Title: JHU QuarkNet Workshop 2024 Monday

Worlds That Aren't Dr. Nima Arkani-Hamed

Newtonian Worldview: $V_0 + X_0$ determine its future (particles)
(ignoring relativity + quantum mechanics)

$$m\,\ddot{\vec{r}}(t) = \vec{F}(\vec{r}, \dot{\vec{r}}, t)$$ Newtonian Clockwork Determinism

Why are laws like Gravitation $\frac{1}{r^2}$ $F = \frac{GM_1M_2}{r^2}$

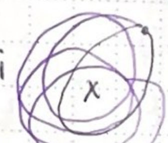
- Density of Field lines $\sim \frac{1}{\text{Area}}$ $\sim \frac{1}{r^2}$

- 3D World: $\frac{1}{r^2}$ 2D: $\frac{1}{r}$ 1D: constant
- 4D: $\frac{1}{r^3}$ nD: $\frac{1}{r^{n-1}}$

What do orbits look like for any dimension? Power

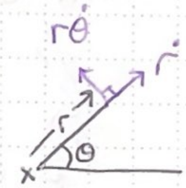
$\frac{1}{r^2} \Rightarrow$ Ellipse

$\frac{1}{r^{2.1}}$



NOT Closed

For Radial Force



correction note $rp \Rightarrow r^p$

Energy $E = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) - \frac{c}{r^p}$ ($p=1$ Kepler) for the potential

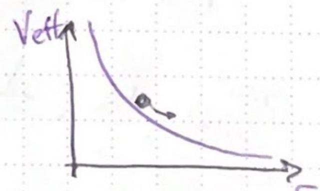
Angular momentum $J = m r r\dot{\theta}$
$= m r^2\dot{\theta}$

$\boxed{\dot{\theta} = \frac{J}{mr^2}}$ substitute into Energy

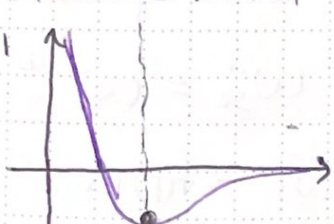
$$E = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\left(\frac{J}{mr^2}\right)^2 - \frac{c}{r^p}$$

$$E = \frac{1}{2}m\dot{r}^2 + \left[\frac{1}{2}\frac{J^2}{mr^2} - \frac{c}{r^p}\right]$$

Effective Potential $V_{eff}(r) = -\frac{c}{r^p} + \frac{J^2}{mr^2}$

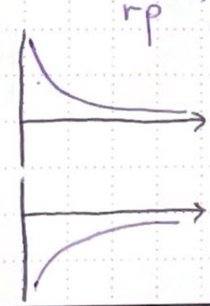


p=1

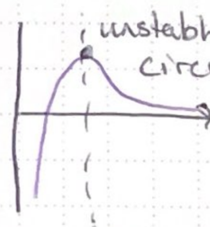


stable circular orbit

p=2



p=2.1



unstable circular orbit

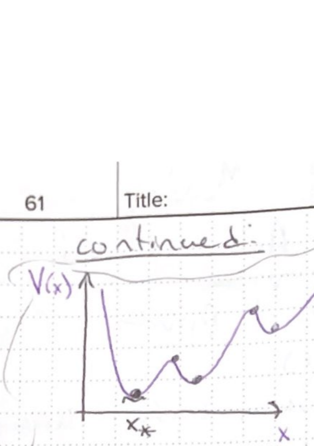
Signature: | Date: 7/22/24 | Team Members: | Continued Page #
Witness: | Date:

continued:

$V(x = x_* + \delta x) = V(x_*) + \delta x\, V'(x_*)$

but $\delta x\, V'(x_*) \Rightarrow 0$ so next term

$+ \frac{1}{2} \delta x^2 V''(x_*)$



$E = \frac{1}{2} m \dot{x}^2 + V(x)$ $\quad x = x_* + \delta x$

$E = \frac{1}{2} m \delta\dot{x}^2 + \frac{1}{2} \underbrace{V''(x_*)}_{k} \delta x^2$ Harmonic Oscillator $\omega^2 = \frac{k}{m} = \frac{V''(x_*)}{m}$

Taylor expansion: go to the first non zero term

$V_{eff}(r) = \frac{1}{2} \frac{J^2}{mr^2} - \frac{C}{r^p}$ $\quad V'_{eff}(r) = -\frac{J}{mr^3} + \frac{pC}{r^{p+1}}$

$r_{circular}$:

$r_{circ}^{p-2} = \frac{mp \cdot C}{J^2}$ $\quad r_{circ} = \left(\frac{mpC}{J^2}\right)^{\frac{1}{p-2}}$ $\quad J = \left(\frac{mpC}{r_{circ}^{p-2}}\right)^{1/2}$

$mr^2 \dot{\theta} = J$ $\quad \dot{\theta} = \underbrace{\frac{1}{m r_{cir}^2}\left(\frac{mpC}{r_{cir}^{p-2}}\right)^{1/2}}_{K}$

General $\omega = K \frac{1}{r^{2+\left(\frac{p-2}{2}\right)}}$

when $p=1$ $\omega = K \frac{1}{r^{3/2}}$

Kepler's 3rd Law

$\dot{\theta} = \frac{J}{m r_{cir}^2} = \omega_{circ}^2 = \frac{J^2}{m^2 r_{cir}^4}$

Small kicks to circular orbit using Taylor Expansion

$V'_{eff}(r) = -\frac{J^2}{mr^3} + \frac{pC}{r^{p+1}} = 0$ $\quad V''(r_{cir}) = \frac{3J^2}{m r_{cir}^4} - \frac{p(p+1)C}{r_{cir}^{p+2}}$

small oscillations from circ orbit $\omega_{so}^2 = \frac{V''_{cir}}{m}$ $\quad$ circular orbit $\omega_{cir}^2 = \frac{m^2 C^4}{J^6}$

when $p=1$

$\omega_{so}^2 = \omega_{cir}^2$

$\omega_{so}^2 = f(p)\, \omega_{cir}^2$

$f(p) = 1$ when $p=1$

leads to closed orbits

Continued: Stabil...

Why don't electron...

$E = \left(\frac{p^2}{2m}\right) - \left(\frac{e^2}{r}\right.$ $\quad \frac{1}{2}mv^2$

$\Delta p \Delta r \sim \hbar$

$E = \frac{\hbar^2}{r^2 2m} - \frac{e^2}{r}$

$r_{atom} \div \frac{\hbar^2}{mr^2} \sim$

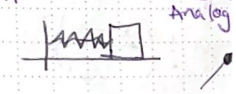
Atoms can ... may exist

Another World

$m\ddot{x} = F$ Newto...

our world has t... reversal invarian... (run a video backw... is a solution

Applied to mass-... Analog...



$m\ddot{x} + kx = 0$

$\omega^2 = \frac{k}{m}$

only 2 real solution

$\omega^6 = \frac{k}{m}$

2 real + 4 complex

Continued : Stability of Matter / Atoms

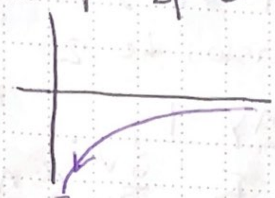
Why don't electrons fall into proton?

$$E = \left(\frac{p^2}{2m}\right) - \left(\frac{e^2}{r}\right)$$ but due to uncertainty

$\frac{1}{2}mv^2$

$\Delta p \Delta r \sim \hbar$ $\qquad p \sim \Delta p \sim \frac{\hbar}{r}$

$$E = \frac{\hbar^2}{r^2 2m} - \frac{e^2}{r}$$



$J \sim \hbar$

Some minimum angular momentum even when not moving

$r_{atom} \hspace{0.5em} \frac{\hbar^2}{mr^2} \sim \frac{e^2}{r} \Rightarrow \quad r_{atom} \sim \frac{\hbar}{me^2}$

Atoms can only exist in 3D higher dimensions may exist but not at the atomic structure

Another World

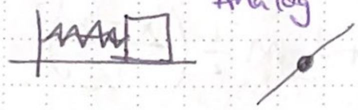
$m\ddot{x} = F$ Newton

our world has time reversal invariance (run a video backwards is a solution)

$F \neq ma$

- $m\dot{x} = f$ Nima
  can't reverse time
- $m\dddot{x} = f$ Nima'
  can't reverse time
- $m\ddddot{x} = f(x, \dot{x}, \ddot{x}, \dddot{x}, t)$
  v a Jerk

Applied to mass-spring Analog



$m\ddot{x} + kx = 0$

$\omega^2 = \frac{k}{m}$

only 2 real solution

$m\ddot{x} = f = -kx$

$m\ddddot{x} + \tilde{k}x = 0$

$\omega^4 = -\frac{k}{m}$

imaginary solution

$x = \sin(\omega t)$
$\dot{x} = \omega\cos(\omega t)$
$\ddot{x} = -\omega^2\sin(\omega t)$
$\dddot{x} = -\omega^3\cos(\omega t)$
$\ddddot{x} = \omega^4\sin(\omega t)$

