# Accelerators: what they are and how they work

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# Accelerators

Physicists have been building accelerators since the 1850s

- H. Geissler creates gas discharge tubes using newly developed vacuum pump and various gases
  - currents of electrons and ions move through the tubes
- accelerators are still found everywhere but are becoming less common
  - fluorescent lighting [being replaced by led bulbs]
  - cathode ray tubes [being replaced by lcd screens]
  - x-ray machines are found in medical/dental offices
  - linear accelerators are found in nearly every large hospital

# Lorentz Force

All accelerators are "engineering applications" of the same Lorentz force that we teach to students in intro E&M courses:

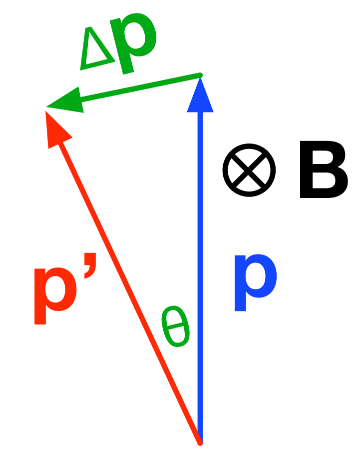
$$\frac{d\vec{p}}{dt} = \frac{d(\gamma m\vec{v})}{dt} = q\left[\vec{E} + \frac{1}{c}\vec{v} \times \vec{B}\right] \qquad \gamma = \frac{1}{\sqrt{1-(v/c)^2}}$$

- charges accelerate along the net electric field
  - gain/lose energy from work done by the E-field
- charges accelerate transversely to their direction in magnetic fields
  - no work done and therefore no energy gain/loss
- this form of the Lorentz force is relativistically covariant
  - works in all inertial frames
  - must use the relativistic definition of momentum [w/ $\gamma$ factor]
- $1/c$ is present when using CGS units

# Motion in uniform [dipole] B fields

In uniform B fields, the charge moves in a circular orbit. Because the energy [$\gamma$] does not change, we can write that for short time intervals:

$$|\Delta\vec{p}| = q|\vec{v}||\vec{B}|\Delta t \quad \rightarrow \quad \theta \simeq \frac{|\Delta\vec{p}|}{|\vec{p}|} = \frac{qB}{\gamma m}\Delta t$$




Summing over many short intervals, we define the orbital period T as the time needed for the angle to reach $2\pi$ radians. It is also the ratio of the circumference and the speed,

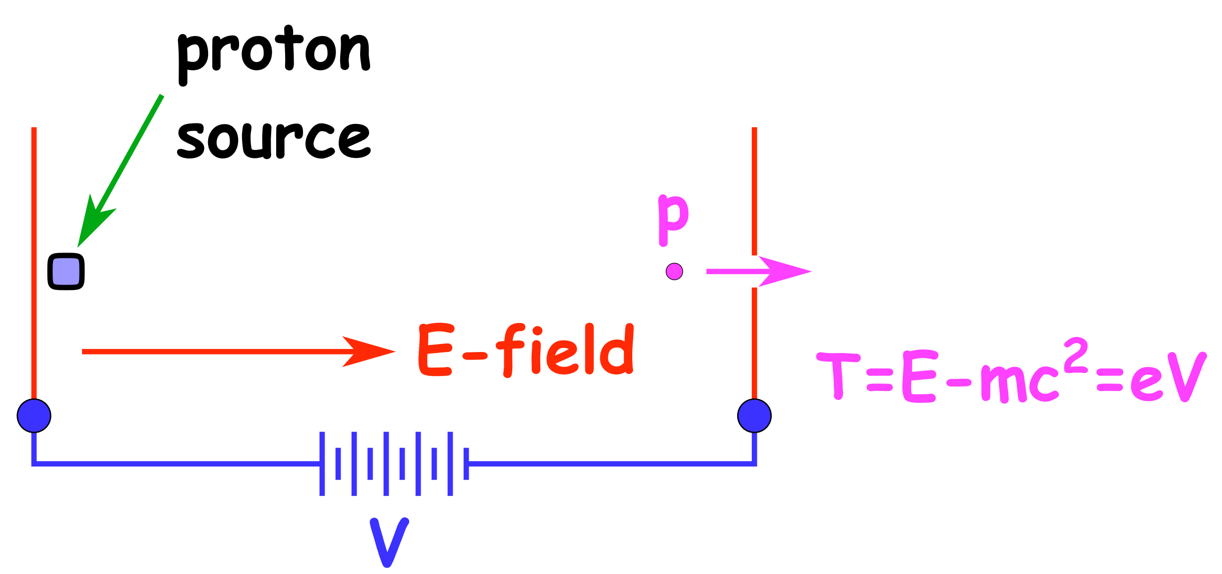
$$T = \frac{\gamma m}{qB}2\pi = \frac{2\pi R}{|\vec{v}|} \quad \rightarrow \quad R = \frac{|\vec{p}|}{qB}$$

- at low energies [$\gamma$~1], T depends only on q/m and B
  - at larger energies when the particle is relativistic, T is linear in $\gamma$
- the orbital radius increases linearly with p and decreases as 1/B

# High Energy Accelerators

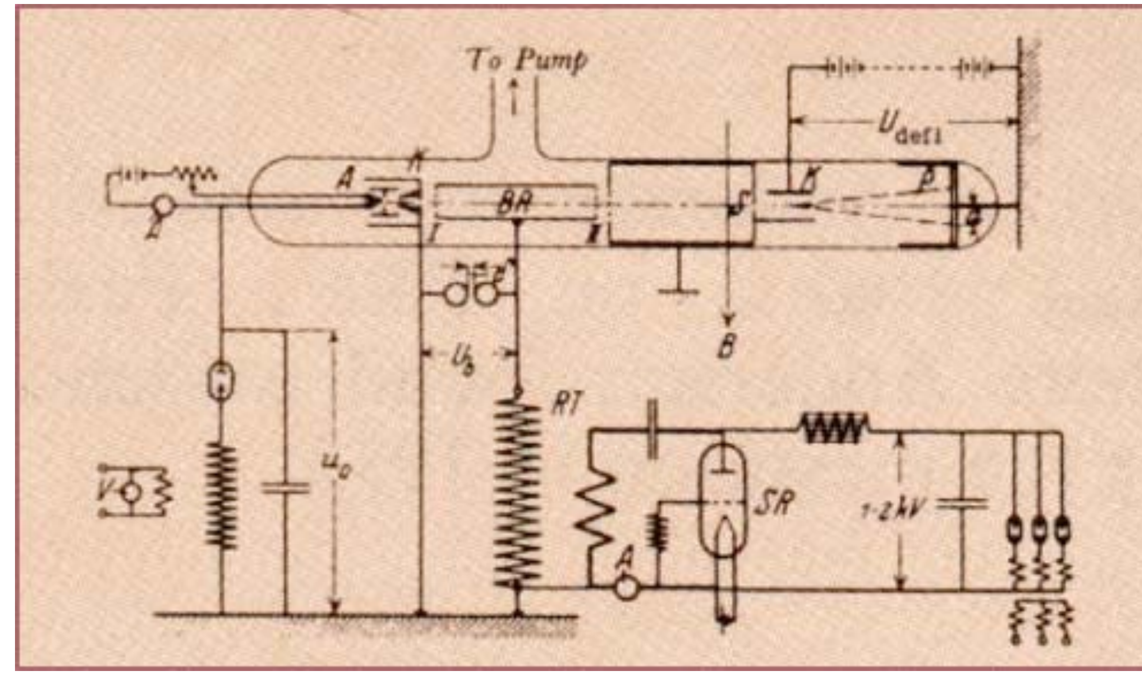
In the late 1920's, a number of physicists were searching for ways to artificially accelerate protons and other nuclei to energies larger than typical alpha sources:

- need protons of energies > few-10 MeV to overcome Coulomb repulsion in the nucleus and initiate nuclear reactions
- first accelerators were just parallel plate capacitors held at high potential

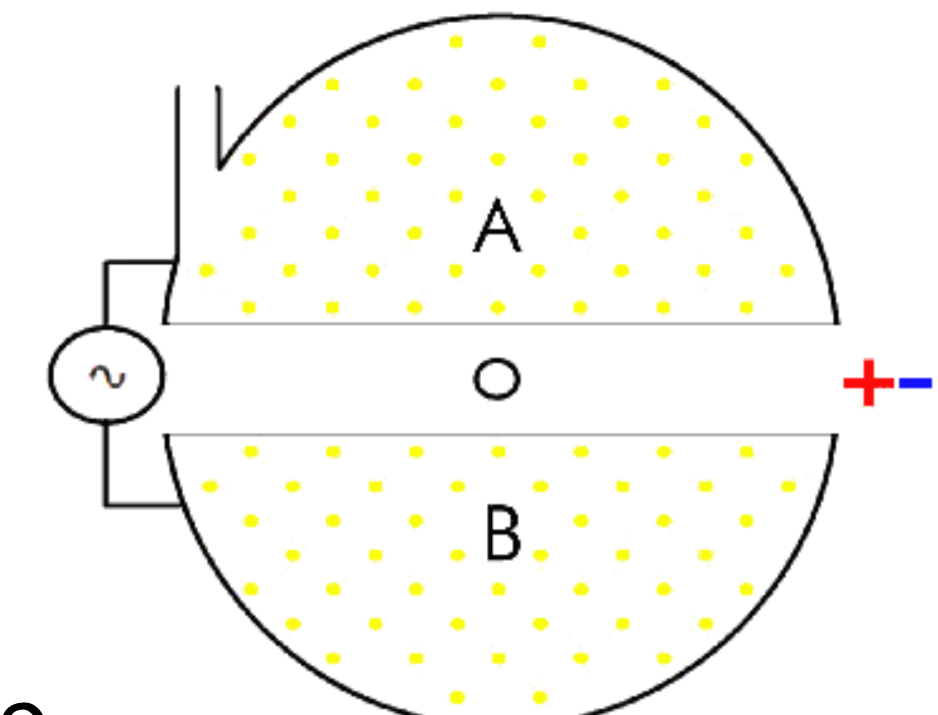



– need very large (MegaVolt) potentials ... very difficult

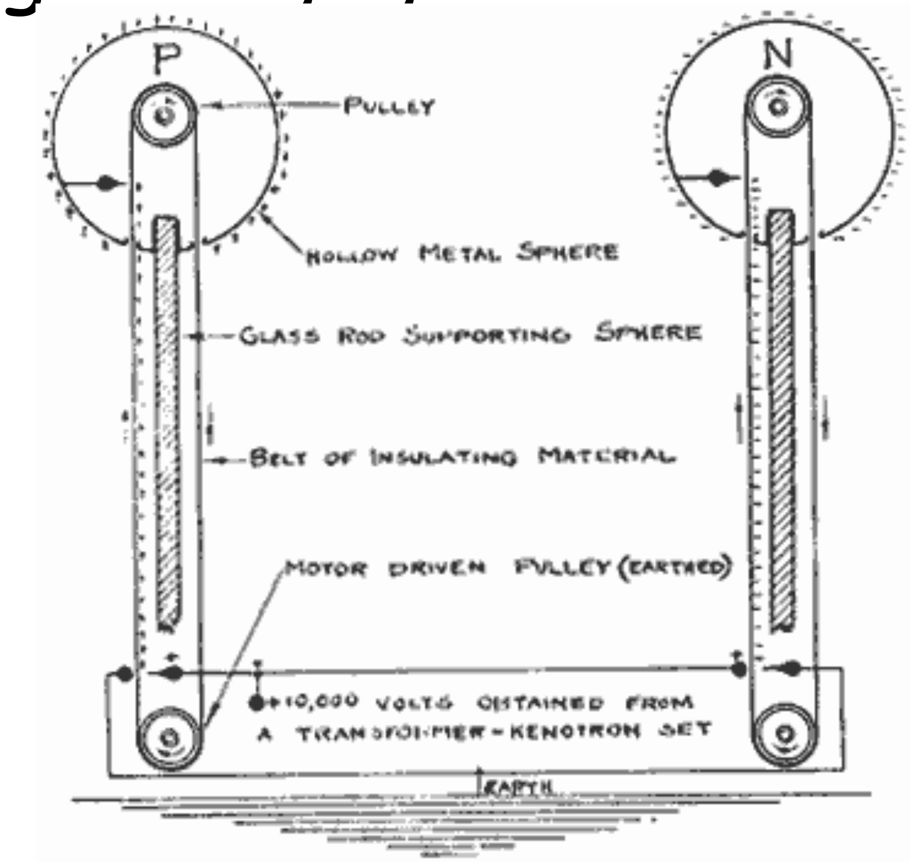
- In 1926, Rolf Wideroe wrote a paper proposing a linear accelerator using an oscillationg field to accelerate charge particles multiple times with a lower potential:
  - only low voltages then available
  - needed to be quite long
  - ionized nuclei reach only 50 keV




