Quantum Physics

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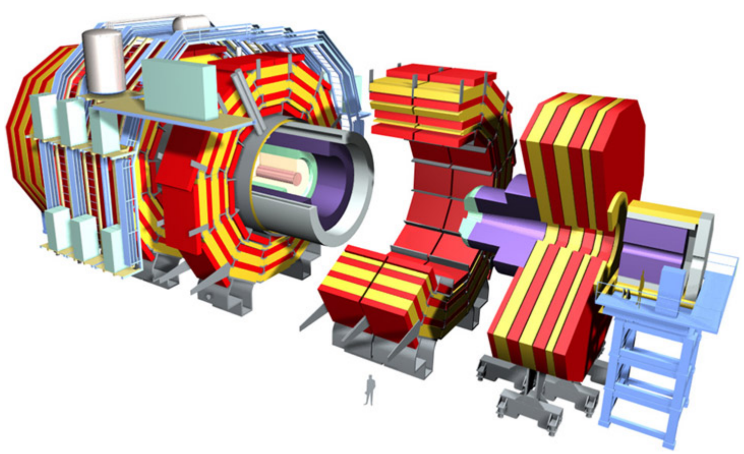
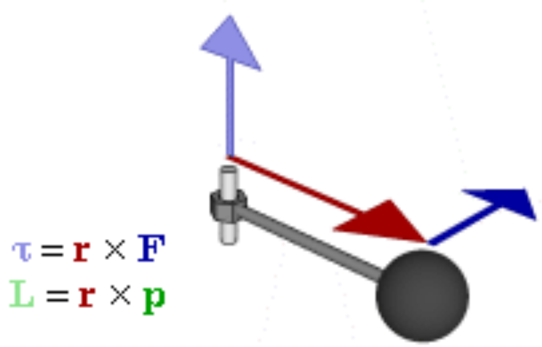


July 25, 2022

Johns Hopkins University

Johns Hopkins University QuarkNet Physics Workshop

Quantum Physics: Spin and Magnetic Moment



- Spin = Intrinsic angular momentum of a particle (system)

Classically: $\vec{L} = \vec{r} \times \vec{p}$ $\qquad L = mrv$

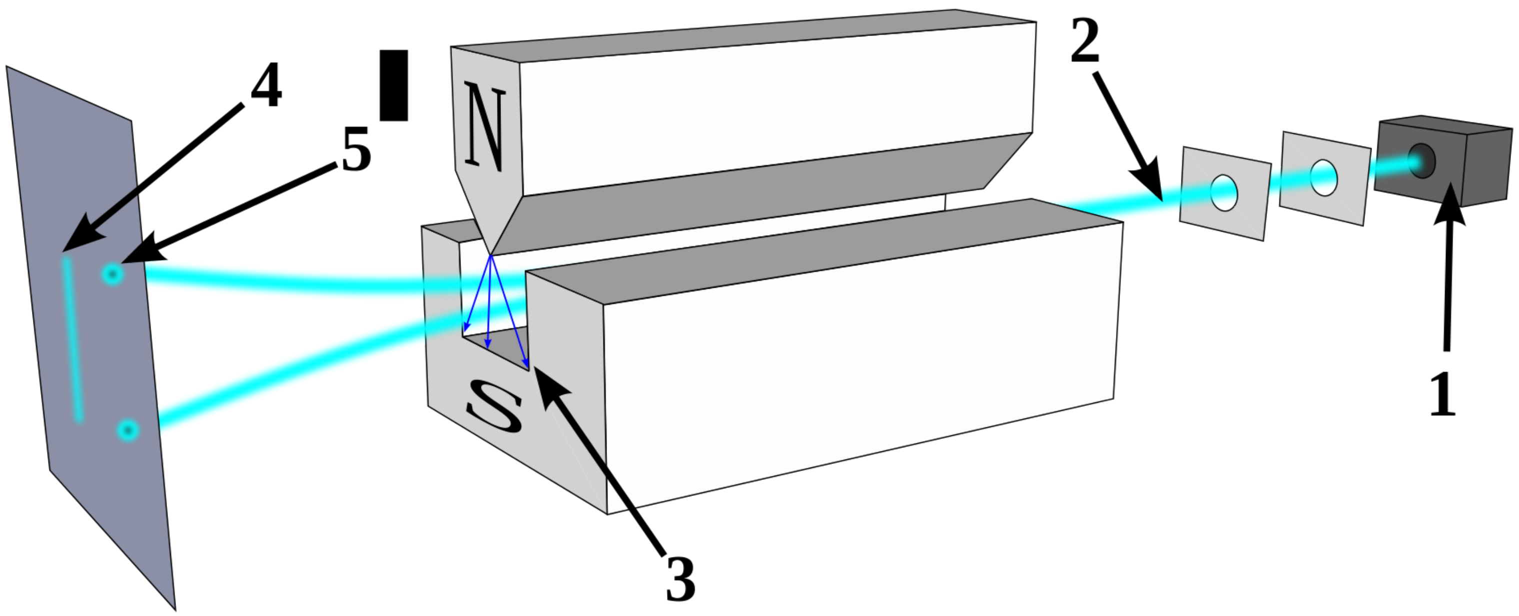
- Magnetic moment = current (I) x loop areas (A)

$$\vec{\mu} = I \times \vec{A} \qquad \mu = \frac{qv}{2\pi r}\pi r^2 = qrv/2$$

$$\mu = g \times \frac{q}{2m} S \qquad g \neq 1 \text{ in QM}$$

Quantum Physics: Stern–Gerlach experiment

1922 (100 years!)



Atom, outer electron interaction energy: $E = -\vec{\mu} \cdot \vec{B}$

$$F_z = \frac{\partial}{\partial z}(\vec{\mu} \cdot \vec{B}) = \mu_z \frac{\partial B_z}{\partial z}$$

$$\mu_z = g \times \frac{q}{2m} S_z \quad \Rightarrow \quad \boxed{S_z = \pm \frac{\hbar}{2}}$$ electron

Quantum Physics: Spin of Electron

electron $$S_z = \pm \frac{\hbar}{2}$$ quantization!

Planck's constant $$\hbar = \frac{h}{2\pi} = 6.5821 \times 10^{-16}\ \mathrm{eV} \cdot s$$

electron's spin

$$S = \frac{\hbar}{2}$$

spin projection on axis z

$$S_z = \pm \frac{\hbar}{2}$$

Foundation of Quantum Physics!

Spin of Elementary Particles

- Until recently, all elementary particles were of two types:

$$S = \frac{\hbar}{2}$$

Fermions (half-integer spin)
occupy space (Fermi statistics: exclusion princ.)
constitute matter (quarks, leptons)

$$S = 1\hbar$$

Bosons (integer spin)
carry interactions (γ photons, g gluons, $W^{\pm}, Z$)

- One can create compose particles of any spin $S = \frac{N\hbar}{2}$, $N = 0,1,2,..$

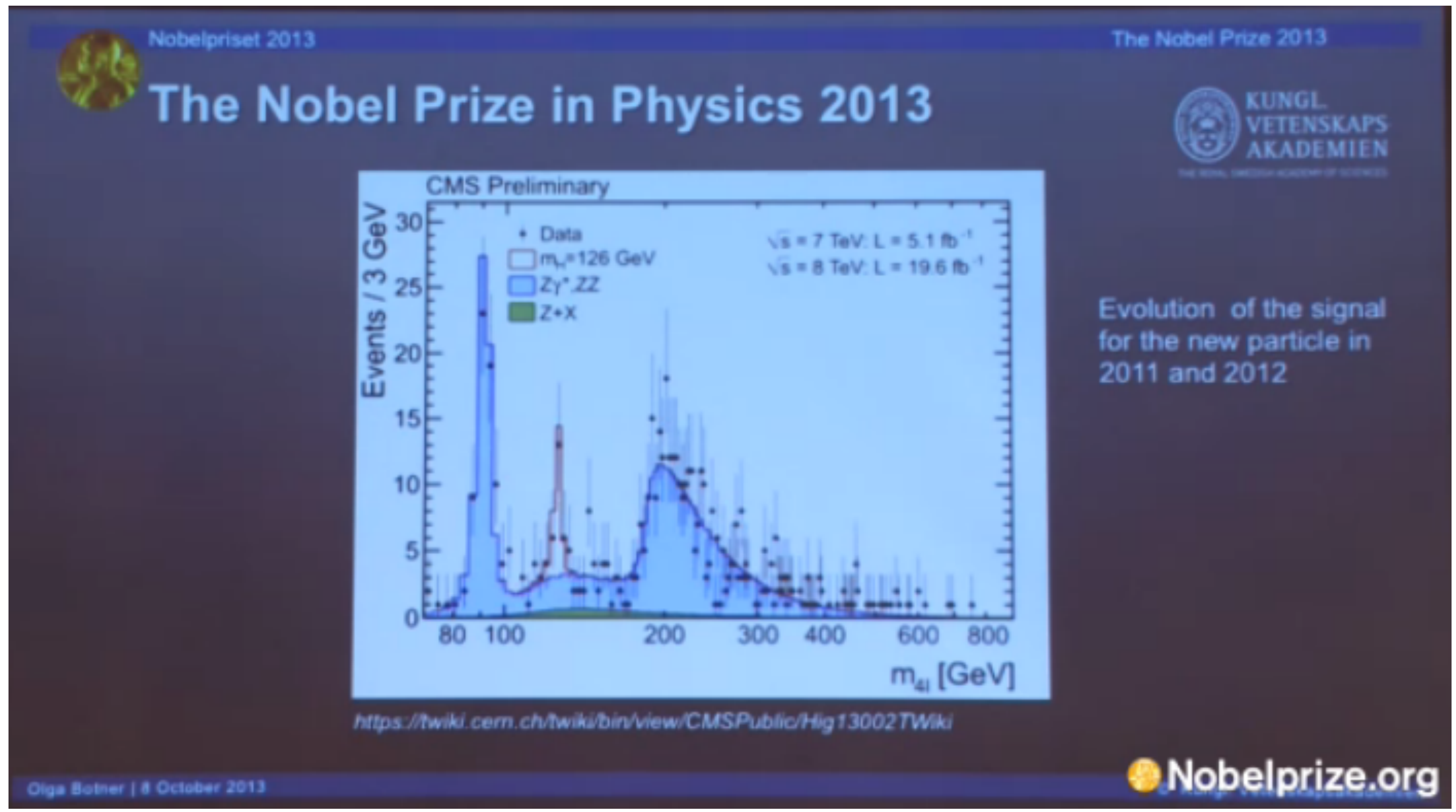
for example π^0 meson made of $q\bar{q}$ has $S = 0$

but there was no elementary particle with no spin, until 2012…

Spin of the Higgs boson?

- Spin = 0

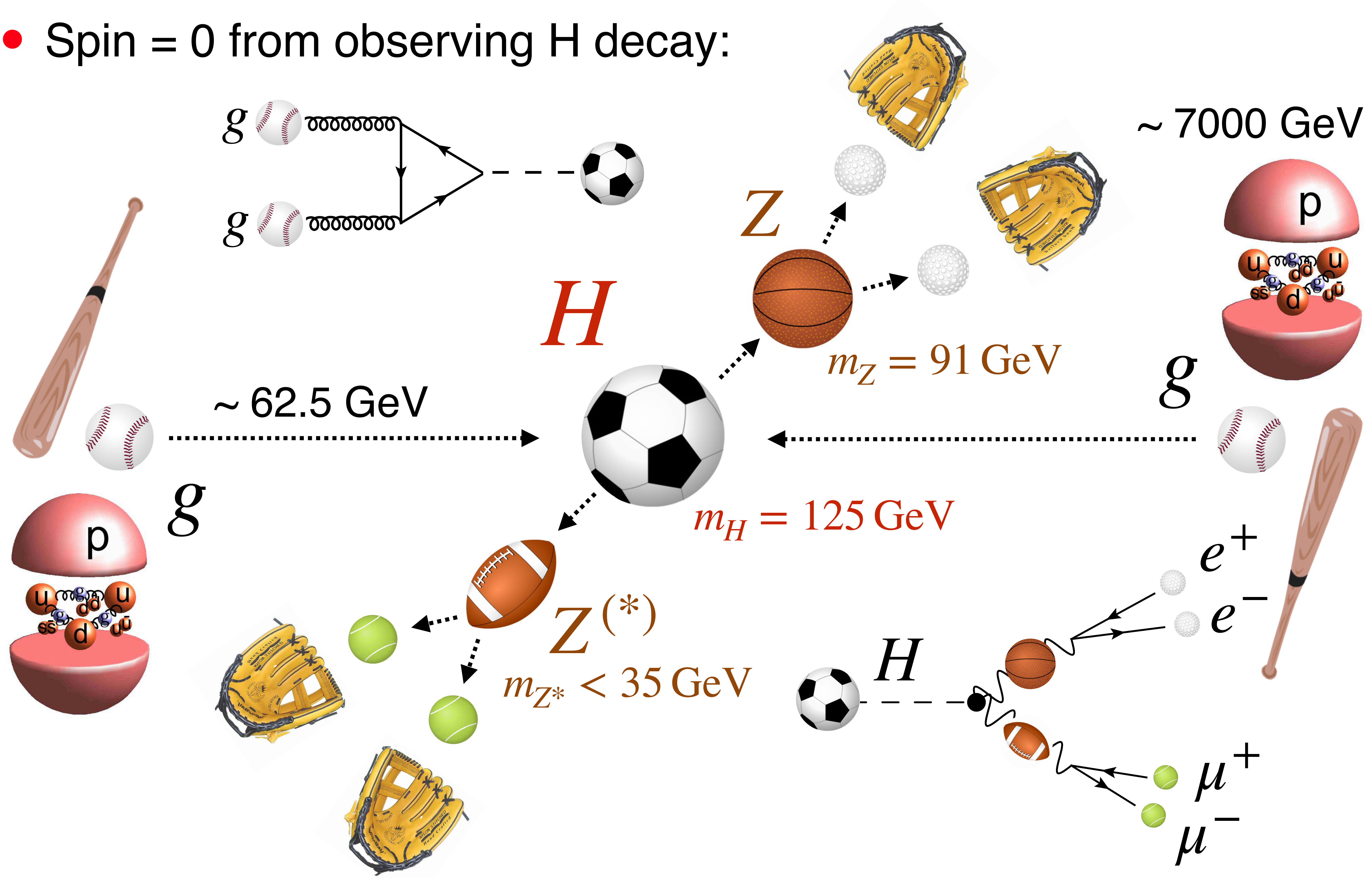
2012 (10 years!)



- The only known elementary particle with no spin !
  - — how do we know it has no spin ?

Spin of the Higgs boson?

