nabla \cdot \rho$$

If you try the same Hamiltonian type stuff with gravity, you get $H = 0$ b/c (?)