- In January 1931, E.O. Lawrence shortens the Wideroe linear accelerator by magnetically bending particle trajectories in a circle
  - time to make 1/2 orbit is indep of E (NR)
  - reverse field across gap each 1/2 orbit
  - particle spirals to larger radius
  - 4.5" cyclotron achieves 80 keV
  - 11" cyclotron achieves 1.1 MeV August
  - 27" cyclotron achieves 3.6 MeV in 1932
  - 37" cyclotron finished in 1937, 60" in 1939

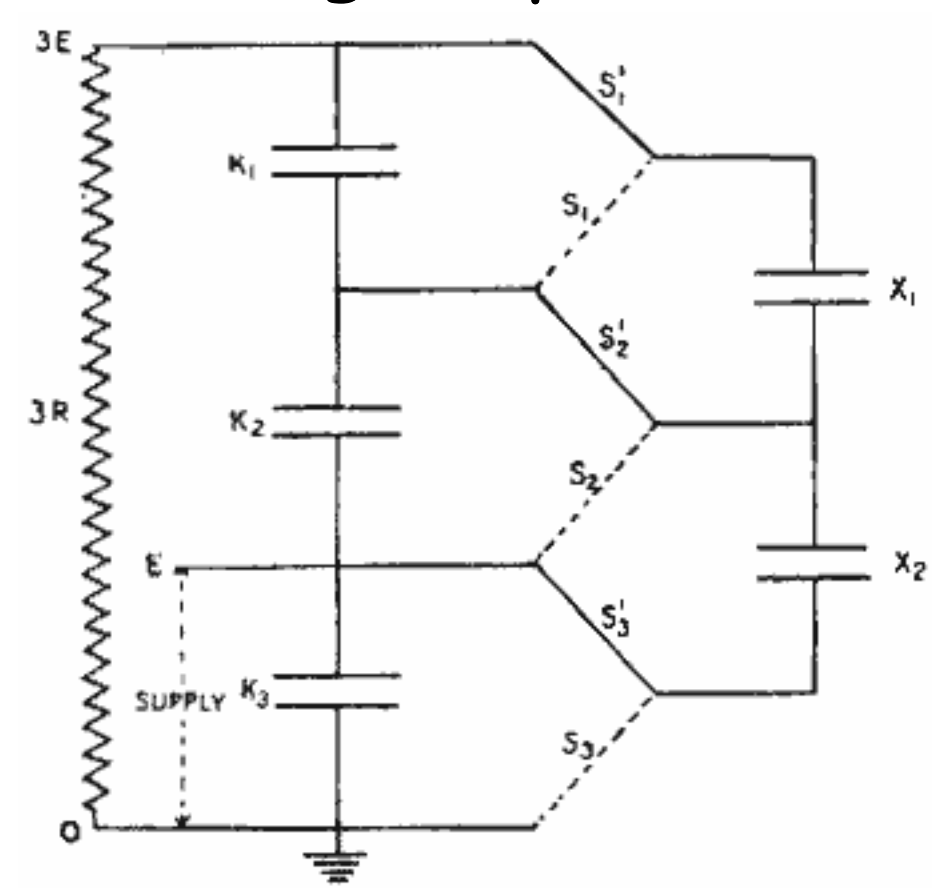


- 1929-1931, Van de Graff invents static high voltage generator:
  - reaches 2x750 kV
  - he is an engineer
  - not used to accelerate particles yet




- In 1932, Cockroft and Walton design a switched voltage amplifier
  - protons reach 800 keV
  - they do a PHYSICS experiment:

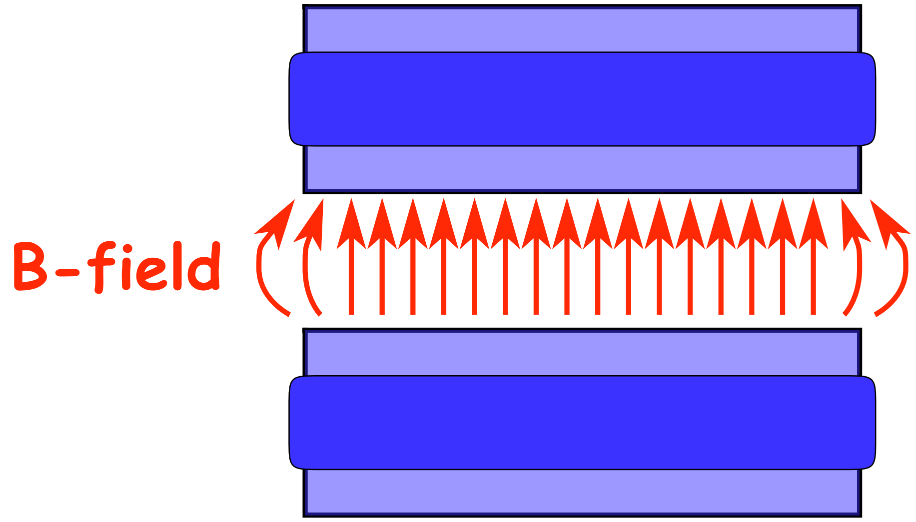
    $p + {}^{3}Li_{7}$ Æ $2 \times {}^{2}He_{4}$
  - discover artificial nuclear transmutation
  - win 1951 Nobel Prize



- Why do Cockcroft and Walton succeed with such a low energy?
  - because of Fermi momentum: caused by confinement of nucleons to a small volume (uncertainty principle):

$$p_F = (9\pi Z/4)^{1/3}\, \hbar / r_A$$

    - ✦ cm energy is increased by motion of target nucleons: overcomes the Coulomb barrier at lower projectile energy
- Why didn't Lawrence do this first?
  - he believed too much in theoretical predictions
  - he wasn't really interested in nuclear physics
  - he was interested in building bigger and better cyclotrons
- Why did the cyclotron work? It shouldn't have ....
  - beams should grow transversely and strike the vacuum chamber
    - ✦ dipole magnets have fringe fields which focus the beams
    - ✦ Lawrence was lucky!!