- Spin = 0 from observing H decay:



Spin of elementary particles

- Spin = 0 H boson (excitation of the vacuum field)
- Spin = $\frac{\hbar}{2}$ $e^{\pm}, \mu^{\pm}, \tau^{\pm}, \nu_e, \nu_\mu, \nu_\tau$, quarks… matter
- Spin = $\hbar$ $\gamma, Z, W^+, W^-, g_1, g_2, g_3, g_4, g_5, g_6, g_7, g_8$ interactions
- Spin = $\frac{3\hbar}{2}$ Not known
  (may be supersymmetric particle, e.g. gravitino)
- Spin = $2\hbar$ Not discovered, expect graviton
- Arguments for higher Spin to be composite particles…

Two events in July

- July 4, 2022 Symposium at CERN to celebrate 10 years of H boson

  — local JHU [article](#) on the topic

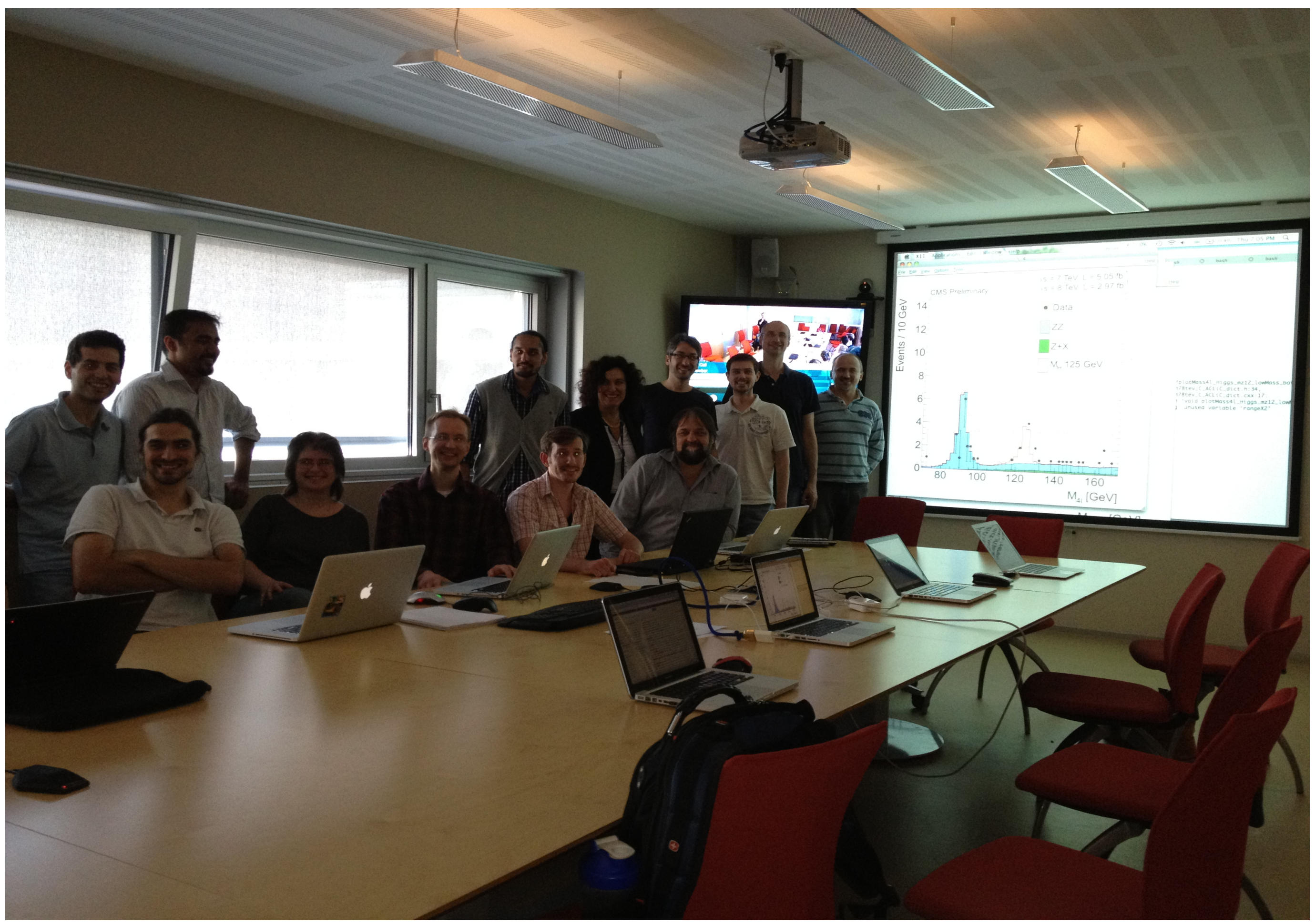
CERN 10 years HIGGS boson discovery

Two events in July

- July 4, 2022 Symposium at CERN to celebrate 10 years of H boson

  — local JHU article on the topic

June 14, 2012, CERN



July 4, 2022, CERN

Two events in July

Two events in July

- July, 2022 Community Summer Study in Seattle (“Snowmass”)

- Big questions and big facilities

  — next Higgs factory ???

- Followup to Snowmass 2001

  Snowmass 2013…

Back to Quantum Physics: Time Evolution

Non-relativistic energy expression: $$E = \frac{\overrightarrow{p}^2}{2m} + V$$

Quantum prescription: $$E \quad \rightarrow \quad i\hbar\frac{\partial}{\partial t}$$

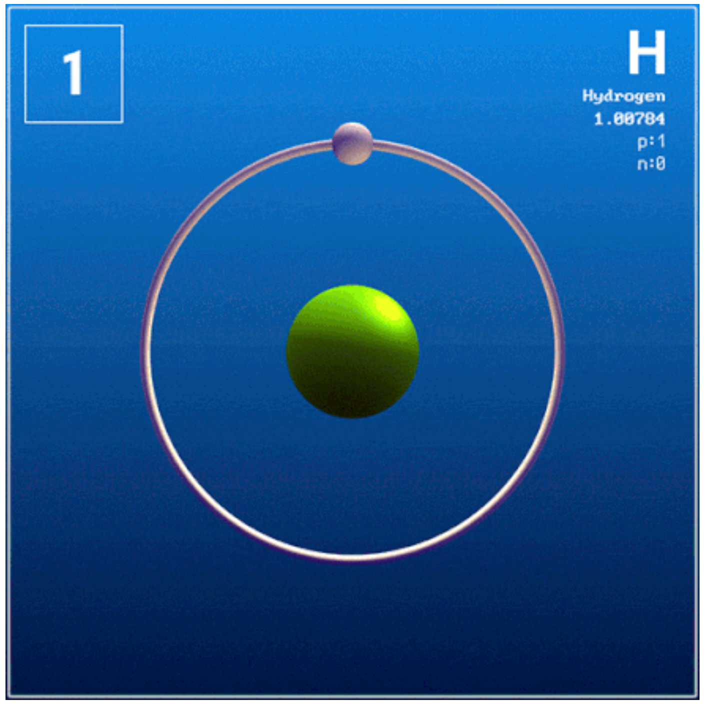
$$\overrightarrow{p} \quad \rightarrow \quad -i\hbar\overrightarrow{\nabla} = -i\hbar\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

Schrodinger equation, for a wave function $\psi(t, x, y, z)$

$$E\psi = \frac{\overrightarrow{p}^2}{2m}\psi + V\psi$$

$$i\hbar\frac{\partial}{\partial t}\psi = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi$$

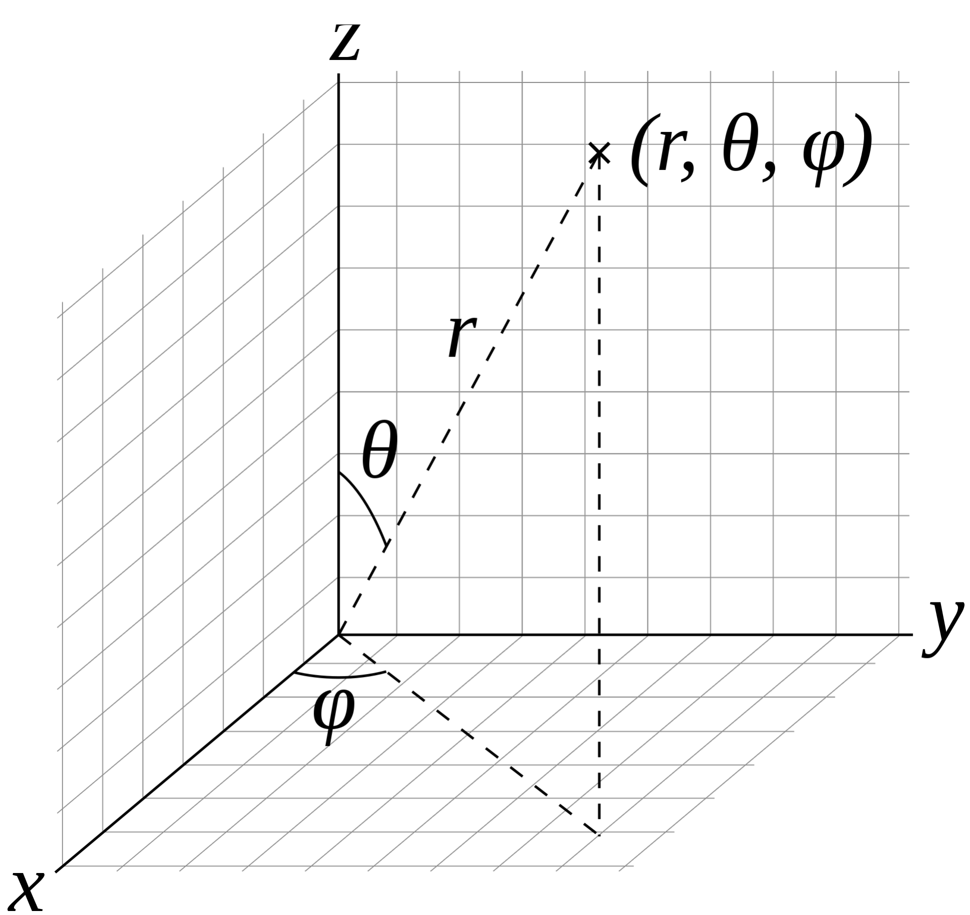
Quantum Physics: Hydrogen Atom



$$i\hbar\frac{\partial}{\partial t}\psi = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi$$

special case:

$$V(x, y, z, t) = V(r) = -\frac{e^2}{4\pi\epsilon_0 r}$$

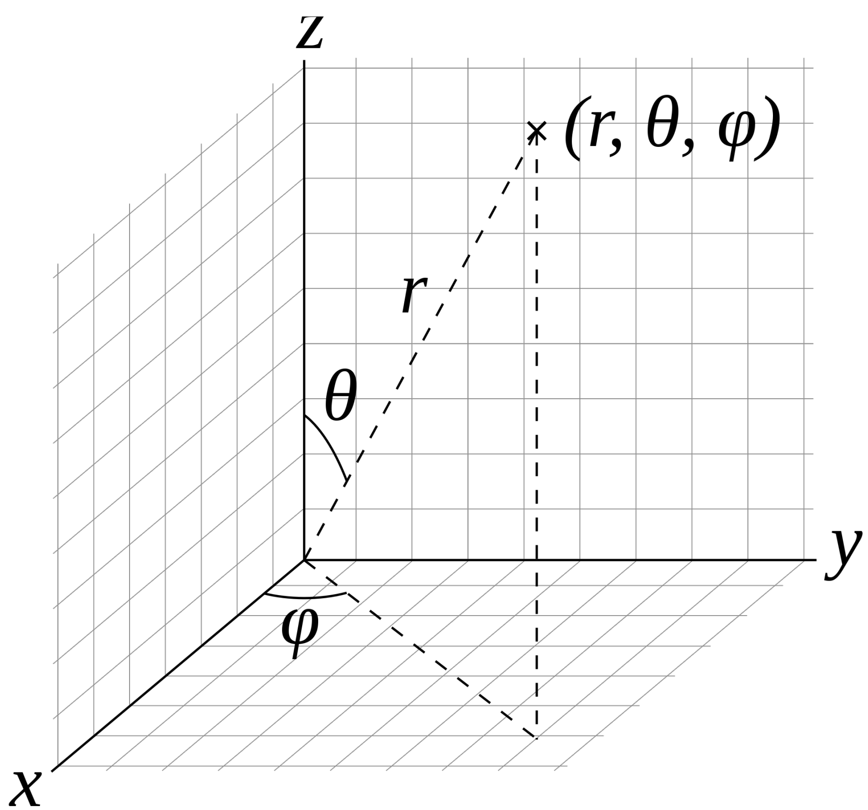


solve in spherical coordinates:

$$\left(-\frac{\hbar^2}{2\mu}\nabla^2 - \frac{e^2}{4\pi\varepsilon_0 r}\right)\psi(r,\theta,\varphi) = E\psi(r,\theta,\varphi)$$

Quantum Physics: Hydrogen Atom

$$-\frac{\hbar^2}{2\mu}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\varphi^2}\right]-\frac{e^2}{4\pi\varepsilon_0 r}\psi=E\psi$$



$$\psi(r,\theta,\varphi)=R(r)\,\Theta(\theta)\,\Phi(\varphi)$$

Quantum numbers: n, ℓ, m

$$\psi_{n,\ell,m}(r,\theta,\varphi)\propto R_{n,\ell}(r)\,Y_{\ell,m}(\theta,\varphi)$$

principal quantum number: $n = 1,2,3,...$

orbital angular momentum: $\ell = 0,1,2,3,... \; < n$

projection of angular momentum: $m = -\ell, (-\ell+1), ..,0,..,(\ell-1), \ell$

Quantum Physics: Hydrogen Atom

$$\psi_{n,\ell,m}(r,\theta,\varphi) \propto R_{n,\ell}(r)\, Y_{\ell,m}(\theta,\varphi)$$

$$|m| \le \ell = 0,1,2,3,\ldots \ < n$$

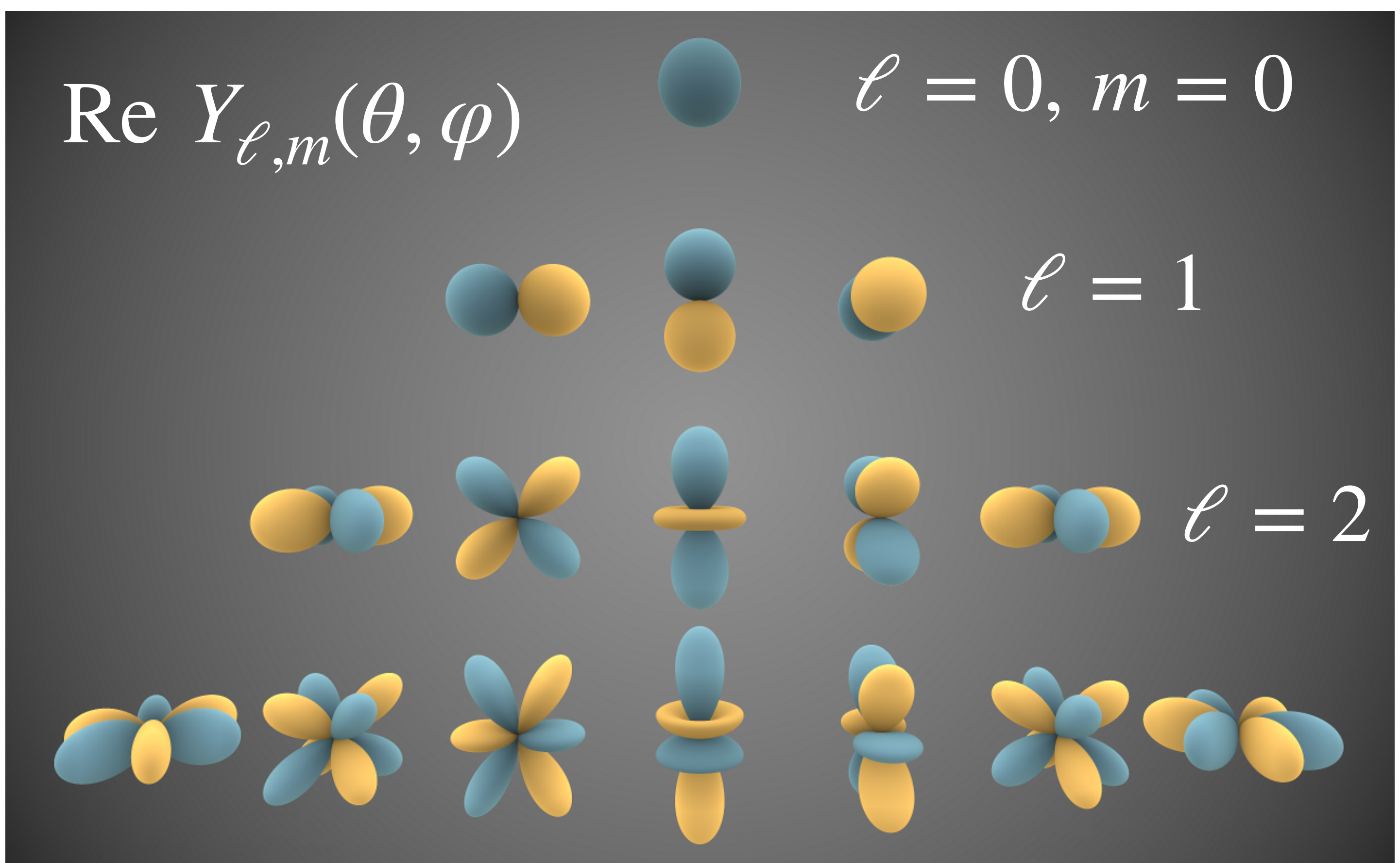
$$E_n = -\frac{\hbar^2}{2ma_0}\frac{1}{n^2}$$

$$n = 1,2,3,\ldots$$

Probability to find electron in (r,θ,φ)

$$|\psi_{n,\ell,m}(r,\theta,\varphi)|^2$$

ground state $R_{1,0}(r) \propto e^{-r/a_0}$



$$Y_0^0(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{1}{\pi}}$$

$$Y_1^{-1}(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{3}{2\pi}}\,\sin\theta\, e^{-i\varphi}$$

$$Y_1^0(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{3}{\pi}}\,\cos\theta$$

$$Y_1^1(\theta,\varphi) = \frac{-1}{2}\sqrt{\frac{3}{2\pi}}\,\sin\theta\, e^{i\varphi}$$

$$Y_2^{-2}(\theta,\varphi) = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\,\sin^2\theta\, e^{-2i\varphi}$$

$$Y_2^{-1}(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{15}{2\pi}}\,\sin\theta\,\cos\theta\, e^{-i\varphi}$$

$$Y_2^0(\theta,\varphi) = \frac{1}{4}\sqrt{\frac{5}{\pi}}\,(3\cos^2\theta - 1)$$

$$Y_2^1(\theta,\varphi) = \frac{-1}{2}\sqrt{\frac{15}{2\pi}}\,\sin\theta\,\cos\theta\, e^{i\varphi}$$

$$Y_2^2(\theta,\varphi) = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\,\sin^2\theta\, e^{2i\varphi}$$

