Quantum Mechanics And Its Consequences

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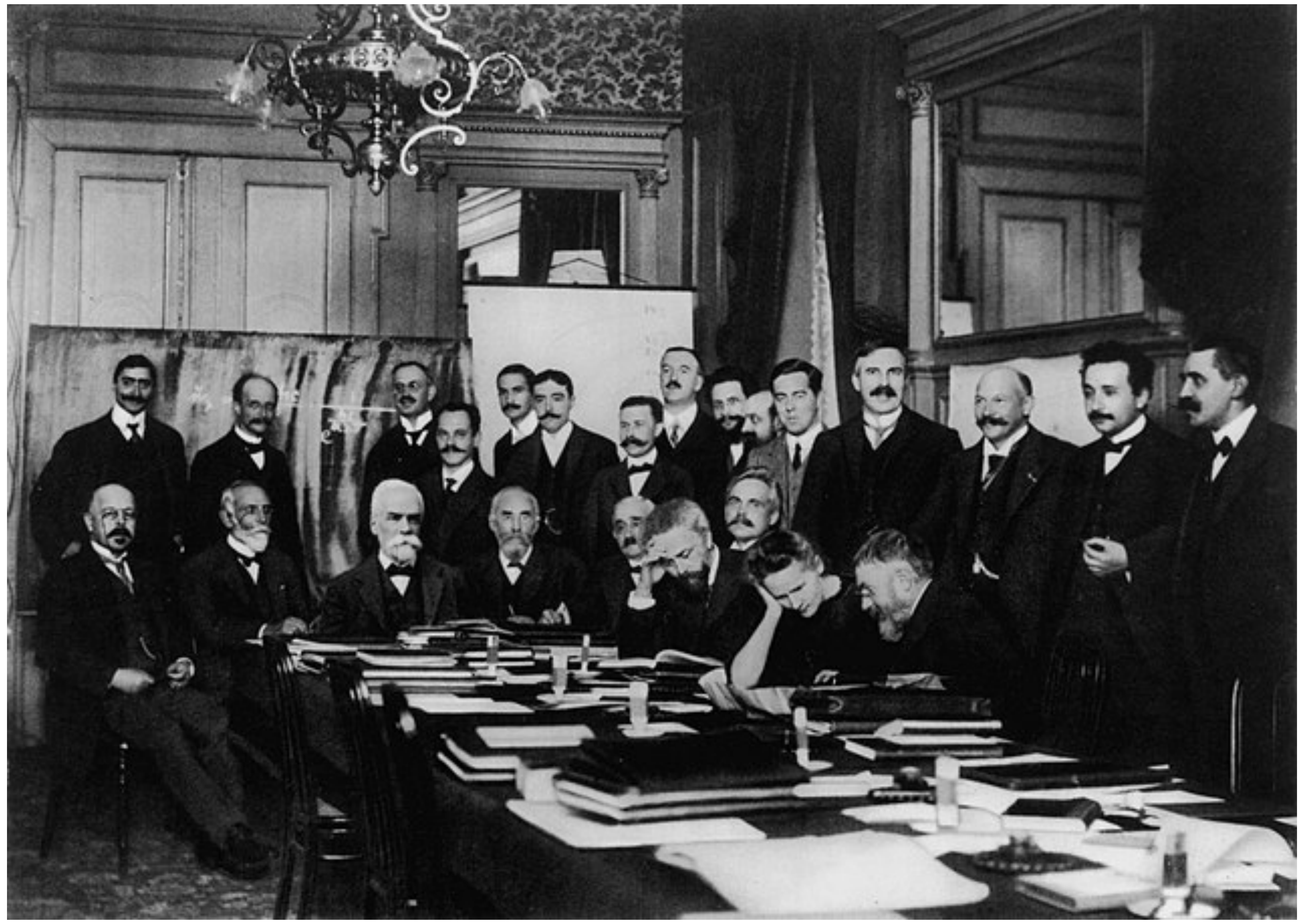
Quantum Mechanics



Quantum Mechanics

Theory built on observations in the 1900s
Why is it the way it is?