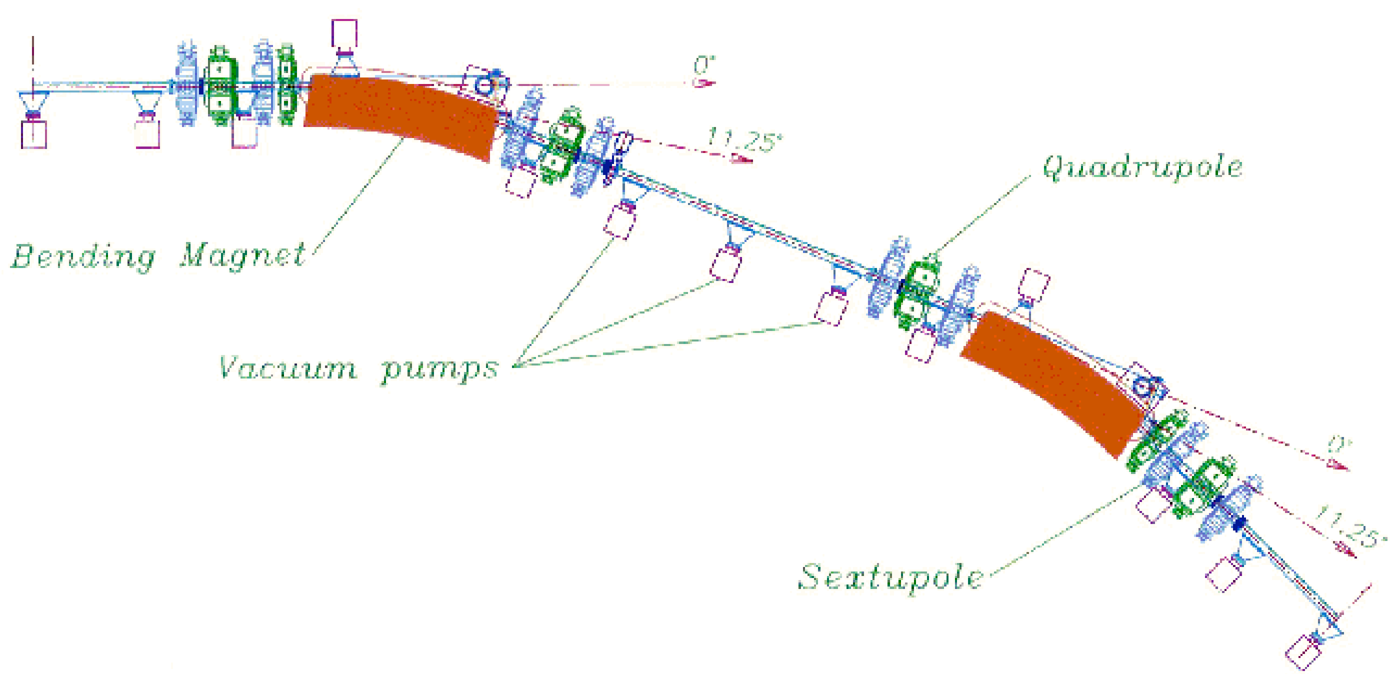
# Accelerator Milestones

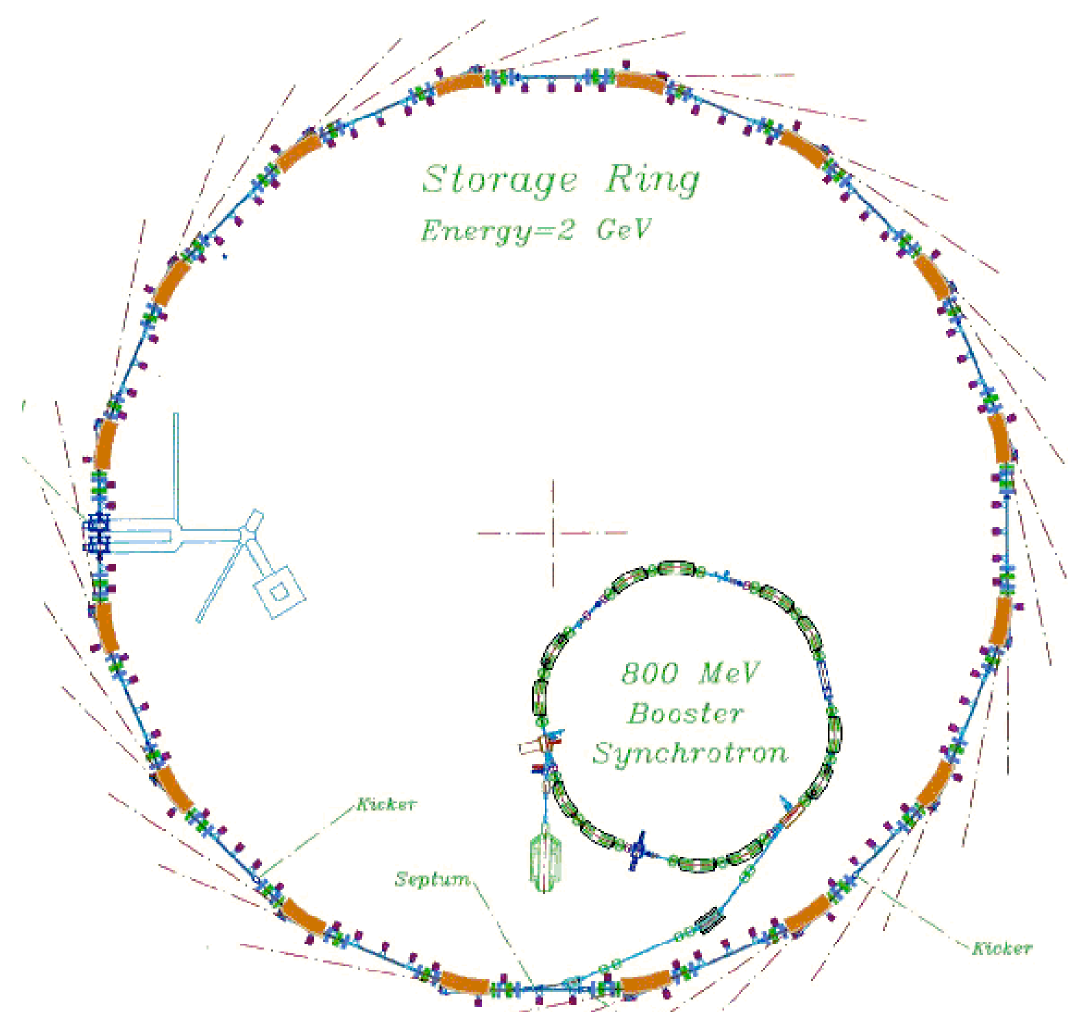
In roughly chronological order:

- high power microwave radar developed during WWII produces new powerful RF sources for post-war accelerators
- by late 1940's, cyclotrons are limited in energy by special relativity (1/2 orbit time is not independent of E):
  - modify RF frequency during acceleration cycle to keep the accelerating field synchronized to particles: Synchrocyclotron
  - trade off continuous low energy beam for pulsed higher energy beam
- build circular accelerator in which field is ramped as particles gain energy (keep orbit radius fixed): Synchrotron is developed in the 1950's (all modern circular accelerators are synchrotrons)
  - much less expensive and more flexible than cyclotrons
  - used to accelerate (and store) protons or electrons