Atomic Physics

Quantum Physics: Atoms

— Particles (electrons) occupy the lowest energy states

— No two identical particles (electrons) may have the same set of quantum numbers (n, ℓ, m, s_z)

(Pauli exclusion principle)

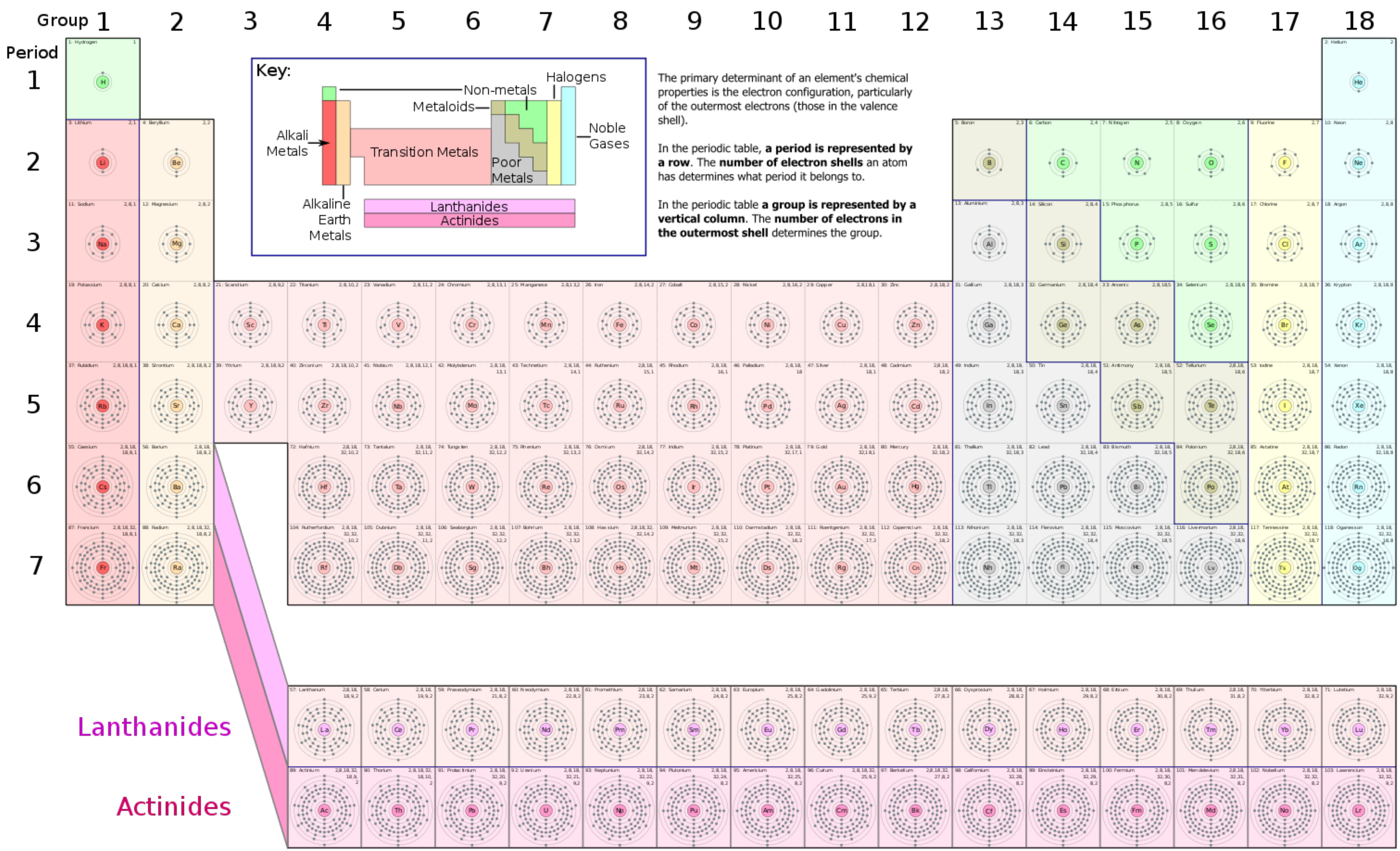
$$|m| \leq \ell = 0,1,2,3,... \; < n$$

$$s_z = \pm \frac{\hbar}{2}$$

Shell name	Subshell name	Subshell max electrons	Shell max electrons
K	1s	2	**2** ← H, He
L	2s	2	2 + 6 = **8**
	2p	6	
M	3s	2	2 + 6 + 10 = **18**
	3p	6	
	3d	10	
N	4s	2	2 + 6 + 10 + 14 = **32**
	4p	6	
	4d	10	
	4f	14	
O	5s	2	2 + 6 + 10 + 14 + 18 = **50**
	5p	6	
	5d	10	
	5f	14	
	5g	18	

Quantum Physics: Atoms

Periodic Table of Elements Showing Electron Shells



Atomic / Nuclear Physics

https://pdg.lbl.gov/2020/reviews/rpp2020-rev-periodic-table.pdf

PERIODIC TABLE OF THE ELEMENTS

1 IA	2 IIA	3 IIIB	4 IVB	5 VB	6 VIB	7 VIIB	8	9 VIII	10	11 IB	12 IIB	13 IIIA	14 IVA	15 VA	16 VIA	17 VIIA	18 VIIIA
1 H hydrogen 1.008																	2 He helium 4.002602
3 Li lithium 6.94	4 Be beryllium 9.012182											5 B boron 10.81	6 C carbon 12.0107	7 N nitrogen 14.007	8 O oxygen 15.999	9 F fluorine 18.998403163	10 Ne neon 20.1797
11 Na sodium 22.98976928	12 Mg magnesium 24.305											13 Al aluminum 26.9815385	14 Si silicon 28.085	15 P phosphorus 30.973761998	16 S sulfur 32.06	17 Cl chlorine 35.45	18 Ar argon 39.948
19 K potassium 39.0983	20 Ca calcium 40.078	21 Sc scandium 44.955908	22 Ti titanium 47.867	23 V vanadium 50.9415	24 Cr chromium 51.9961	25 Mn manganese 54.938044	26 Fe iron 55.845	27 Co cobalt 58.933195	28 Ni nickel 58.6934	29 Cu copper 63.546	30 Zn zinc 65.38	31 Ga gallium 69.723	32 Ge germanium 72.630	33 As arsenic 74.921595	34 Se selenium 78.971	35 Br bromine 79.904	36 Kr krypton 83.798
37 Rb rubidium 85.4678	38 Sr strontium 87.62	39 Y yttrium 88.90584	40 Zr zirconium 91.224	41 Nb niobium 92.90637	42 Mo molybdenum 95.95	43 Tc technetium (97.907212)	44 Ru ruthenium 101.07	45 Rh rhodium 102.90550	46 Pd palladium 106.42	47 Ag silver 107.8682	48 Cd cadmium 112.414	49 In indium 114.818	50 Sn tin 118.710	51 Sb antimony 121.760	52 Te tellurium 127.60	53 I iodine 126.90447	54 Xe xenon 131.293
55 Cs caesium 132.90545196	56 Ba barium 137.327	57–71 LANTHA- NIDES	72 Hf hafnium 178.49	73 Ta tantalum 180.94788	74 W tungsten 183.84	75 Re rhenium 186.207	76 Os osmium 190.23	77 Ir iridium 192.217	78 Pt platinum 195.084	79 Au gold 196.966569	80 Hg mercury 200.592	81 Tl thallium 204.38	82 Pb lead 207.2	83 Bi bismuth 208.98040	84 Po polonium (208.98243)	85 At astatine (209.98715)	86 Rn radon (222.01758)
87 Fr francium (223.01974)	88 Ra radium (226.02541)	89–103 ACTINIDES	104 Rf rutherford. (267.12169)	105 Db dubnium (268.12567)	106 Sg seaborgium (269.12863)	107 Bh bohrium (270.13336)	108 Hs hassium (269.13375)	109 Mt meitnerium (278.1563)	110 Ds darmstadt. (281.1645)	111 Rg roentgen. (282.16912)	112 Cn copernicium (285.17712)	113 Nh nihonium (286.18221)	114 Fl flerovium (289.19042)	115 Mc moscovium (290.19598)	116 Lv livermorium (293.20449)	117 Ts tennessine (294.2105)	118 Og oganesson (294.21392)

Series															
Lanthanide series	57 La lanthanum 138.90547	58 Ce cerium 140.116	59 Pr praseodym. 140.90766	60 Nd neodymium 144.242	61 Pm promethium (144.91276)	62 Sm samarium 150.36	63 Eu europium 151.964	64 Gd gadolinium 157.25	65 Tb terbium 158.92535	66 Dy dysprosium 162.500	67 Ho holmium 164.93033	68 Er erbium 167.259	69 Tm thulium 168.93422	70 Yb ytterbium 173.054	71 Lu lutetium 174.9668
Actinide series	89 Ac actinium (227.02775)	90 Th thorium 232.0377	91 Pa protactinium 231.03588	92 U uranium 238.02891	93 Np neptunium (237.04817)	94 Pu plutonium (244.06420)	95 Am americium (243.06138)	96 Cm curium (247.07035)	97 Bk berkelium (247.07031)	98 Cf californium (251.07959)	99 Es einsteinium (252.08298)	100 Fm fermium (257.09511)	101 Md mendelevium (258.09844)	102 No nobelium (259.10103)	103 Lr lawrencium (262.10961)