Postulates

Two Postulates of Quantum Mechanics

Probability

Linearity

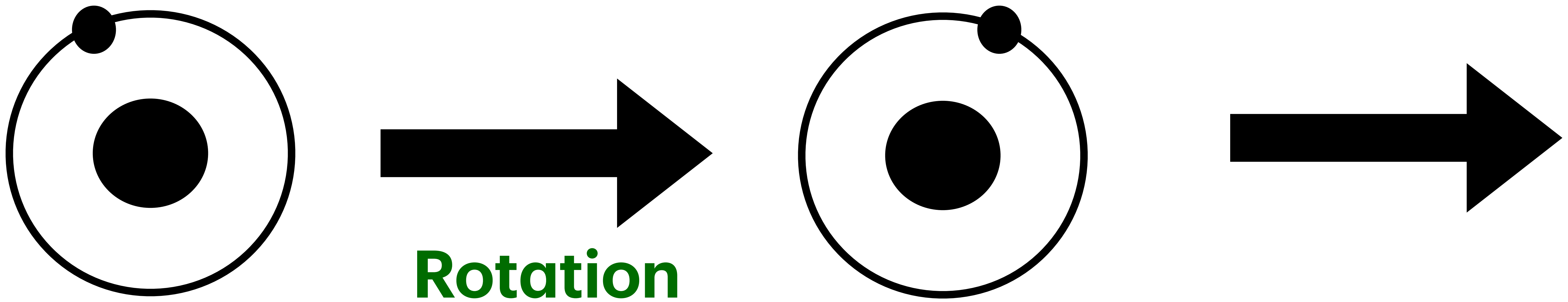
Probability

Finite system has a finite set of energies

Continuous observables and symmetries

} Deterministic Observables?

Could an electron in an atom have a well defined position?



Infinite Degeneracy

Quantum Mechanics

Sacrifice Determinism.

Preserve finite set of energy states, continuous symmetries and observables

Bell Inequalities, Kochen-Specker, SSC Theorems

The Postulates of Quantum Mechanics

Quantum State: $|\Psi>$

Time Evolution: Schrodinger Equation $i\frac{d|\Psi\rangle}{dt} = H|\Psi\rangle$

Time Evolution Operator $U = e^{-iHt}$

H: Hermitean Hamiltonian => U is unitary. Reversible.

Linear Time Evolution

$$|\Psi\rangle = \sum_k c_k|\Psi_k\rangle \text{ allowed solution}$$

No Restriction on physical size of $|\Psi_k>$

Could be macroscopic object

The "Problems" of Schrodinger

$$i\frac{d|\Psi\rangle}{dt} = H|\Psi\rangle$$

Time evolution is deterministic - no probability

Unitary time evolution is reversible

BUT: We see probabilities when we measure! Cannot Reverse Measurement

$$|\Psi\rangle = \sum_k c_k|\Psi_k\rangle \text{ allowed solution}$$

Macroscopic $|\Psi_k>$

Why do we not "see" macroscopic superpositions?

Invoke Additional Postulates

The Measurement Postulate

$|\Psi>$: Quantum State. Undergoes unitary evolution $$i\frac{d|\Psi\rangle}{dt} = H|\Psi\rangle$$

But then, "measurement" of some operator O occurs

"Stop" Hamiltonian Evolution

$$|\Psi\rangle = \sum_k c_k|f_k\rangle \quad O|f_k\rangle = \lambda_k|f_k\rangle$$

Wavefunction "collapses" to an eigenstate $|f_k>$ of O

Born Rule: Probability to obtain λ_k is $|c_k|^2$

Subsequent time evolution is with $|f_k>$ - forget $|\Psi>$ completely

No macroscopic superpositions because macroscopic system is always "measured"

No doubt that this is what we experience

The "Measurement Problem"
What is "Measurement"?

$|\Psi\rangle$: Quantum State.   Undergoes unitary evolution $i\frac{d|\Psi\rangle}{dt} = H|\Psi\rangle$

But then, "measurement" stops the Schrödinger Equation!

"Picks" random outcome and some eigenstate - resets norm of quantum state and then time evolve new state!

Why are electrons and protons in a detector doing a "measurement" suddenly acting differently i.e. not undergoing unitary evolution?

How is the wavefunction "collapsing" and getting you the outcome λ_k?

What determines the size of a macroscopic body that somehow cannot be in superposition?

Shut Up and Calculate

Only Axiom of Quantum Mechanics: Schrodinger Equation

Quantum State: |Ψ>

Time Evolution: $$i\frac{d|\Psi\rangle}{dt} = H|\Psi\rangle$$

No “Measurement” Postulates.