# Synchrotrons

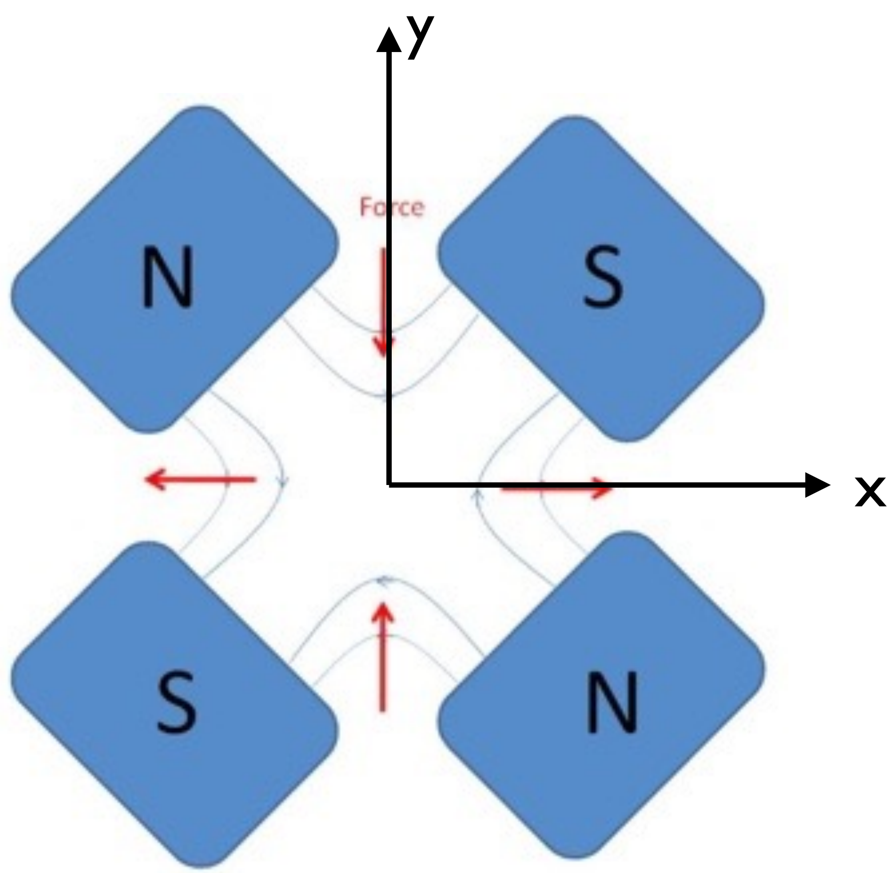
The synchrotron is a ring made of discrete elements: think cyclotron but with the middle removed. In early synchrotrons, the bending magnets were designed to also focus the beam using the non-linear edge fields [like the cyclotron]. The strength of the focusing was limited and the beam sizes [and therefore machine apertures] were large. All later synchrotrons use dedicated magnetic lenses which can strongly focus the beam. The beams and apertures are much smaller. Smaller beams make colliding beam machines possible too.



# Magnetic Lenses

Modern synchrotrons [and storage rings] use quadrupole magnets to focus the beam. A quadrupole magnet has two N pole faces and two S pole faces:

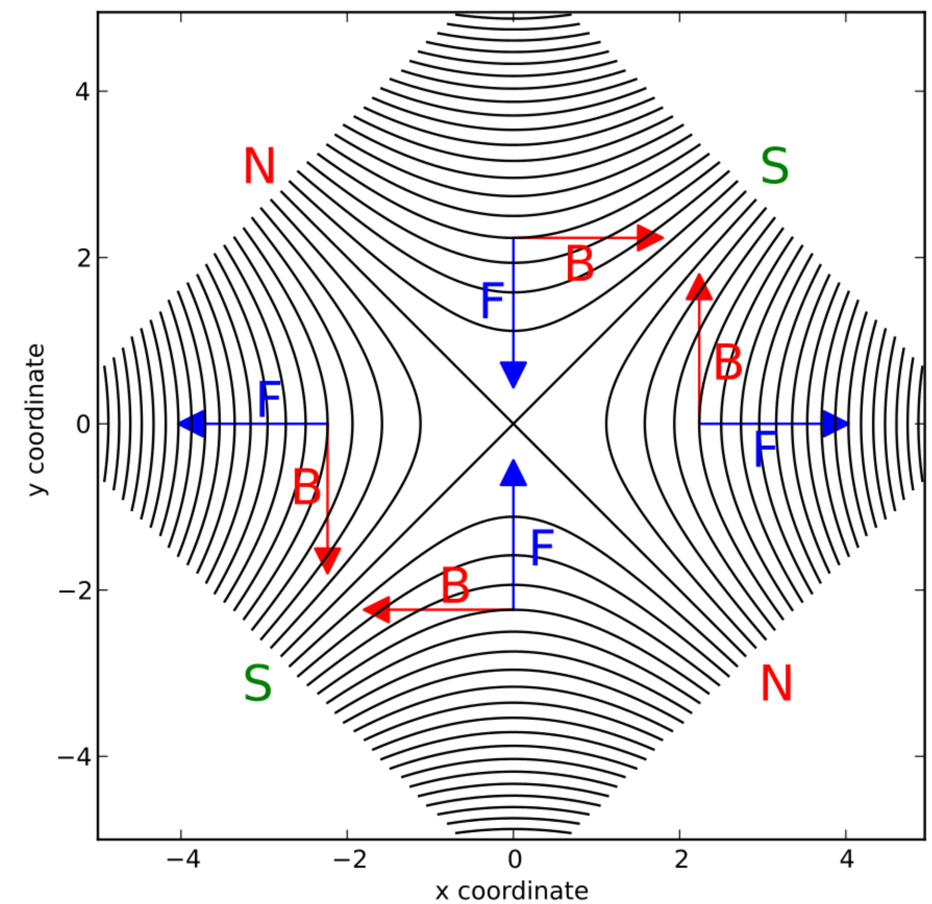



- the B-field vanished at the center of the magnet [by symmetry]
- the horizontal field increases with vertical distance y from the center
  - changes sign for $y<0$
- the vertical field increases with horizontal distance x from the center
  - changes sign for $x<0$

The field strength is characterized by a gradient G [T/m]. Assuming that the charged particle is traveling into the page [-z direction],

$$\vec{B} = G\left[y\hat{x} + x\hat{y}\right] \quad \rightarrow \quad \vec{F} = q\vec{v} \times \vec{B} = qG|\vec{v}|\left[x\hat{x} - y\hat{y}\right]$$