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	5g	18	

← *H, He*

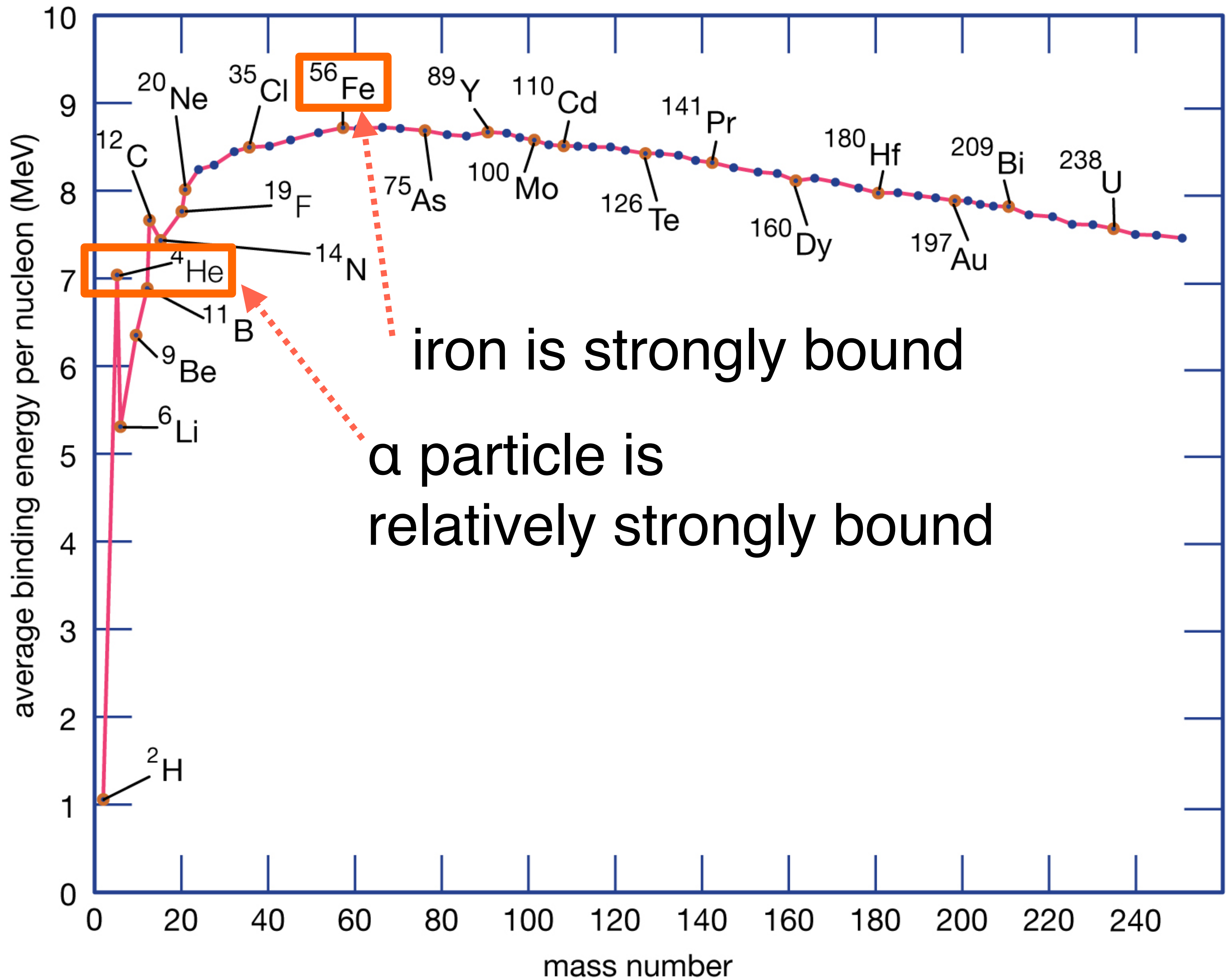
Nuclear Physics

Nuclear binding energy

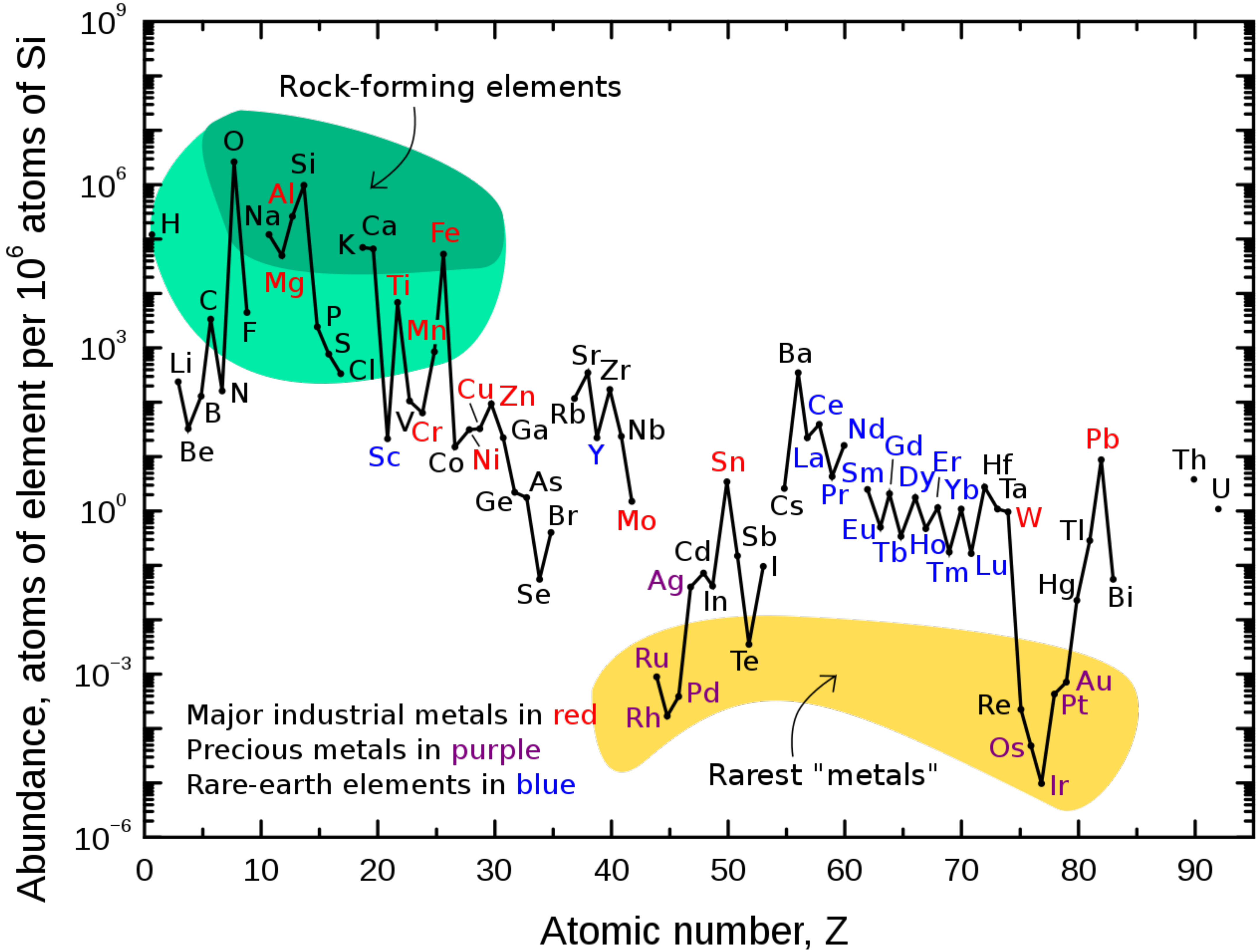
$$B(A,Z) = [Z(M_p + m_e) + (A - Z)M_n - M(A,Z)] \cdot c^2$$

8 MeV →

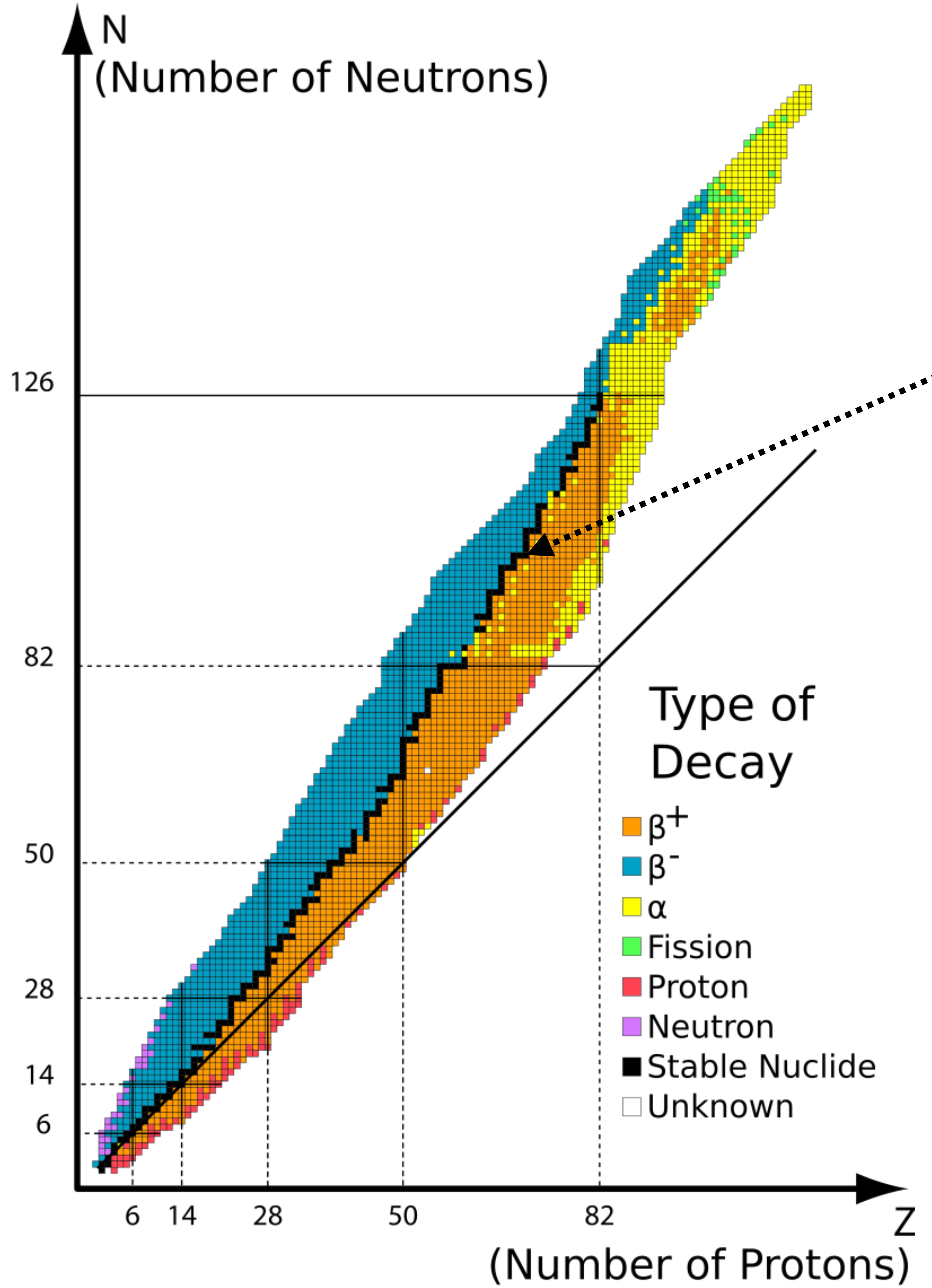
$$\frac{B(A,Z)}{A}$$



Abundance of the chemical elements on Earth

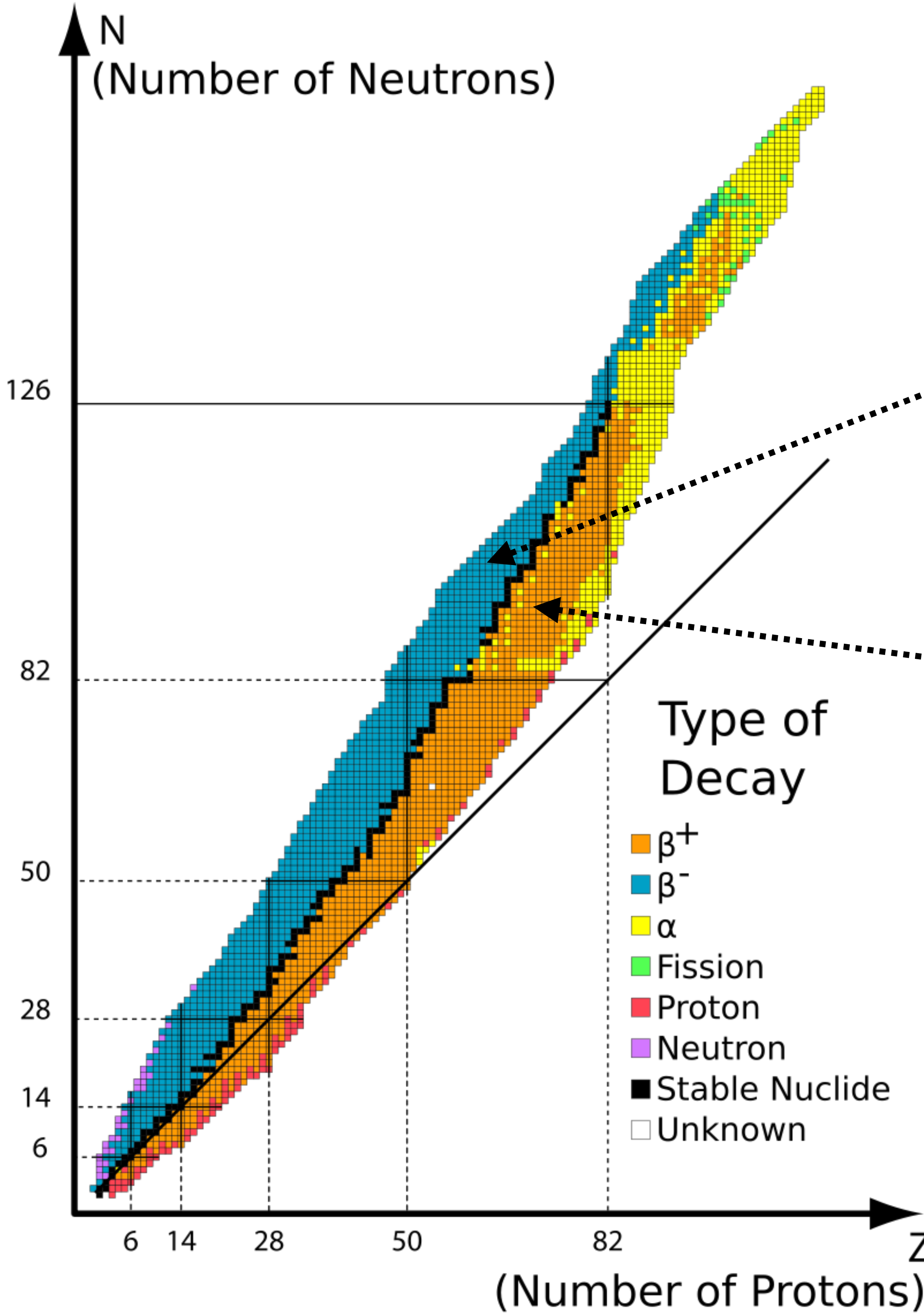


Stable nuclide (nuclear species)



stable isotopes
(do not decay)

Types of decay (weak force)



Weak interactions:

too many neutrons:

$$n \rightarrow p + e^- + \bar{\nu}_e$$

including double-β decay

too many protons:

$$p \rightarrow n + e^+ + \nu_e$$

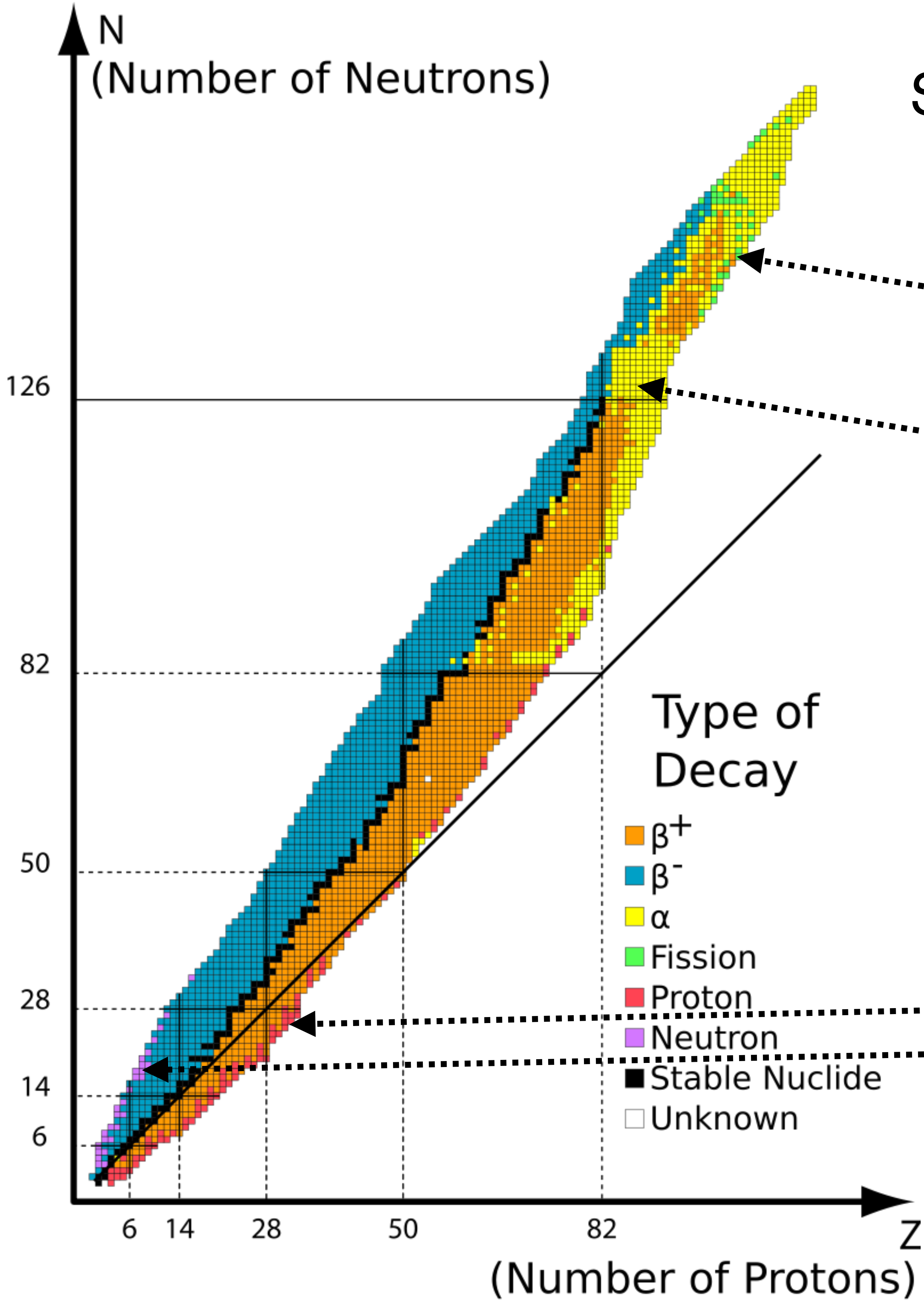
including double-β decay

$$^{106}_{48}Cd \rightarrow \, ^{106}_{46}Pd + 2e^+ + 2\nu_e$$

also electron capture

$$p + e^- \rightarrow n + \nu_e$$

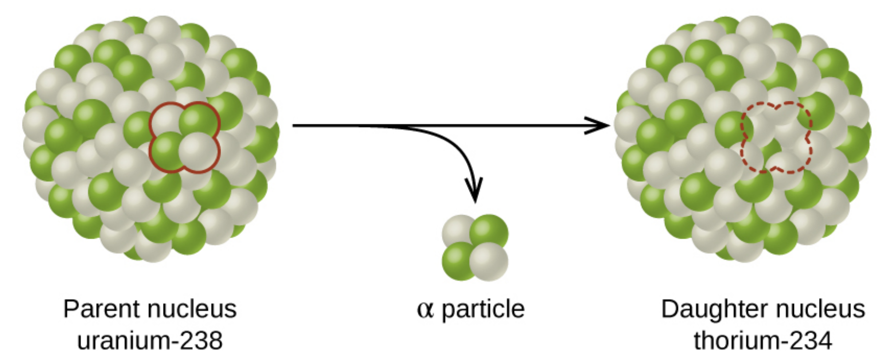
Types of decay (strong force)