From Deterministic equation, derive the phenomenology of measurement

(1) Probabilistic Outcomes

(2) Wavefunction “Collapse”

(3) Do not “see” macroscopic superpositions

(4) Born Rule

Ingredients

Claim: Measurement is simply interactions between various systems

(e.g. quantum system and measuring device)

All of these systems are made of atoms - so they all obey quantum mechanics

Time evolution given by Schrodinger.

What kinds of interactions are there in the Hamiltonian?

$$i\frac{d|\Psi\rangle}{dt} = H|\Psi\rangle$$

Requirement: Want causality.

Hamiltonians are local!

$$H \supset \bar{\Psi}(x)\,\Psi(x)\,\phi(x)$$

$$H \not\supset \bar{\Psi}(x_1)\,\Psi(x_2)\,\phi(x_3)$$

Inner Products of Macroscopic Bodies

Suppose B is a macroscopic object - that is B is composed of a large number (N) of particles

The states $|\Psi\rangle$ are product states of a large number of individual particle states

$$|\Psi\rangle = |\Psi_1\rangle \otimes \ldots |\Psi_N\rangle$$

Key Point

For any local operator H, super easy for the inner products $\langle\Phi|H|\Psi\rangle = 0$

This is true even under time evolution of $|\Phi\rangle$ and $|\Psi\rangle$

Relevant for what makes a “detector”

Why is this true?

Inner Products of Macroscopic Objects

Super easy for the inner products <Φ|H|Ψ> = 0

$$H = \sum_i O_i$$

$$|\Psi\rangle = |\Psi_1\rangle \otimes \ldots |\Psi_N\rangle \qquad |\Phi\rangle = |\Phi_1\rangle \otimes \ldots |\Phi_N\rangle$$

$$\langle\Phi|H|\Psi\rangle = \sum_i \left(\Pi_{i\neq j}\langle\Phi_j|\Psi_j\rangle\right)\langle\Phi_i|O_i|\Psi_i\rangle$$

$$\approxeq \sum_i \langle\Phi_j|\Psi_j\rangle^{N-1}\langle\Phi_i|O_i|\Psi_i\rangle$$

For large N, easily zero!

Inner Products of Macroscopic Objects

Super easy for the inner products $\langle\Phi|H|\Psi\rangle = 0$

$$H = \sum_i O_i$$

$$|\Psi\rangle = |\Psi_1\rangle \otimes \ldots |\Psi_N\rangle \qquad |\Phi\rangle = |\Phi_1\rangle \otimes \ldots |\Phi_N\rangle$$

$$\langle\Phi|H|\Psi\rangle = \sum_i \left(\Pi_{i \neq j} \langle\Phi_j|\Psi_j\rangle\right) \langle\Phi_i|O_i|\Psi_i\rangle$$

Also zero if $\langle\Phi_j|\Psi_j\rangle = 0$ for some j

Super easy for this to happen - gas molecule scatters off me!