- this quadrupole focuses the beam vertically, defocuses horizontally
- can also change the polarity to focus horizontally, defocus vertically

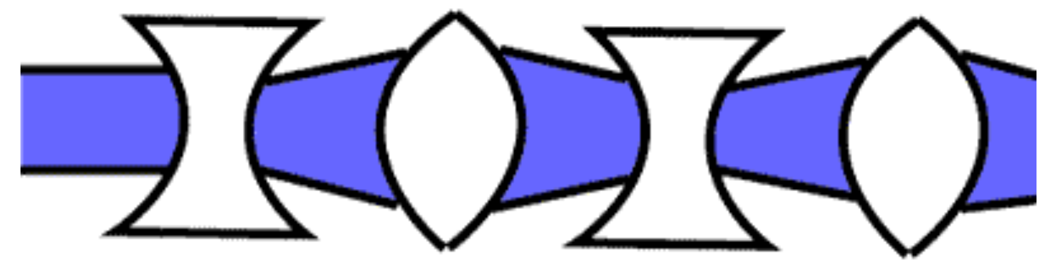



Calculating the angular deflection passing through a magnet of length L,

$$\theta_x = \frac{\Delta p}{p} = \frac{qGvx}{\gamma m v}\Delta t = \frac{qGL}{\gamma m v}x = \frac{x}{f}, \qquad f = \frac{p}{qGL}$$

The angular deflection is linear in x: it looks like an optical lens with focal length f!

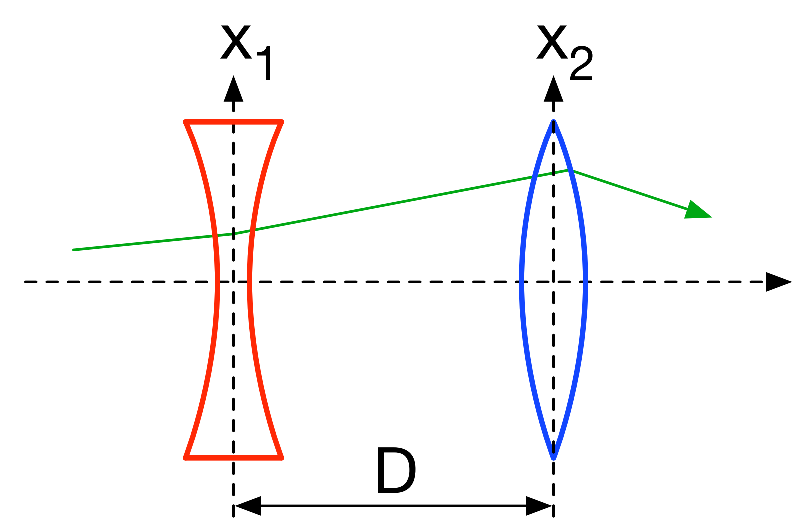
In accelerators, quadrupole lenses are arranged so that they alternately focus (H)/defocus (V) and defocus(H)/focus(V). In each plane, the net effect is to focus.



Proof: consider two thin lenses of focal length $f$ and separation $D$. The first is a defocusing lens and the second is a focusing lens. A thin lens changes the angle of a ray by $\pm x/f$. Let's consider a ray entering at $x_1$ with angle $\theta_1$. It propagates to the second lens and exits with angle $\theta_2$,

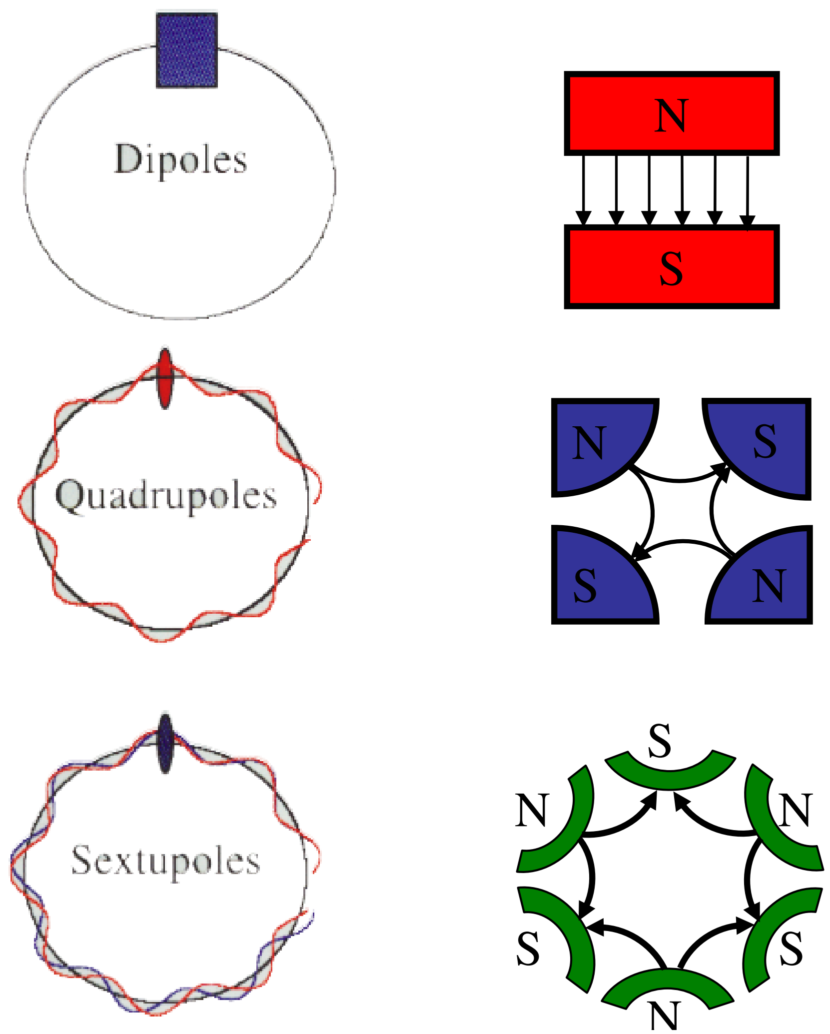
$$x_2 = x_1 + D\left[\theta_1 + \frac{x_1}{f}\right]$$

$$\theta_2 = \theta_1 + \frac{x_1}{f} - \frac{x_2}{f} = \theta_1\left[1 - \frac{D}{f}\right] - \frac{Dx_1}{f^2}$$




- $D/f < 1$ for any real system
- the output angle is always reduced by $x_1/F$,
  - the doublet lens ALWAYS focuses with focal length $F=f^2/D$
- the same result follows from putting the focusing lens first

Higher order multipole magnets are used to correct chromatic and other aberrations. Sextapoles are inserted into places of high dispersion [different momenta are separated spatially] to correct the chromatic effects