Strong interactions:

too many nucleons:
fission (more next)

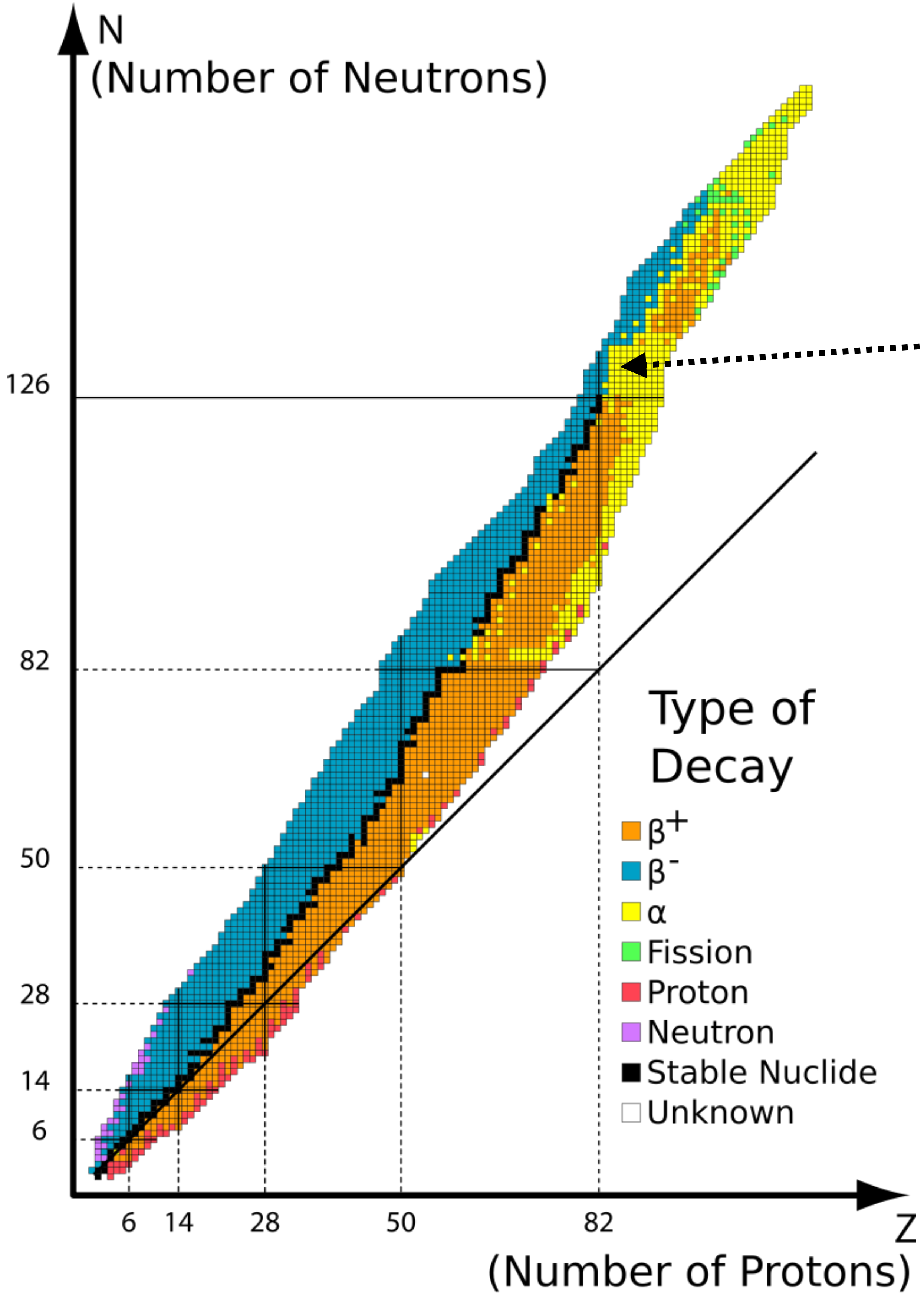
too many nucleons:
eject α particle (2n2p)