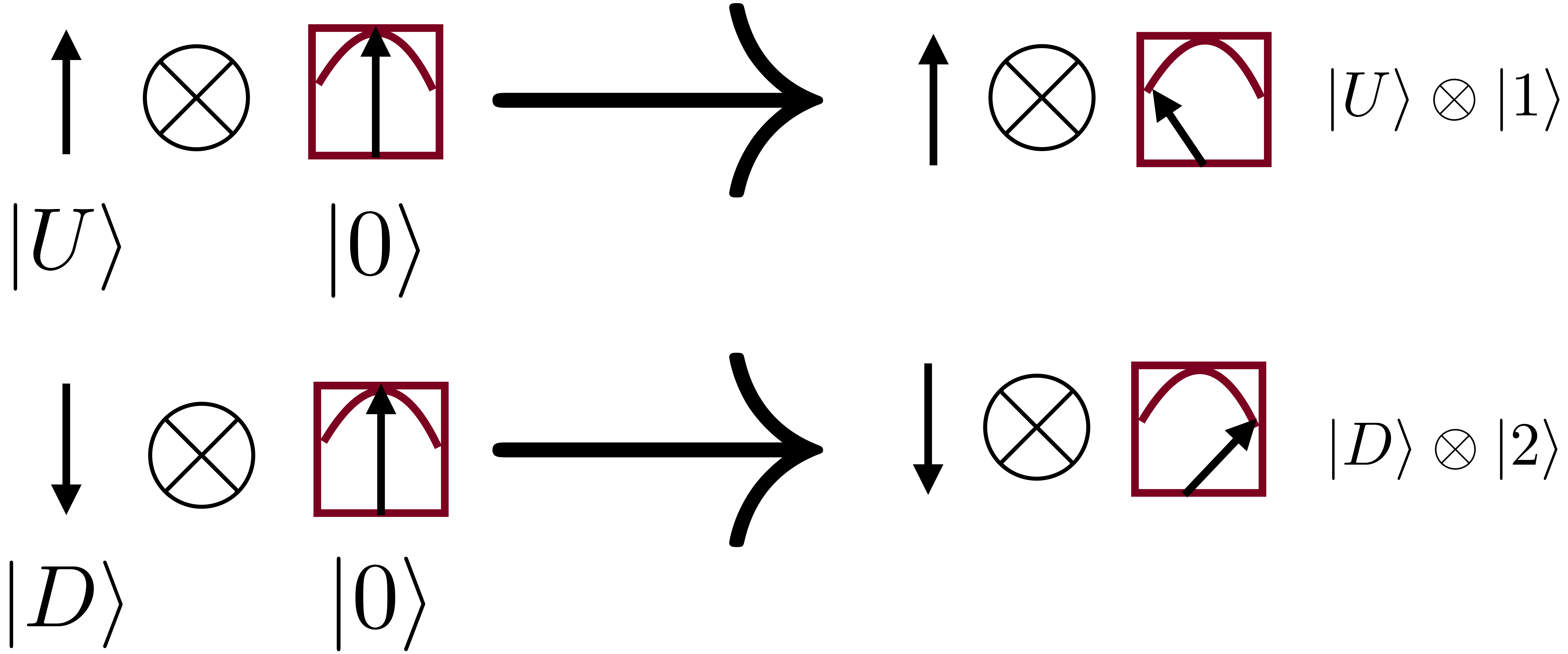
What happens when I do a measurement?

How does this Human interact with a spin?

Toy Model of Human's Brain

Brain contains say M registers to deal with spins. Initially, all registers are in quantum state |o>

Interaction Hamiltonian between spin and Human: Register state changes



$$\langle 1|2\rangle = 0 \quad \langle 1|H|2\rangle = 0$$

What happens when I do a measurement?

Instead of one spin, give the human N spins all in state |U>

$$|UU\dots U\rangle|00\dots 0\rangle \rightarrow |UU\dots U\rangle|11\dots 1\rangle$$

In Addition, human has a “Predictivity" circuit C_k

This register looks at k-1 spins and asks if it can predict the next one

If it can, it says |NP> (i.e. non probabilistic)
If not, it says |P> (i.e. probabilistic)

$$C_k|UU\dots U\rangle|11\dots 1\rangle \rightarrow |UU\dots U\rangle|11\dots 1\rangle|NP\rangle$$

No probability here - “eigenstate of operator being measured”

Probabilistic Outcomes

Now send α|U> + β|D>

Per the Hamiltonian that describes the interaction:

$$(\alpha|U\rangle + \beta|D\rangle) \otimes |0\rangle \rightarrow \alpha|U\rangle|1\rangle + \beta|D\rangle|2\rangle$$

Prediction of Linear Quantum Mechanics

NOT |U>|1> OR |D>|2>. But the entangled superposition.

$$(\alpha|U\rangle + \beta|D\rangle) \otimes |0\rangle$$

Nice factorized interpretation

$$\alpha|U\rangle|1\rangle + \beta|D\rangle|2\rangle$$

Entangled State - no factorized interpretation. Correlated States

“Collapse” of the Wave Function

Measurement has created entanglement between system and macroscopic human

$$(\alpha|U\rangle + \beta|D\rangle) \otimes |0\rangle \rightarrow \alpha|U\rangle|1\rangle + \beta|D\rangle|2\rangle$$

|1> and |2> are orthogonal states of the macroscopic detector

For macroscopic detectors, we have seen that <1(t)|H|2(t)> = 0

Immediate Consequence

Want to track subsequent evolution of |U>|1>? We can totally ignore the existence of |D>|2>

$$|\,(\alpha|U\rangle + \beta|D\rangle)\,|0\rangle \rightarrow \alpha|U\rangle|1\rangle + \beta|D\rangle|2\rangle \rightarrow \alpha|U\rangle|\text{SR plays cricket}\rangle + \beta|D\rangle|\text{SR eats Indian Food}\rangle$$