Bending (following reference trajectory)

Focusing the beam

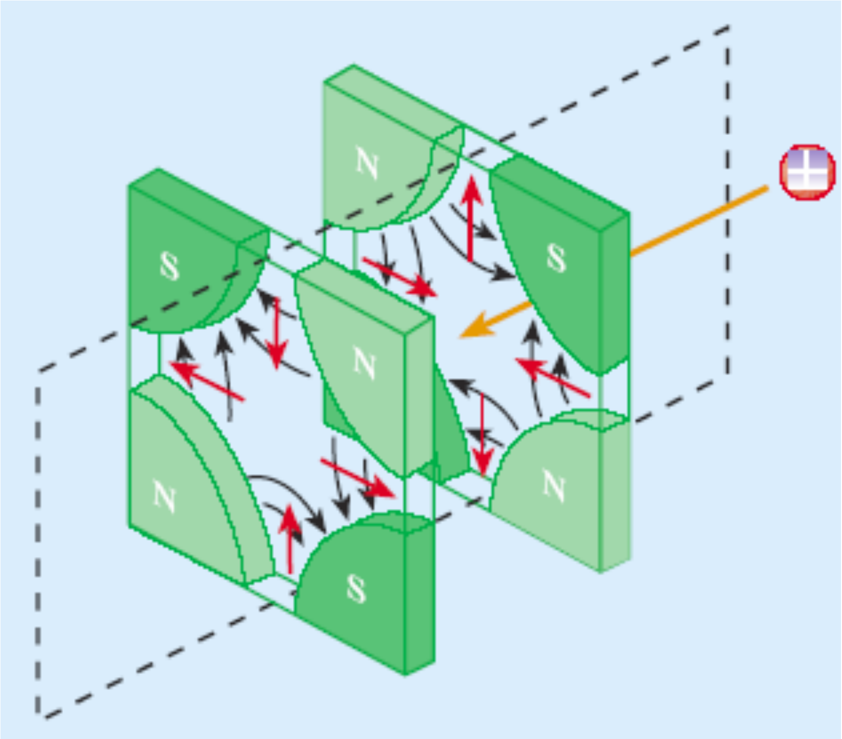
"Chromatic compensation"

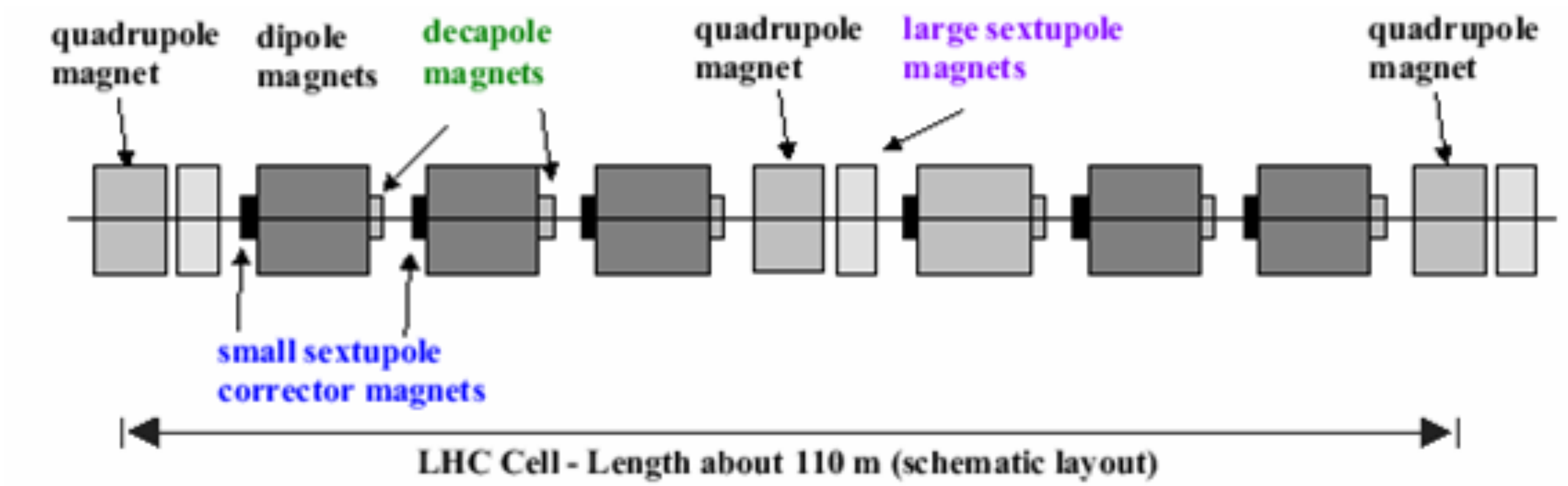
from Holmes, Henderson, Zhang

- this level of control is not possible in the older machines with combined function magnets
- the design of an accelerator lattice is something of an art

# LHC Magnet Lattice

To keep the beams from diffusing/wandering transversely, they are focused by a series of quadrupole lenses integrated into the magnet lattice



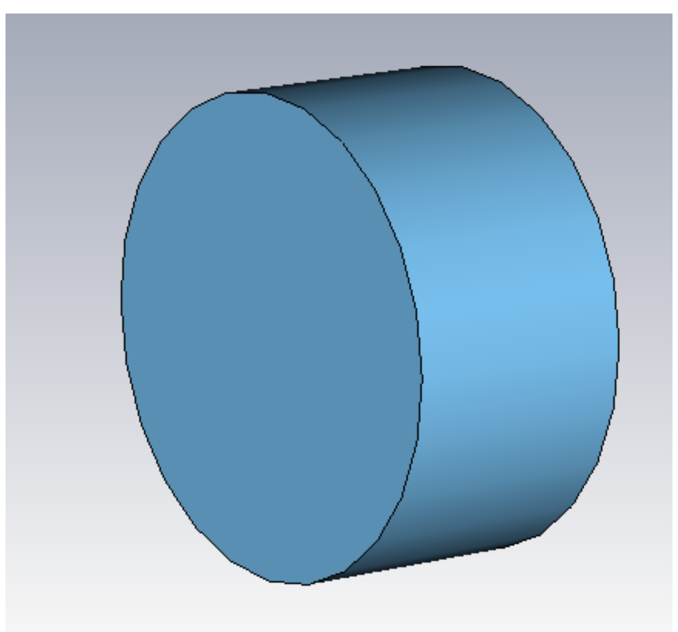




- A total of 858 quadrupoles focus the beams in both planes
  - quadrupole lenses are used in the final focii to demagnify the beams and being them into collision
- Sextapole and decapole magnets provide correction for aberrations
  - each f0d0 cell is 100m long