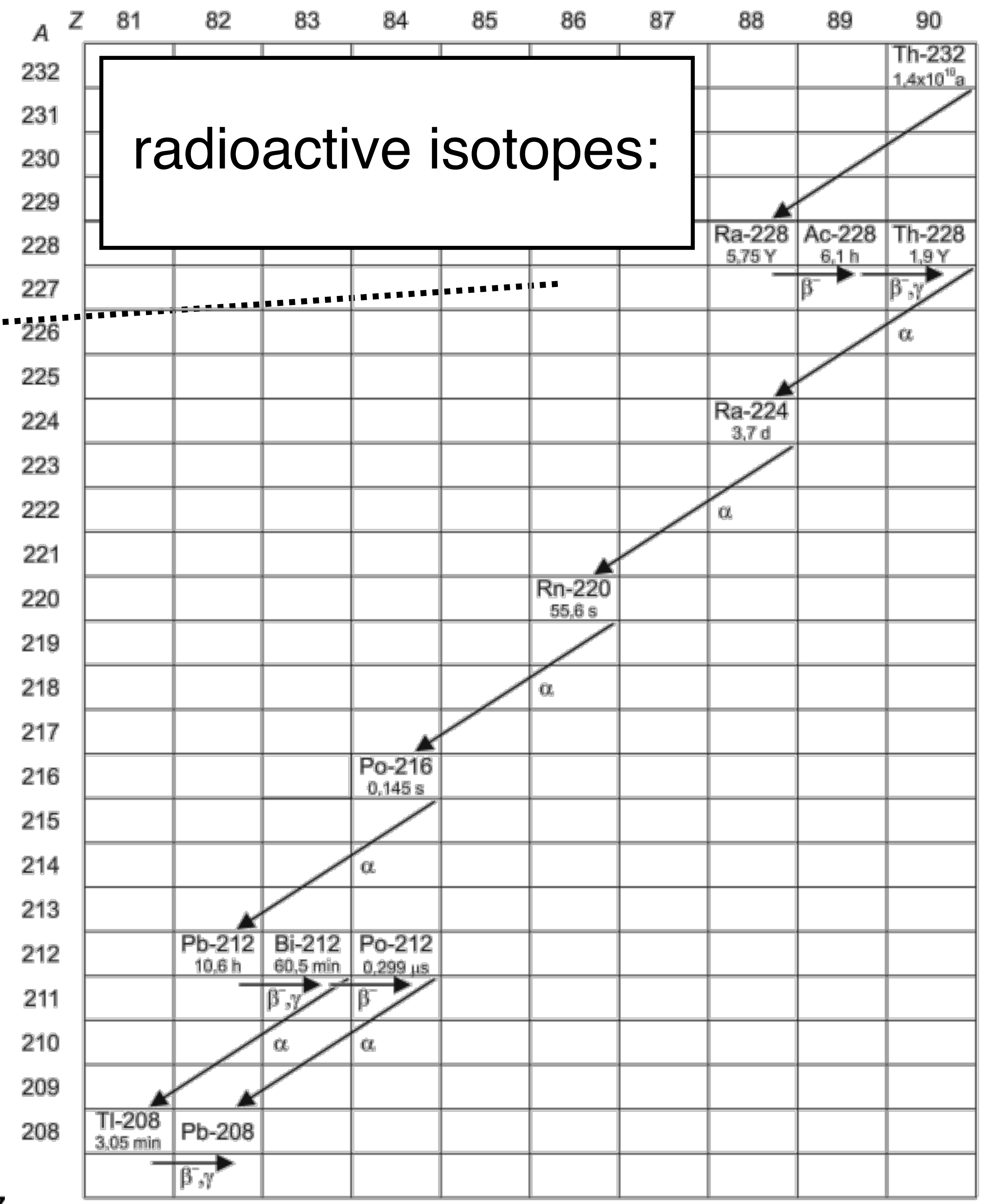


too many protons:
eject proton

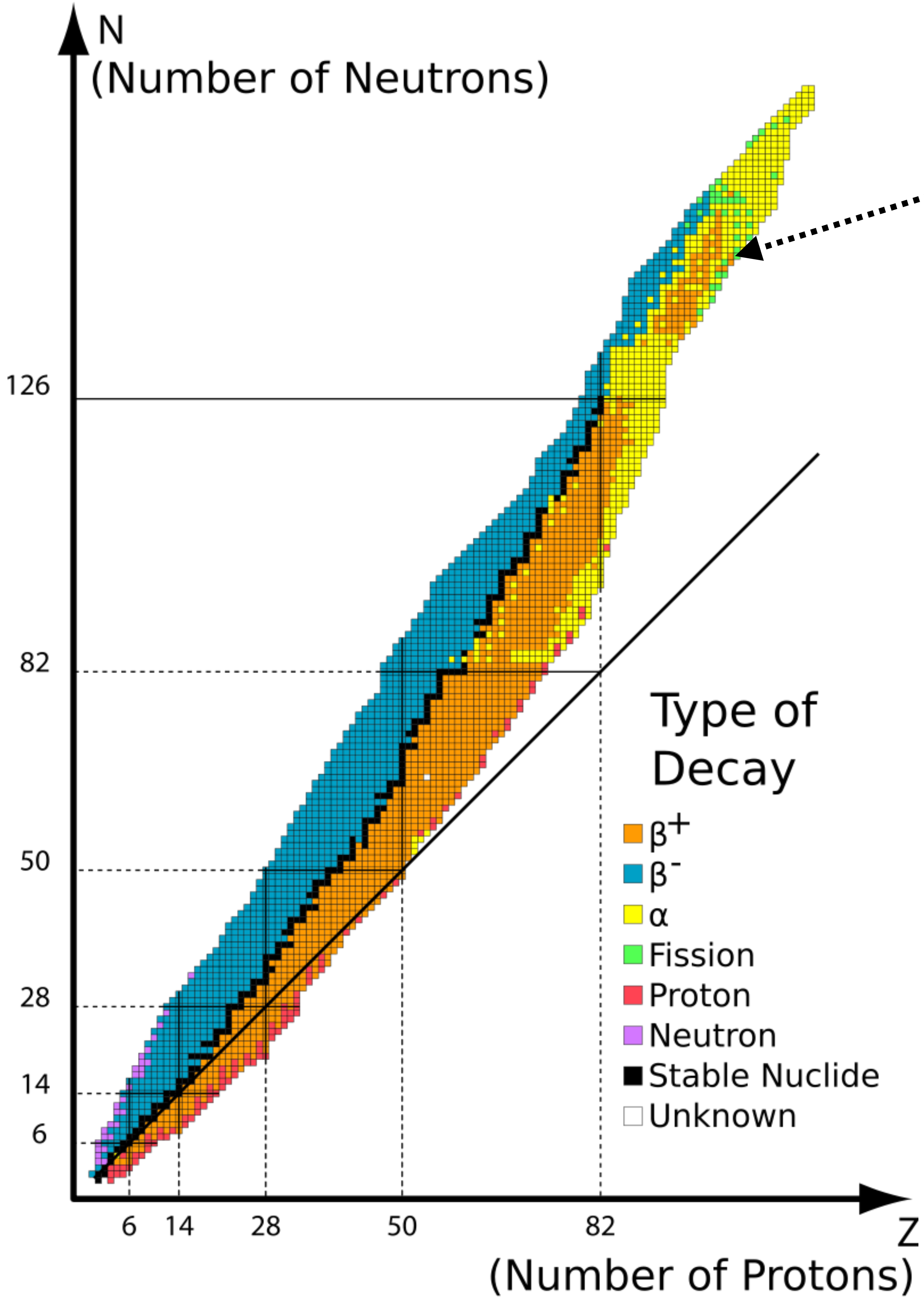
too many neutrons:
eject neutron

Types of decay



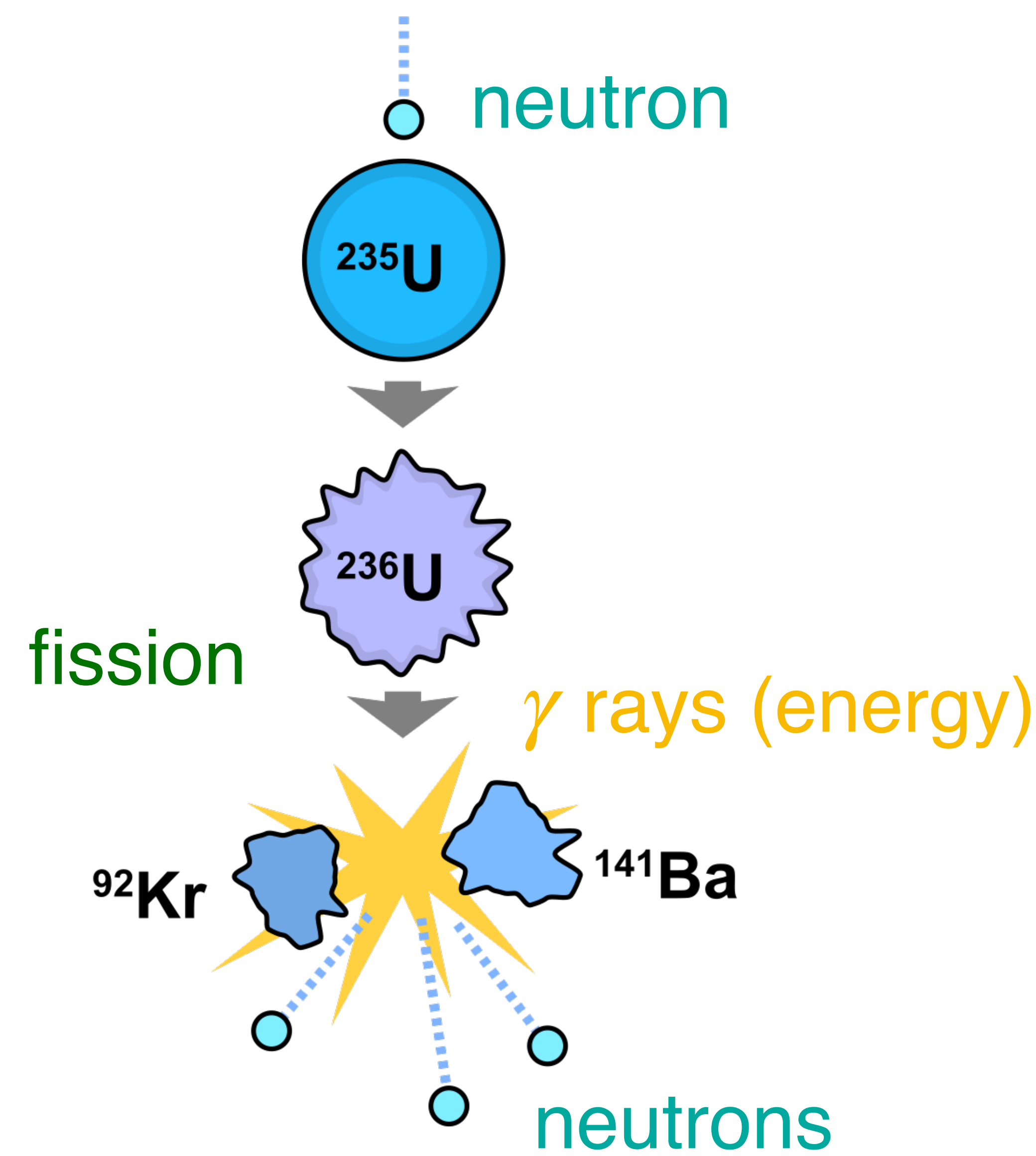


Fission



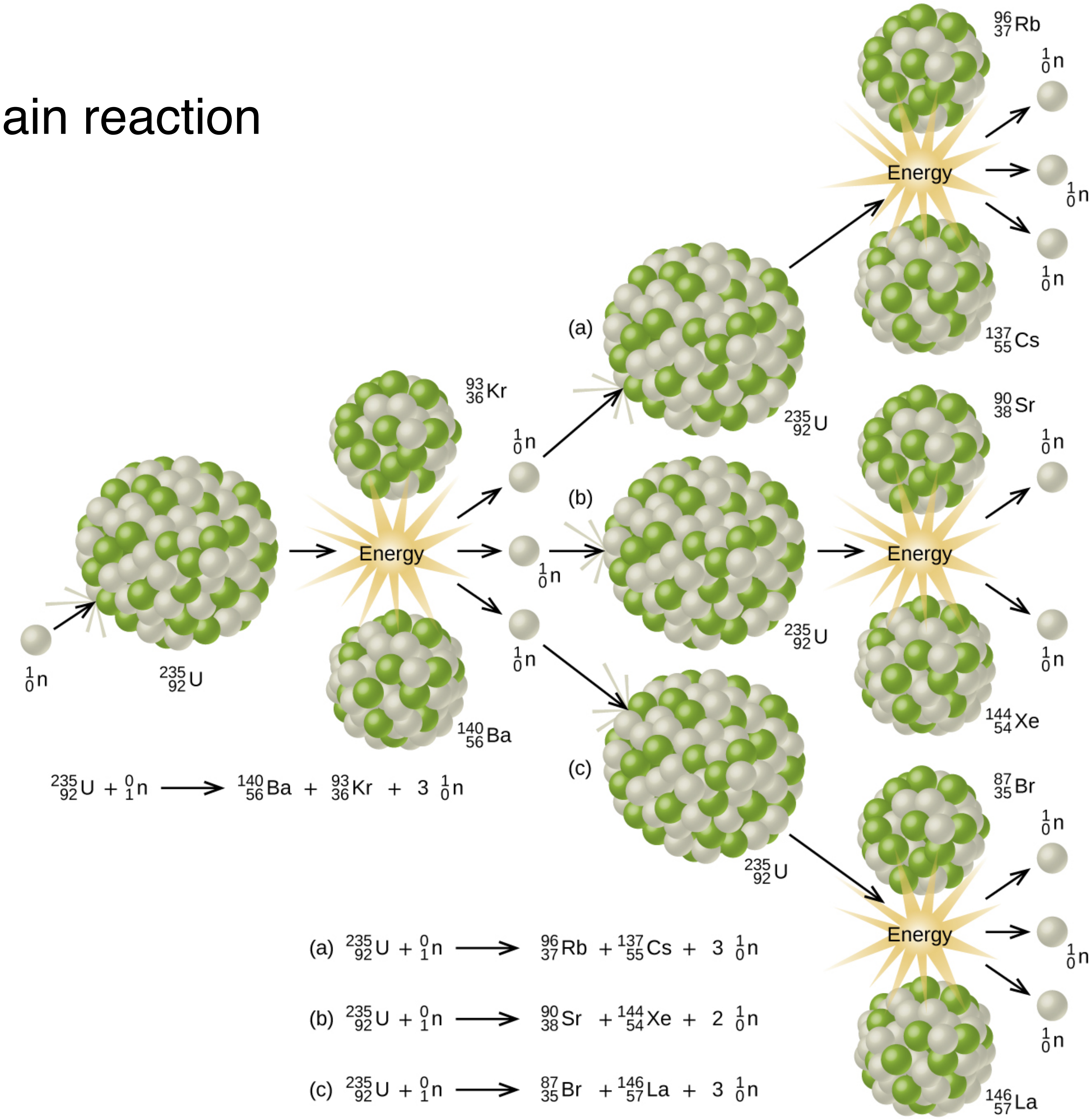
too many nucleons: fission

induced nuclear fission



Fission

Chain reaction



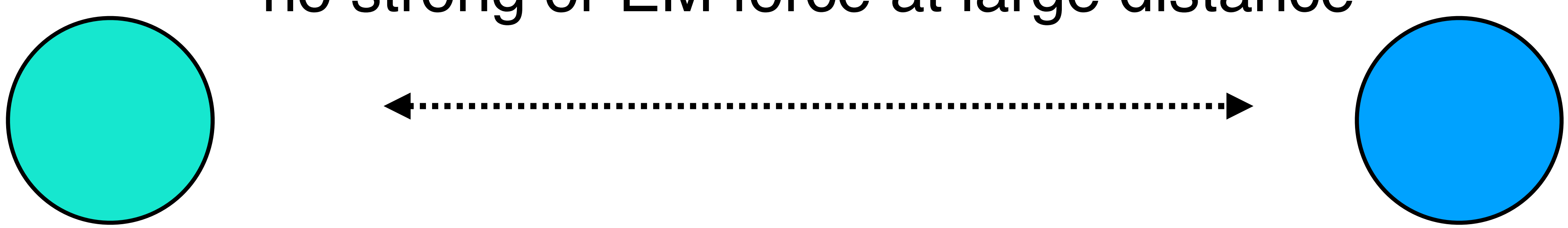
Nature of Nuclear Force

Nuclear binding energy - key in understanding nuclear processes

$$B(A,Z) = [Z(M_p + m_e) + (A - Z)M_n - M(A,Z)] \cdot c^2$$

Nuclear force - based on strong force, but works differently than binding force of quarks and baryons

no strong or EM force at large distance



neutron
color-neutral
charge-neutral

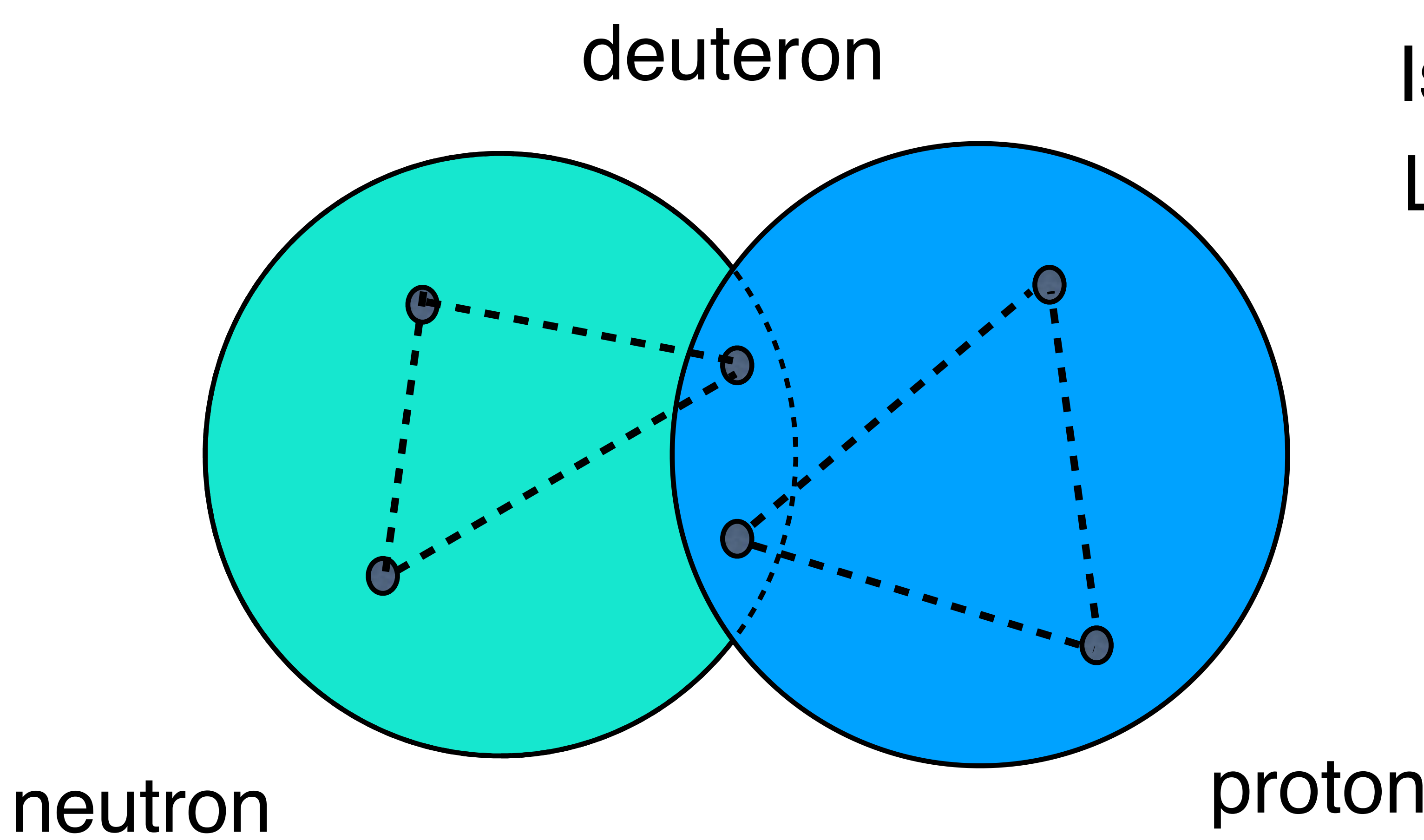
proton
color-neutral

Nature of Nuclear Force

Nuclear binding energy - key in understanding nuclear processes

$$B(A,Z) = [Z(M_p + m_e) + (A-Z)M_n - M(A,Z)] \cdot c^2$$

strong force attraction and repulsion at shorter distances:

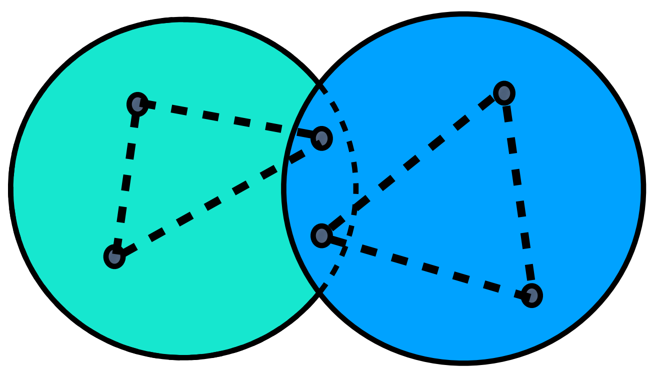


Isospin=0

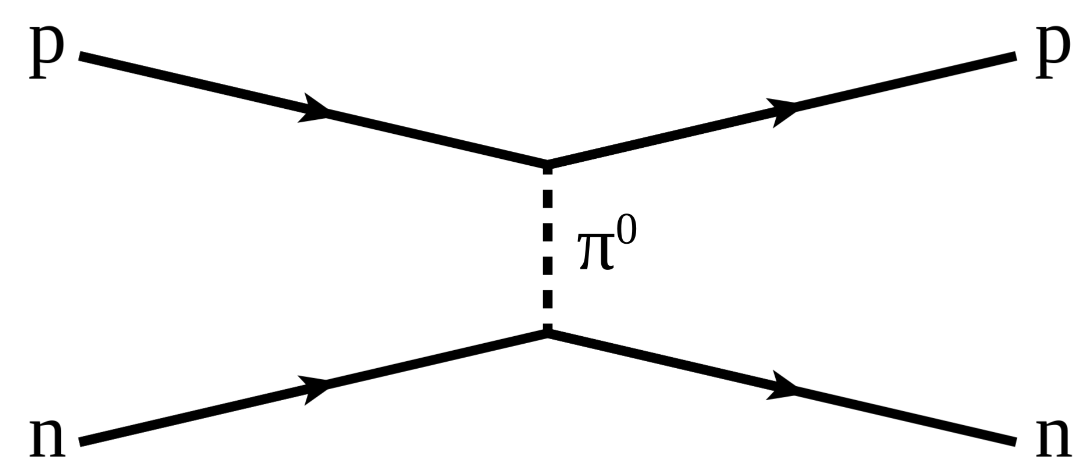
L=0 (96%)

Nature of Nuclear Force

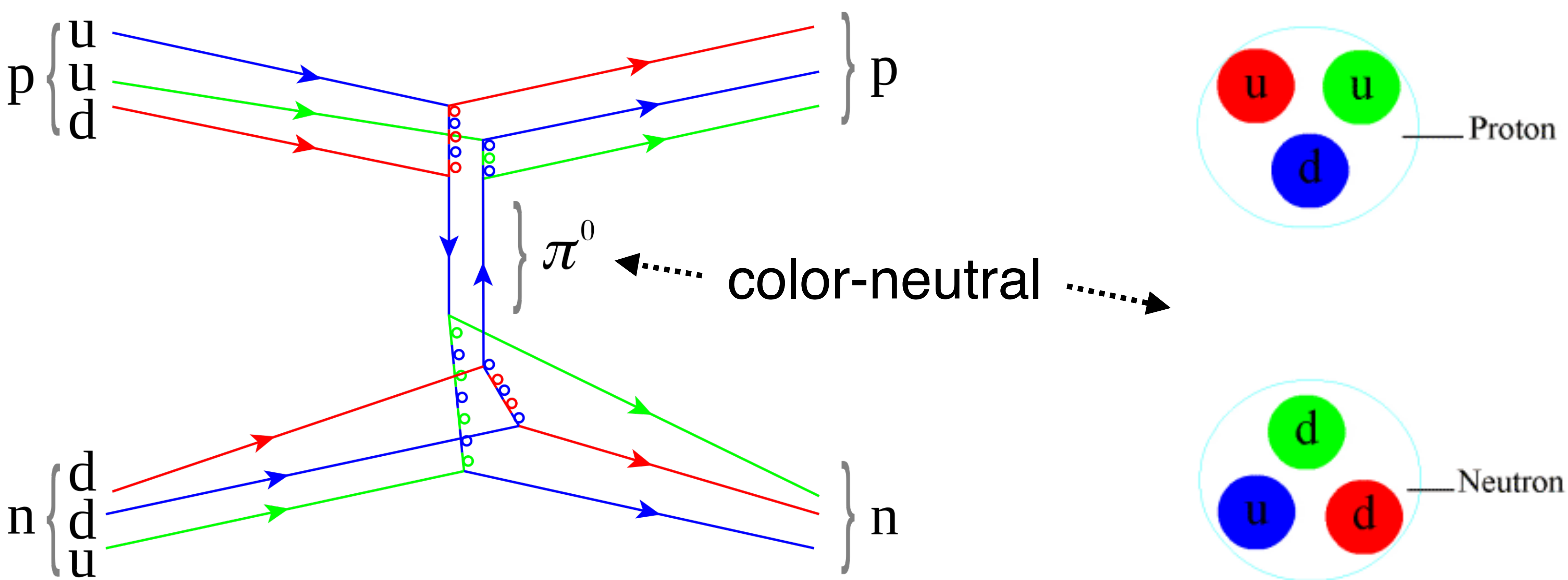
Particle Physics perspective:



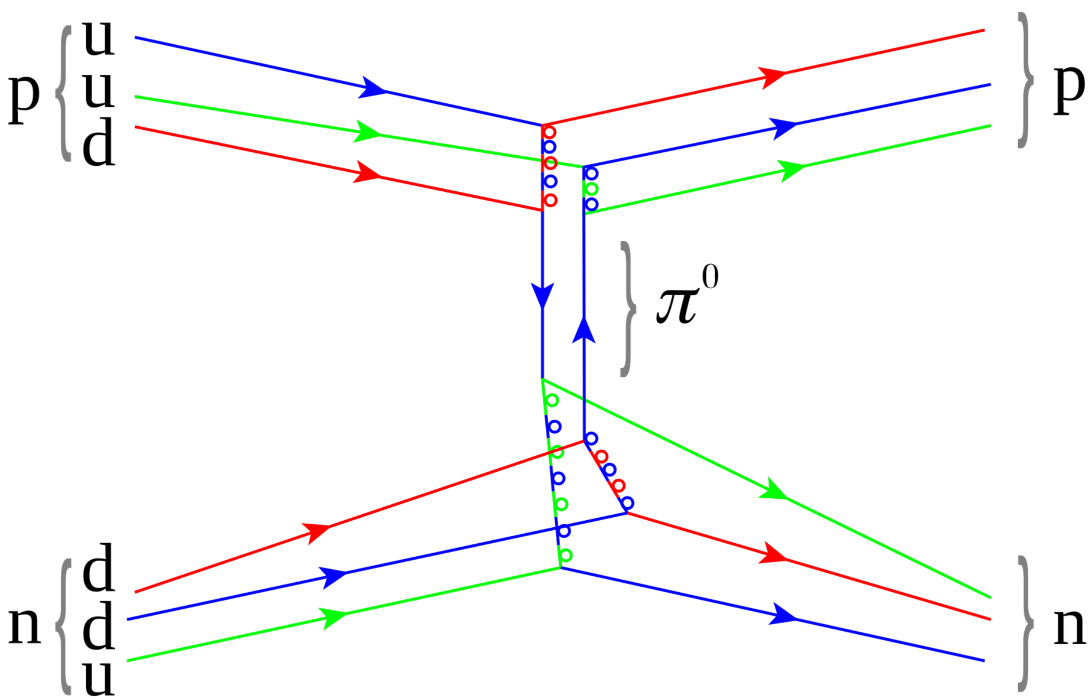
quark exchange



meson exchange



Nature of Nuclear Force

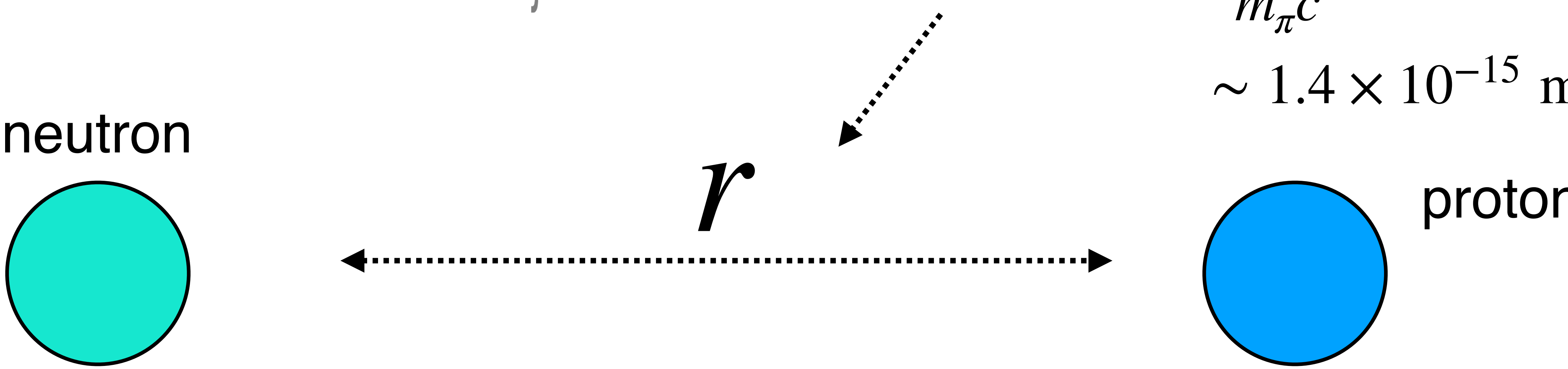


Yukawa potential at larger distances:

$$V(r) = g \cdot \frac{e^{-\frac{m_\pi c}{\hbar} r}}{r}$$

$$\text{range } d \sim \frac{\hbar}{m_\pi c} \sim 1.4 \text{ fm}$$

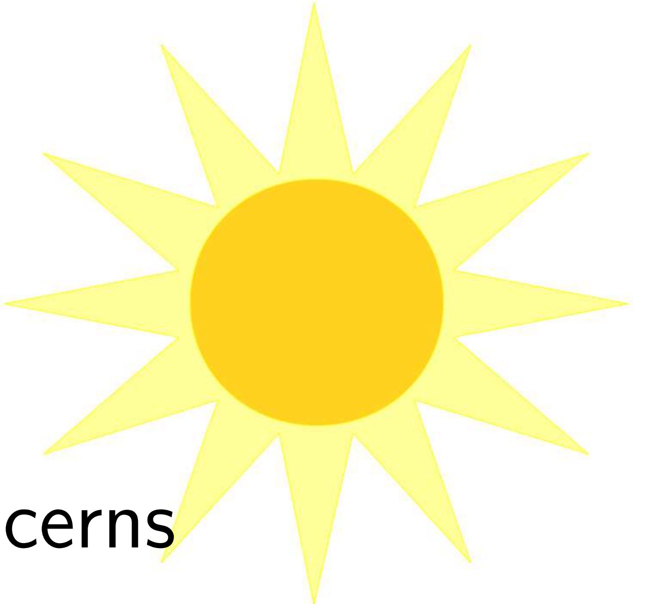
$$\sim 1.4 \times 10^{-15} \text{ m}$$



Compare for $q\bar{q}$ (colored): $V_{\text{QCD}}(r) = -\frac{4\alpha_S}{3r} + kr$

Nuclear Energy

Energy Sources



- Fossil fuel (current $\sim 86\%$)

  petroleum, coal, natural gas

  – energy from the Sun stored in the past
  – limited supply 40–400 years, environmental concerns

- Renewable energy (current $\sim 7\%$)

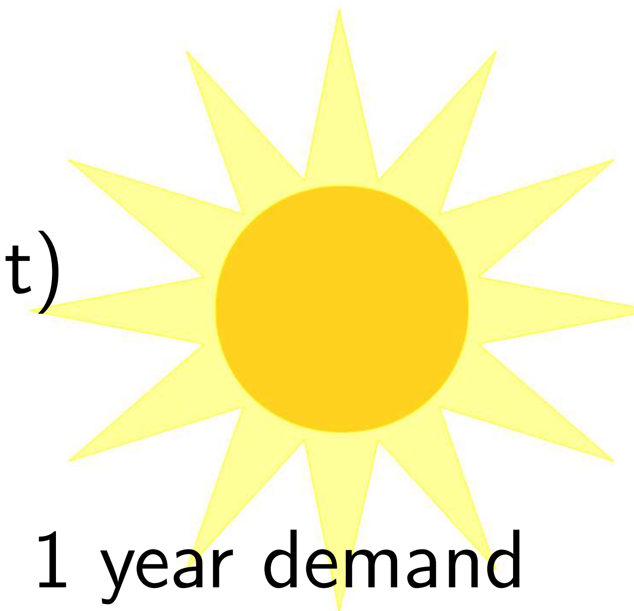
  sunlight, wind, hydro, biomass (&wood, waste),..

  – one way or another, mostly convert present Sun energy

- Nuclear energy (current $\sim 7\%$)

  – uranium-235, plutonium-239 (fission)
  – supply 100's years (fission), safety concerns
  – there is also fusion, but need technology

Energy Source: Sun as a "Nuclear Reactor"



- Both fossil fuel and renewable energy

  mostly pass energy from the Sun (past or present)

  Sun – huge nuclear fusion reactor

  supply: billions of years, 1 hour flux on Earth = 1 year demand

- Challenge with renewable energy technological:

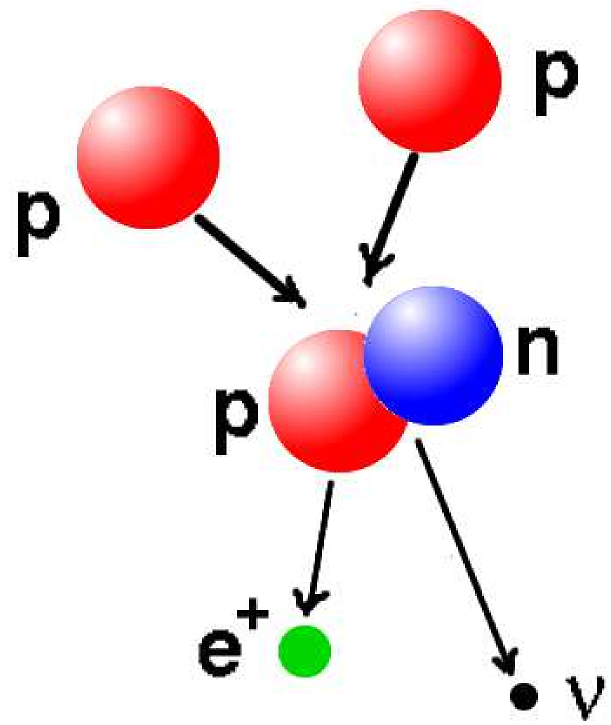
  collect enough Sun light

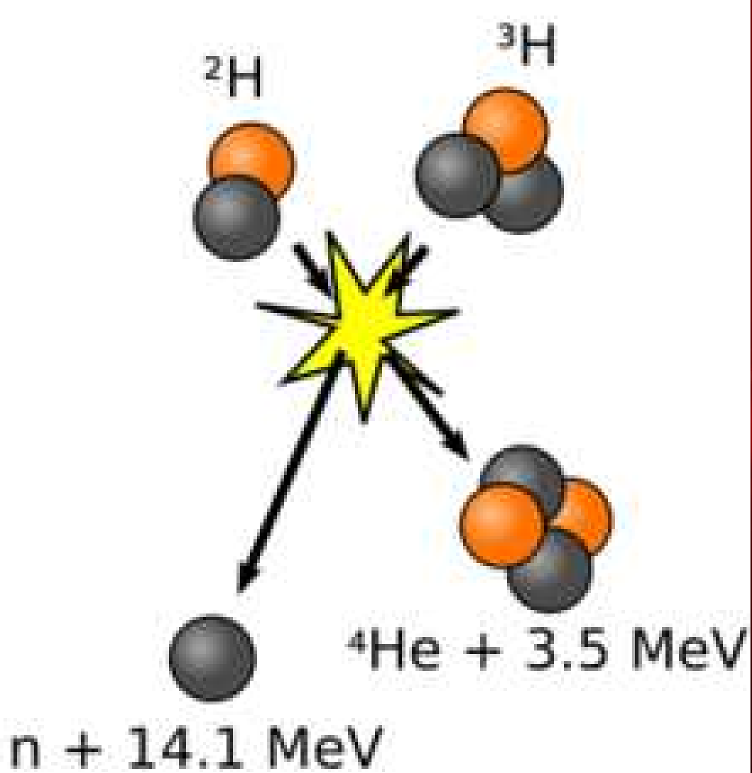
  effectively convert and store collected energy

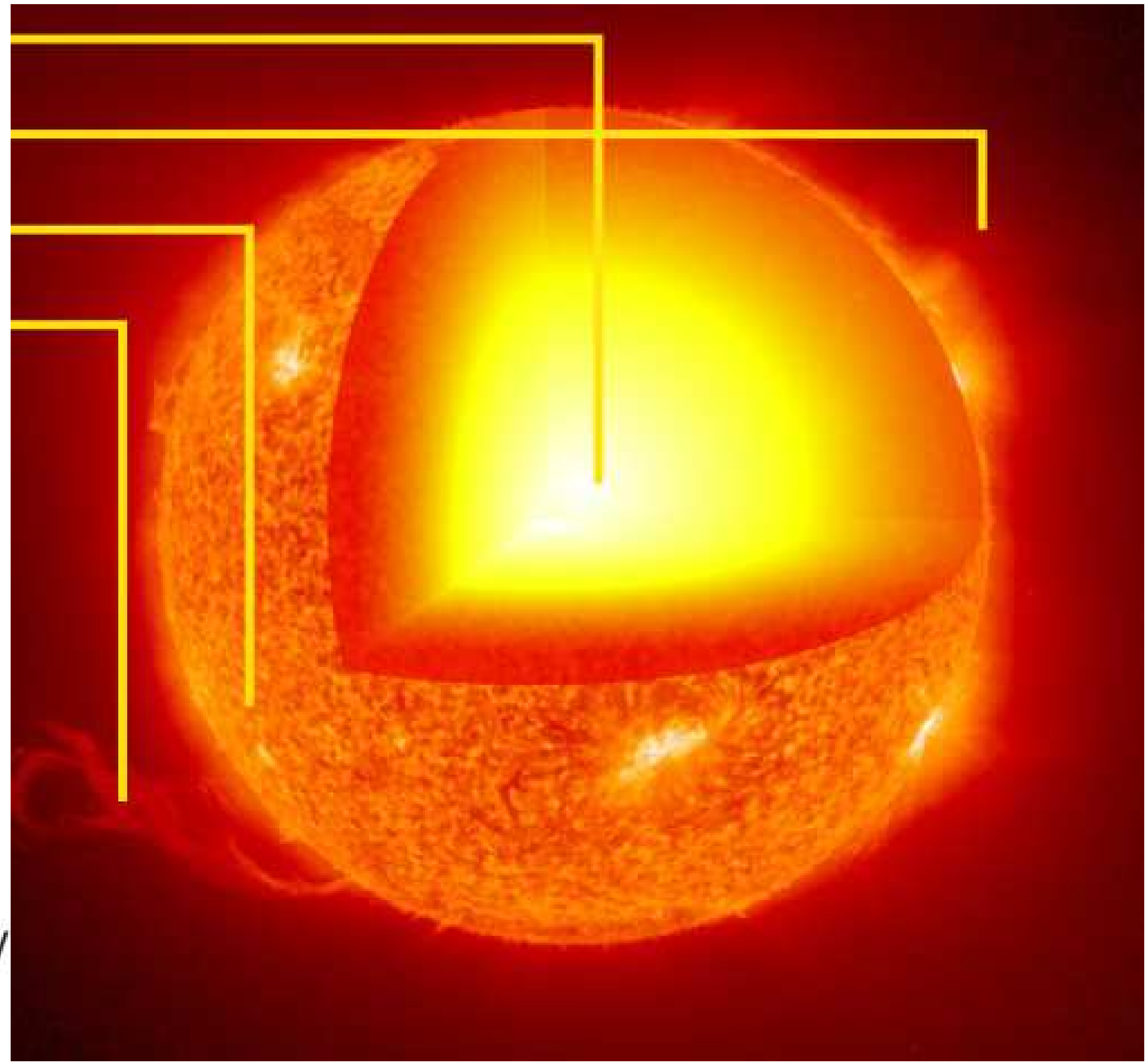
  examples: photosynthesis by green plants;
  solar power panels

  beyond the scope of this discussion

Sun as a "Nuclear Reactor"