Only way for this NOT to be true is if <1(t)|H|2(t)> != 0

If we want to track |U>|1>, can say wavefunction “collapsed” to |U>|1>

Probabilistic Outcomes

Send $\alpha|U\rangle + \beta|D\rangle$ again

$$(\alpha|U\rangle + \beta|D\rangle) \otimes (\alpha|U\rangle|10\rangle + \beta|D\rangle|20\rangle) \rightarrow$$

$$\alpha^2|UU\rangle|11\rangle + \alpha\beta|DU\rangle|21\rangle + \beta\alpha|UD\rangle|12\rangle + \beta^2|DD\rangle|22\rangle$$

Now think about the predictivity circuit $|C_2\rangle$ - this bit looks at the first bit in the spin register after the first interaction and tries to guess what the next bit would be

What does this look like?

$$C_2|UD\rangle|12\rangle \rightarrow |UD\rangle|12\rangle|P\rangle$$

For some large number of spins

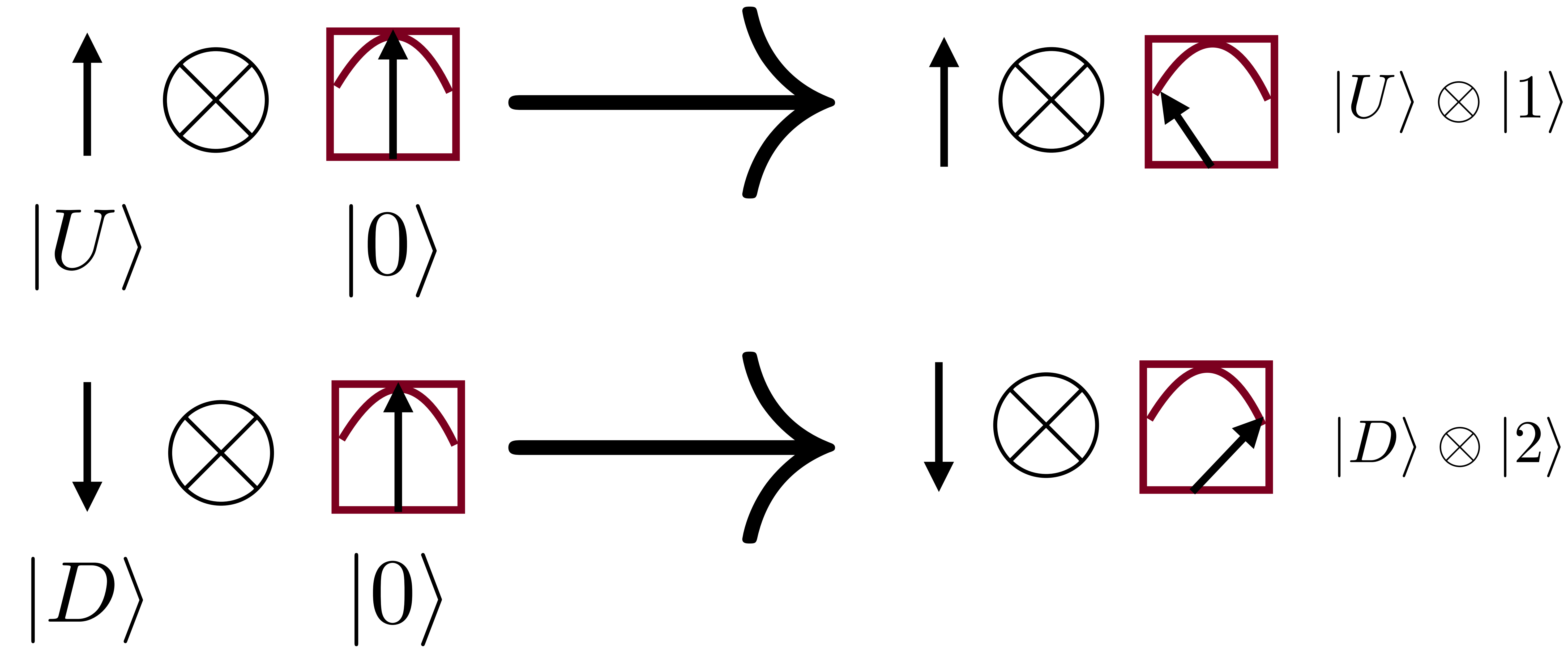
$$C_k|UDUUDU\ldots\rangle|121121\ldots\rangle \rightarrow |UDUUDU\ldots\rangle|121121\ldots\rangle|P\rangle$$

Why can't we "see" Macroscopic Superpositions?

$$|\Psi\rangle = \sum_k c_k |\Psi_k\rangle \text{ allowed solution}$$

Why can't we see it?