$\omega^6 = \frac{k}{m}$

2 real + 4 complex solutions ready to blow up

Why is energy conserved? Counterfactual 3

1 Dimensional $F = -\frac{dV}{dx}$ $\quad m\ddot{x} = -\frac{dV}{dx}$ ~~trick~~

$\dot{x}\left[m\ddot{x} + \frac{dV}{dx}\right] = 0$ Trick Gauranteed

$\frac{d}{dt}\frac{1}{2}m\dot{x}^2 + V(x) = 0$ Trick can keep expanding

going from 1 Dimension to 2

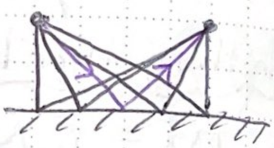
$m\ddot{x} = F_x \quad m\ddot{y} = F_y \quad E = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) + V(x,y)$

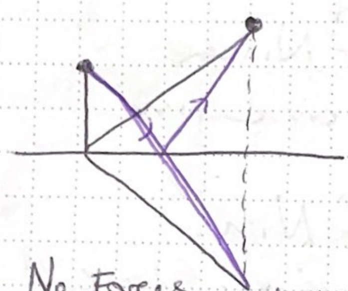
$$\boxed{F_x = \frac{\partial V}{\partial x} \qquad F_y = \frac{\partial V}{\partial x}}$$ not gauranteed

Principle of Least Action

Principle of Least Time: light travels in a path to minimize time.

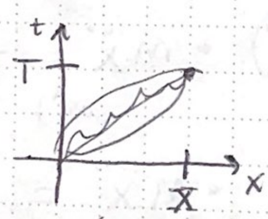
Ex Reflection: All paths possible



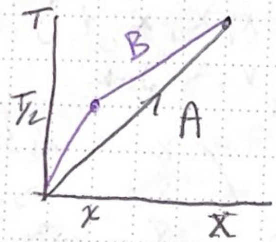


snell's law
impacts speed changes angles.

No Forces 1-D



$V = \frac{X}{T}$

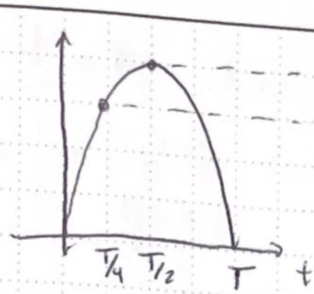


$A = \frac{1}{2}m\left(\frac{X}{T}\right)^2 T = \frac{1}{2}m\frac{X^2}{1}$

$B = \frac{1}{2}m\left(\frac{T}{2}\right)\left(\frac{x}{T/2}\right)^2 + \frac{1}{2}m\frac{T}{2}\left(\frac{X-x}{T/2}\right)^2$

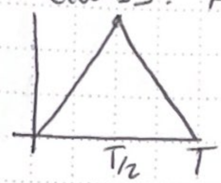
Minimum Path B: $4x - 4(X-x) = 0 \qquad x = \frac{X}{2}$

projectile



why a parabola?

Guess: Average KE − Ave PE



$$\text{Av. KE} = \frac{1}{2} m \left(\frac{H}{T/2}\right)^2 \frac{T}{2} \times 2 = \frac{2mH^2}{T}$$

$$\text{Av PE} = mg \tfrac{1}{2} T H$$

$$\frac{2mH^2}{T} - \frac{mgHT}{2}$$

Minimized when $\frac{d}{dH} \% = 0$

$$\frac{4mH}{T} - \frac{1}{2} mgT = 0 \qquad H = \frac{1}{8} g T^2$$

If you put a particle on a potential, Average Kinetic − Average Potential Energy which will then follow Newton's Laws of Motion

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Classical: World is deterministic

Quantum Mechanical: World is not deterministic, Amplitude of outcome.

Feynman $\sum_{\text{paths}} e^{i \frac{S_{\text{path}}}{\hbar}}$ Action $S = \int dt \left[\frac{1}{2} m \dot{x}^2 - V(x)\right]$

$\therefore$

classical trajectory path dominates because the other paths cancel out

$\hbar \approx T \times \text{Energy}$

when $S_{\text{Typical}} \gg \hbar$

Action is Big Classical

Action is Small Quantum Mechanics
comparable to $\hbar$