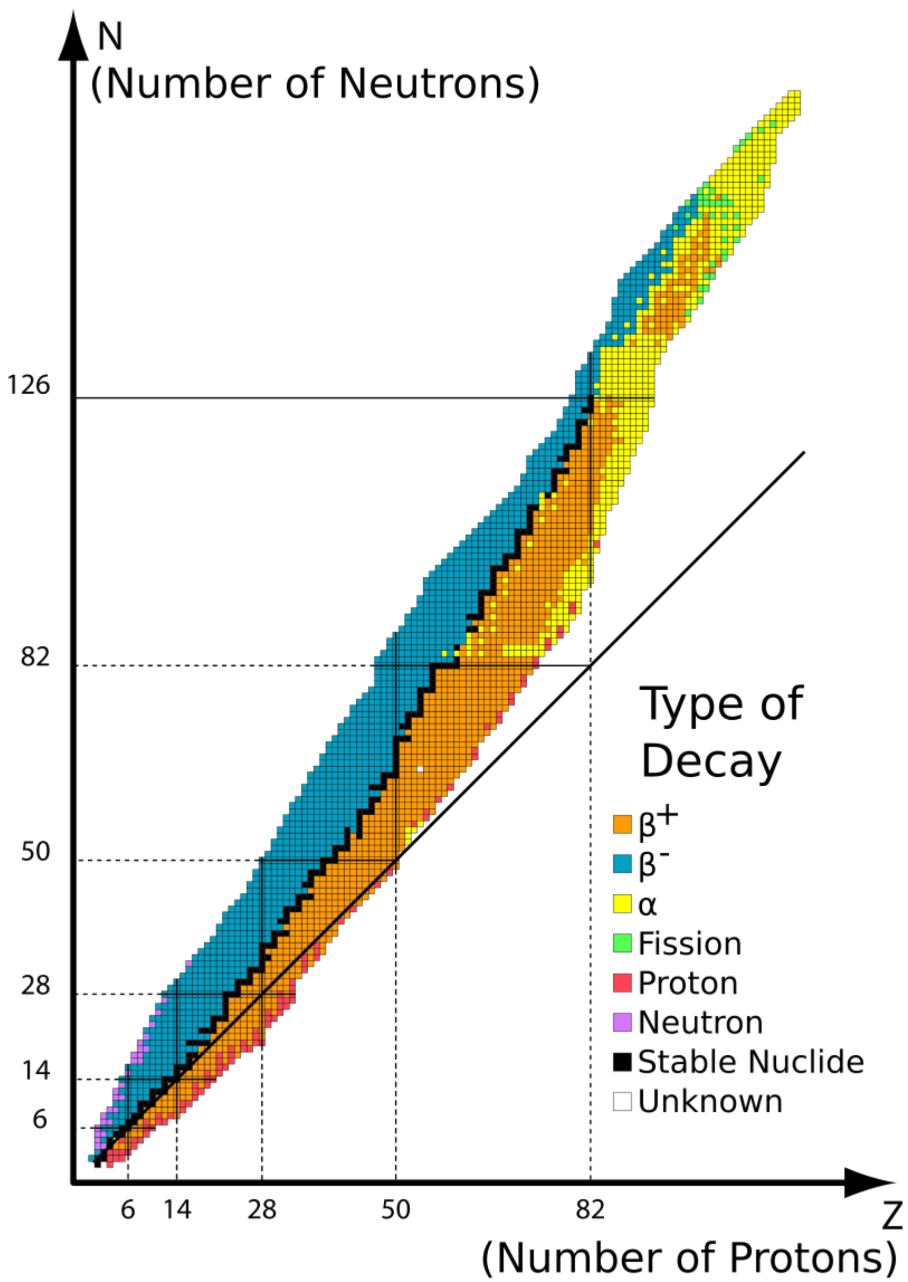


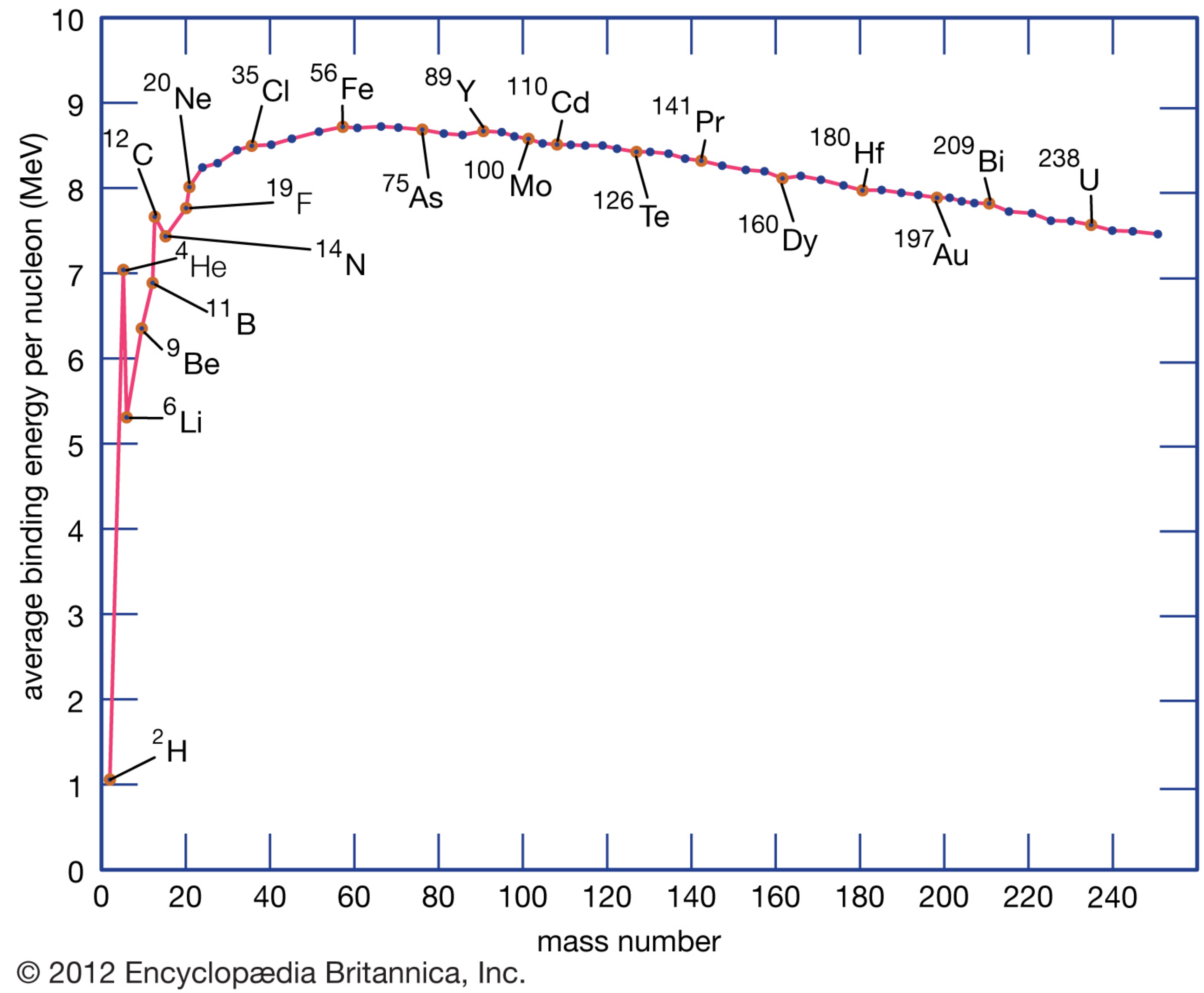


Stable nuclide (nuclear species)

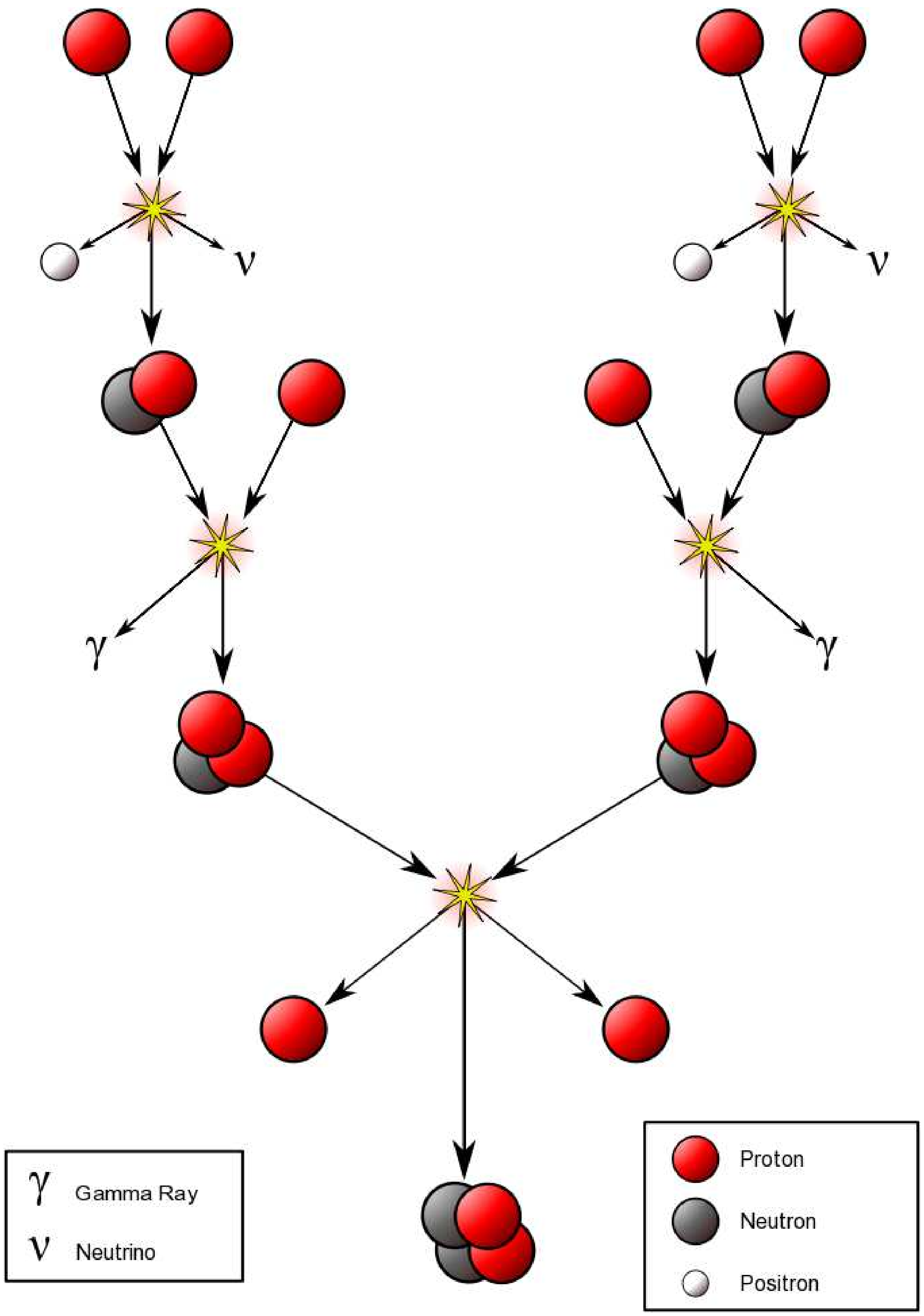
Nuclear binding energy - key in understanding nuclear processes

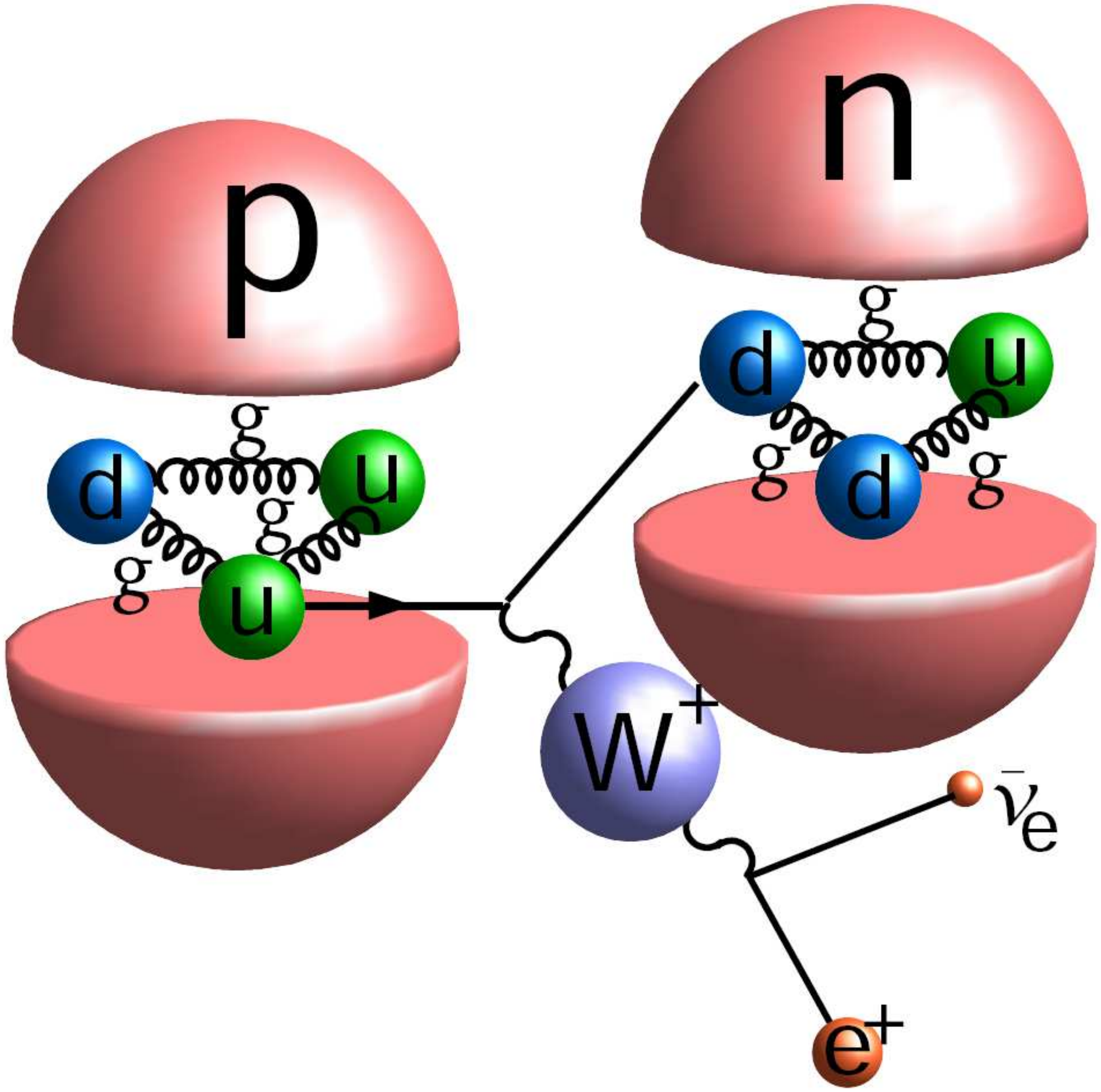
$$B(A,Z) = [Z(M_p + m_e) + (A - Z)M_n - M(A,Z)] \cdot c^2$$



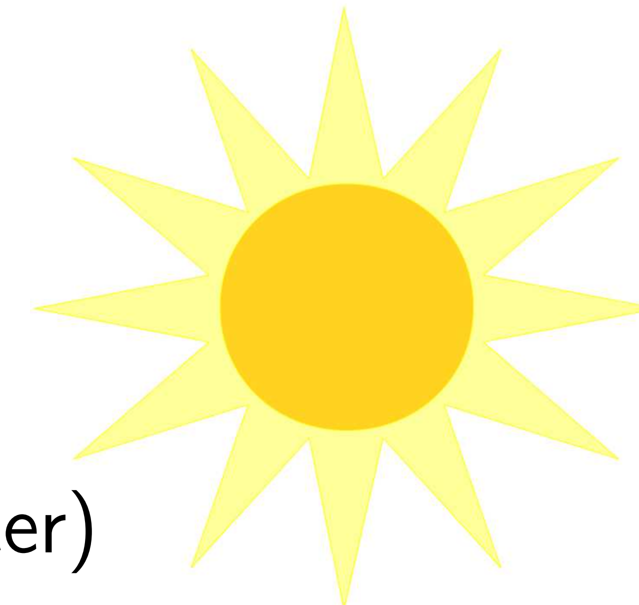


Sun as a "Nuclear Reactor"





Energy Source: Fuel



- combustion

  burn fuel (carbon)

  $$CH_4 + 2\ O_2 \rightarrow CO_2 + 2\ H_2O + \text{energy}$$

  (methane) + (oxygen) → (carbon dioxide) + (water)

- nuclear fission

  $$n + {}^{235}U \rightarrow {}^{92}Kr + {}^{141}Ba + 3\ n + \text{energy}$$

- nuclear fusion

  $${}^{2}H + {}^{3}H \rightarrow {}^{4}He + n + \text{energy}$$

- antimatter annihilation

  ${}^{1}H^{+}$ (matter) + ${}^{1}H^{-}$ (antimatter) → energy

  science fiction (e.g. see Angels and Demons with Tom Hanks)

Nuclear Energy: Present

- Nuclear fission reactor