For spins, given states |U> and |D>, we could find an interacting Hamiltonian which permits:



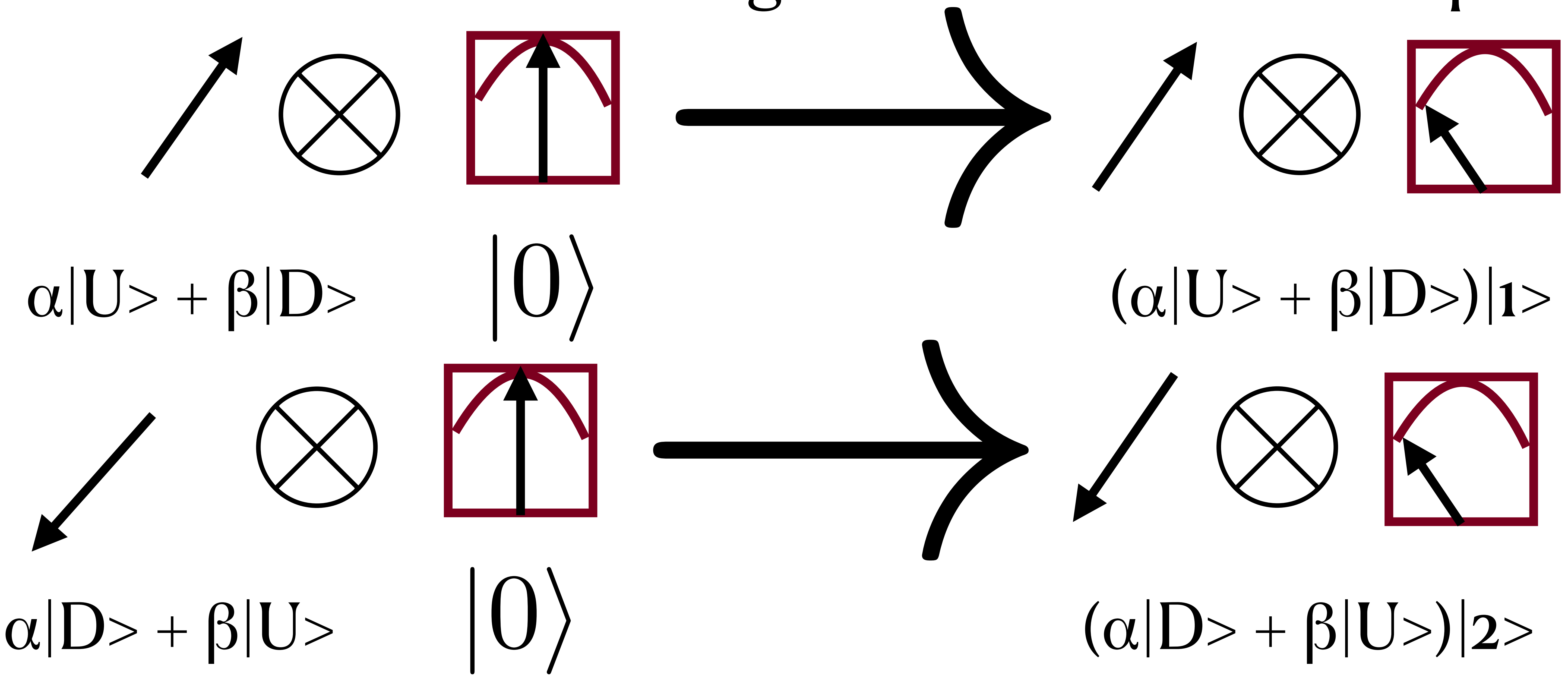
That is, given the state |U> or |D>, we "see" them because our registers suitably change

Can I "see" the state α|U> + β|D>? What does that mean?

Why can't we "see" Macroscopic Superpositions?

Can I "see" the state α|U> + β|D>? What does that mean?

