**Milliken’s “Nut” Drop Experiment**

Robert A. Milliken performed a set of experiments which gave two important results:

1. Electric charge is quantized. All electric charges are integral multiples of a unique elementary charge *e*.
2. The elementary charge was measured and found to have the value *e* = 1.6 x 10-19 Coulombs.

Of these two results, the first is the most significant since it makes an absolute assertion about the nature of matter. We now recognize *e* as the elementary charge carried by the electron and other elementary particles.

**Millikan’s Oil-Drop Experiment**

In Robert Millikan’s well-known experiment to determine the charge on the electron, small oil drops are introduced between two parallel electrodes. Viewed through a microscope, the pinpoint specks of light marking the location of the droplets can be seen falling downward under the influence of three forces: gravity, a buoyant force, and a friction force. The droplets quickly reach their terminal velocity due to the friction force, and this terminal velocity can be measured with a length scale and a stopwatch or a video camera.

The application of an appropriate voltage difference across the electrodes produces an electrostatic force on any non-neutral droplets, and those droplets can be made to rise upward. Measurements of the terminal velocity of the drops as they rise and fall allow for a determination of the electric charge on the droplet. While this experiment can be done by students in a laboratory, it is generally finicky and difficult to do in a short period of time.

**Simplifying the Experiment – The ‘Nut-Drop’ Experiment**

A main difficulty that Millikan faced was not knowing the number of electrons added to or missing from the oil drops. This is a fairly profound and unusual problem in a physics experiment, where the independent variable is usually well-known. In this experiment, you are to determine the mass of a screw nut using a technique similar to Millikan’s. The oil drops are replaced by a collection of identical plastic water-tight containers that hold an unknown number of screw nuts (“electrons”) hidden inside. Depending upon the number of nuts inside, a container will either rise or fall when placed in water. Too few nuts correspond to a positively charged oil drop and too many correspond to a negatively charged oil drop. So in this experiment, the buoyant force on the container is constant (taking the place of the force of gravity in Millikan’s experiment), and the force of gravity changes (taking the place of the electrostatic force) (see Fig. 1). A number of these containers are allowed to rise or fall through a known distance in a clear tube of water, and a measurement of their terminal velocities can lead to a determination of the mass of an individual nut, even though the number of nuts in the containers is unknown.

Figure 1: a) Falling

 *Ff FB*

 *Ff*

 *‘M’g*

 *Mg*

b) Rising

 *FB*

 *FE*

 *Ff ‘M’g*

 *Ff Mg*

Fig. 1: Schematic showing the forces acting on a) a sinking container and a falling oil drop, and b) a rising container and a rising oil drop. The varying electric force in the oil-drop experiment is compared to the varying gravitational force in the nut-drop experiment, showing how the mass takes the place of the charge of the electron. ‘*M*’ is the effective mass of the oil drop, taking account of the buoyant force.

M

V0

M

V0

**Forces on the Container:**

As the container moves downward in the water, the net force on it is

$F=Mg-F\_{f}-F\_{B}=Ma$*,* (1)

where *M* is the total mass of the container plus the nuts, *g* is the acceleration of gravity, *Ff* is the friction force, *FB* is the buoyant force, and *a* is the acceleration of the container. (Downward acceleration is positive.)

Millikan’s oil drops were small spherical particles moving slowly through air (< 0.01 m/s), so they experienced a drag force proportional to their velocity according to Stokes’ law. The friction force on a rapidly moving, flat-nosed object should be nearly proportional to the velocity squared:

$F\_{f}=Cv^{2}$, (2)

where *C* is the drag coefficient for the container.

The buoyant force is equal to the weight of the water displaced by the container:

$F\_{B}=ρ\_{w}V\_{0}g$*,* (3)

where *ρw* is the density of the water and *V0* is the volume of the container. Once the container reaches its terminal velocity, *vterm*, the acceleration is zero, and we find that

$v\_{term}^{2}={\left(Mg-F\_{B}\right)}/{C}=\left(M-ρ\_{w}V\_{0}\right)\left(^{g}/\_{C}\right)$*.* (4)

But the total mass of the container and the nuts is

$M=m\_{0}+Nm\_{n}$, (5)

where *m0* is the mass of the container alone, *mn* is the mass of a single nut, and *N* is the number of nuts in the container. So the terminal velocity squared is linearly related to the number of nuts in the container:

$v\_{term}^{2}=m\_{n}\left(^{g}/\_{C}\right)N+\left(m\_{0}-ρ\_{w}V\_{0}\right)\left(^{g}/\_{C}\right)$ (6)

**Determining N:**

A plot of *v2term* versus *N* should produce a graph with a slope of *mn*(*g*/*C*). *g* is certainly known, but *C* is not, so that quantity must be determined for this container before the experiment can be completed, just as Millikan needed to do. Note that the buoyant force does not play a direct role in the determination of *m*n. This is similar to the original experiment, where the buoyant force was folded into the effective mass of the oil drop.

Millikan was able to use Stokes’ results to determine the drag force on a sphere in a viscous medium. In the Nut Drop, the drag coefficient must be determined experimentally. Measuring the total mass of a few containers and determining their terminal velocities will lead to a graph of *v2term* versus *M*, whose slope can be used to find *g*/*C* from Eq. (4).

The remaining problem is to determine *N* for each of the containers. Luckily, the precise values of *N* can be replaced with a series of smaller integers, since only the slope of the *v2term* versus *N* line is needed. The data are just sorted into groups that have the same terminal velocity, and then each group can be assigned a value of *N*, starting with *N* = 1 for the group with the smallest terminal velocity. The *v2term* -versus-*N* plot can then be made and the slope determined. Analyzing the sinking containers separately from the rising containers is helpful.

**Procedure:**

Part I: Calculating Drag Coefficient

1. Set up the clear pipe filled with water and securely attach to table or lab bench. Ensure that there is a catch basin for any lost water.
2. Place two lines on the pipe one meter apart and equal distances from the ends of the pipe.
3. Obtain containers of known mass and sort into “sinkers” and “floaters.”
4. Drop a sinker into the pipe and measure the time for the container to travel one meter. By the time it gets to the first line, the container should have reached terminal velocity and therefore will fall at a constant velocity. Use a magnet to retrieve the container from the bottom of the pipe.
5. Repeat measurements to ensure accurate values.
6. Repeat with additional sinkers of known mass. Ensure that containers always fall with the same leading edge (top or bottom of the container). Excess air bubbles will affect the quality of the data.
7. Calculate the average velocity squared for each of the sinkers with known mass.
8. Construct a graph of velocity squared vs. mass and determine the equation of the line of best fit.
9. Obtain the floaters and repeat steps 4 – 8. Except the floaters will need to be released from the bottom of the pipe to allow them to float to the top. Use the magnet to drag the containers the bottom of the pipe and release them by pulling the magnet perpendicular to the pipe. Ensure that the containers have the same leading edge as used before for the sinkers.
10. Use the equation from each of the graphs and $v^{2}=\left(^{g}/\_{C}\right)M-\left(ρ\_{w}V\_{0}\right)\left(^{g}/\_{C}\right)$ to determine the quantity $ ^{g}/\_{C}$. $Slope=\left(^{g}/\_{C}\right)$

Part II: Quantized Nature

1. Using containers filled with unknown number of nuts and unknown total mass determine the terminal velocity squared for each container. Analyze the sinkers and floaters separately.
2. Bin the *v2* and create a histogram for each the sinkers and the floaters. The distribution of these squared velocities should show that the velocities are grouped. (Bin width = 0.025 (m/s)2)
3. Assign N values for the groups, starting at N=1 for the group with the lowest velocity.

Part III: Determining the Mass of One Nut

1. Construct a graph of *v2* vs *N* for each the sinkers and floaters. Determine the equation for the line of best fit.
2. Use the equation of the line from each graph and $v^{2}=m\_{n}\left(^{g}/\_{C}\right)N+\left(m\_{0}-ρ\_{w}V\_{0}\right)\left(^{g}/\_{C}\right)$ to determine the mass of a single nut. $Slope=m\_{n}\left(^{g}/\_{C}\right)$
3. Ask for the actual average mass of a nut from your instructor and calculate percent error for mass of one nut for sinkers and floaters.

**Conclusion:**

1. What errors may have been encountered?
2. In part II of the lab, did you see any “holes” in the data? What may have caused this? Would Milliken have had the same issue? Explain.
3. ???