Want to find an interacting Hamiltonian which maps:



If such a Hamiltonian existed, we would have "seen" the spin in the state α|U> + β|D>

For spins, this is easy - all you need to do, for example, is to rotate some magnetic field and align it along the direction of the spin and you can create this Hamiltonian

Why can't we "see" Macroscopic Superpositions?

What about superpositions of macroscopic states, each localized at different points in space?

$$|\Psi\rangle = \sum_k c_k |\Psi_k\rangle$$

To see this state, we need a Hamiltonian which yields:

$$\left(\sum_k c_k |\Psi_k\rangle\right) \otimes |0\rangle \to \left(\sum_k c_k |\Psi_k\rangle\right) \otimes |1\rangle \qquad \left(\sum_k d_k |\Psi_k\rangle\right) \otimes |0\rangle \to \left(\sum_k d_k |\Psi_k\rangle\right) \otimes |2\rangle$$

If such a Hamiltonian existed, we would "see" the superposition

But: world only has local Hamiltonians!

Cannot create required Hamiltonian that needs to act on multiple points in space at once

Unlike the case of spin - local object!

The Born Rule

Given the state α |U> + β |D>, if we make measurements on N copies of this state, the probability of getting the outcome |U> is $|\alpha|^2$ and probability of getting |D> is $|\beta|^2$

When the spins interact with the detector, we realize all outcomes - not just one or the other

$$(\alpha|U\rangle + \beta|D\rangle)^N \otimes |000...0\rangle \to \sum_{k=0}^{N} C\left(N,k\right)\alpha^k \beta^{N-k} |U\rangle^k |D\rangle^{N-k} |1\rangle^k |2\rangle^{N-k}$$

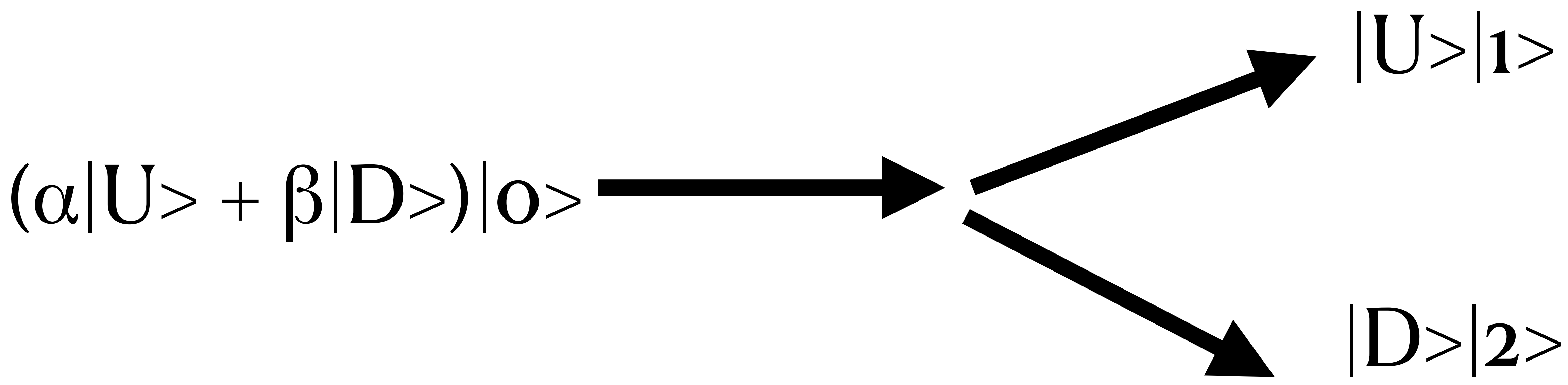
Thus, even if a != 1, the interaction will create the state |UU...U>|11...1> where the human sees all the spins being up

Probability is a counterfactual question: given some outcome (i.e. what the human sees), we ask if that outcome was unlikely or it is what was expected

How can we quickly figure out the statistics of this distribution?

The Born Rule

Stern Gerlach

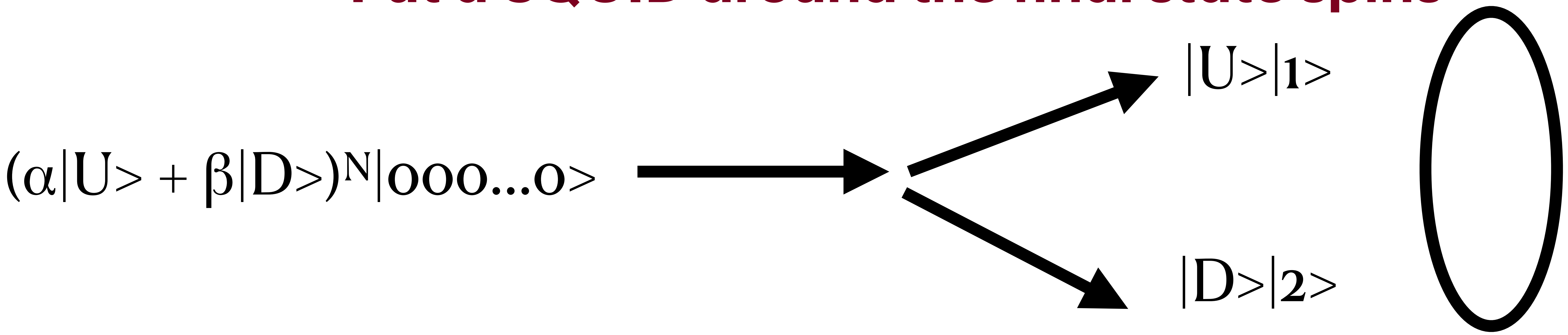


$|0\rangle$, $|1\rangle$, $|2\rangle$ refer to position of spin

Now Send N spins

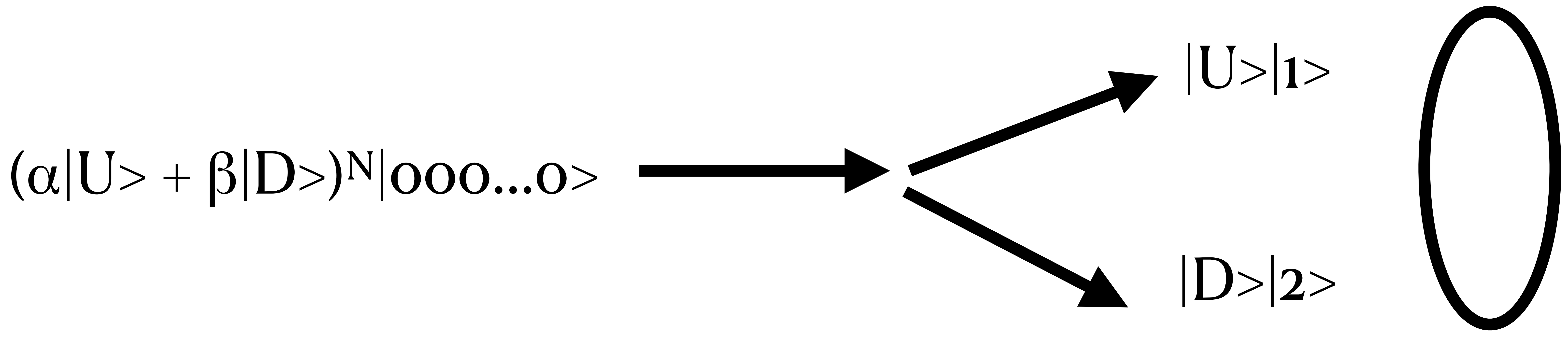
$$(\alpha|U\rangle + \beta|D\rangle)^N \otimes |000...0\rangle \to \sum_{k=0}^{N} C\,(N,k)\,\alpha^k \beta^{N-k} |U\rangle^k |D\rangle^{N-k} |1\rangle^k |2\rangle^{N-k}$$

Put a SQUID around the final state spins



SQUID responds to coherent electromagnetic field sourced by spins

The Born Rule



SQUID responds to coherent electromagnetic field sourced by spins

In the limit of large N, the electromagnetic field sourced by these spins is a coherent state

This state is fully characterized by its expectation value

$$\langle \Psi | A | \Psi \rangle = \sum_{j=0}^{\infty} C\left(N, k\right) |\alpha|^{2k} |\beta|^{2(N-k)} \left(F\left(|U\rangle^{k}, |D\rangle^{N-k}\right)\right)$$

Binomial distribution

Conclusions

Only Axiom of Quantum Mechanics: Schrodinger Equation

No "Measurement" Postulates.

Require local Hamiltonians. Accept that everything is made of atoms, subject to superposition. Concrete toy models for measurement.

Shut up and Calculate.

(1) Probabilistic Outcomes

(2) Wavefunction "Collapse"

(3) Do not "see" macroscopic superpositions

(4) Born Rule