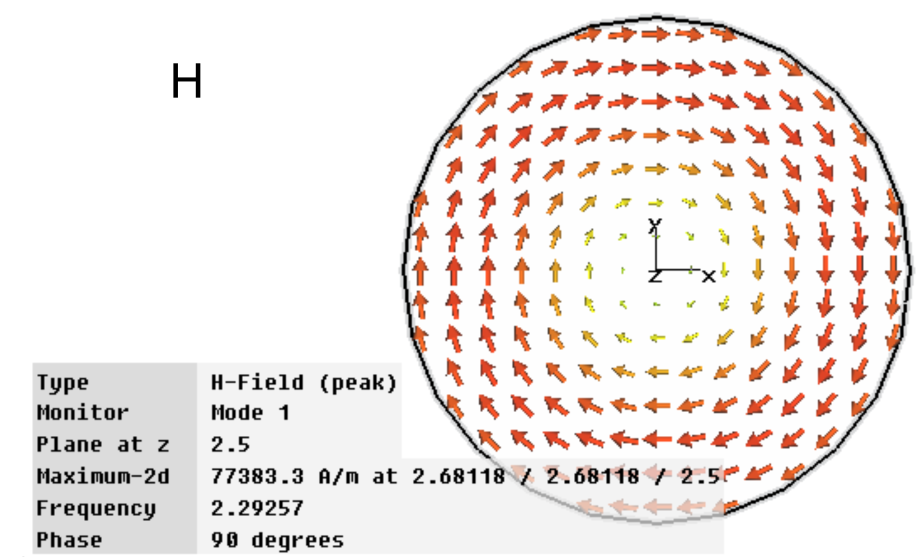
# RF Cavities

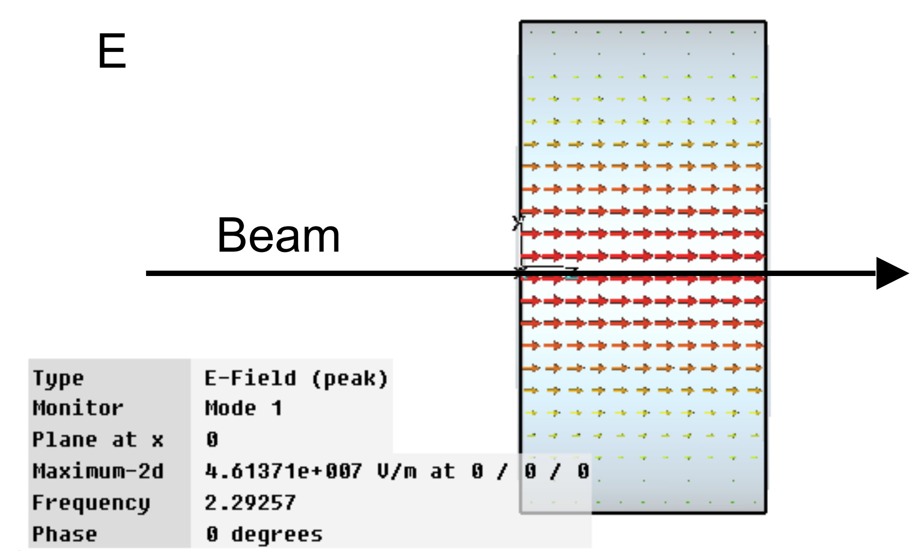
To use the new high power Radio Frequency sources [developed for Radar], need to use wave guides and cavities

- freely propagating EM waves have transverse E/B fields which won't accelerate charges
- use standing wave modes in RF cavities that have longitudinal E-fields
- make multiple passes through cavities to gain energy



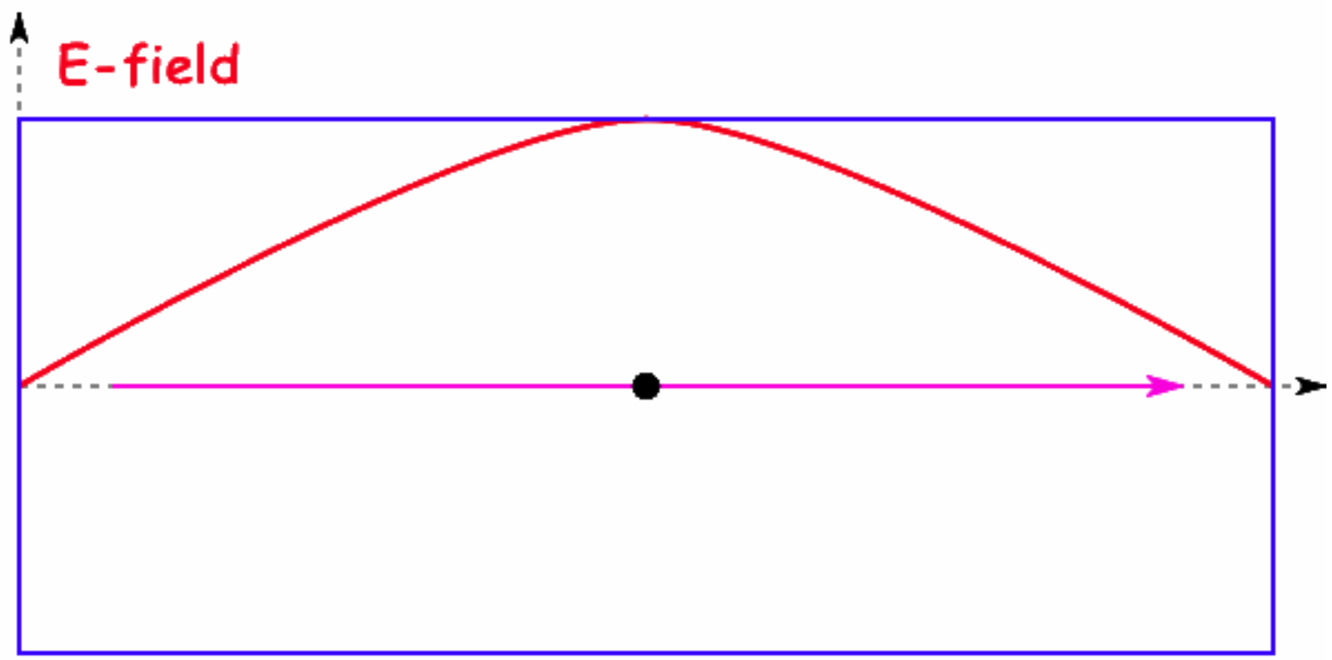
$TM_{010}$ Mode

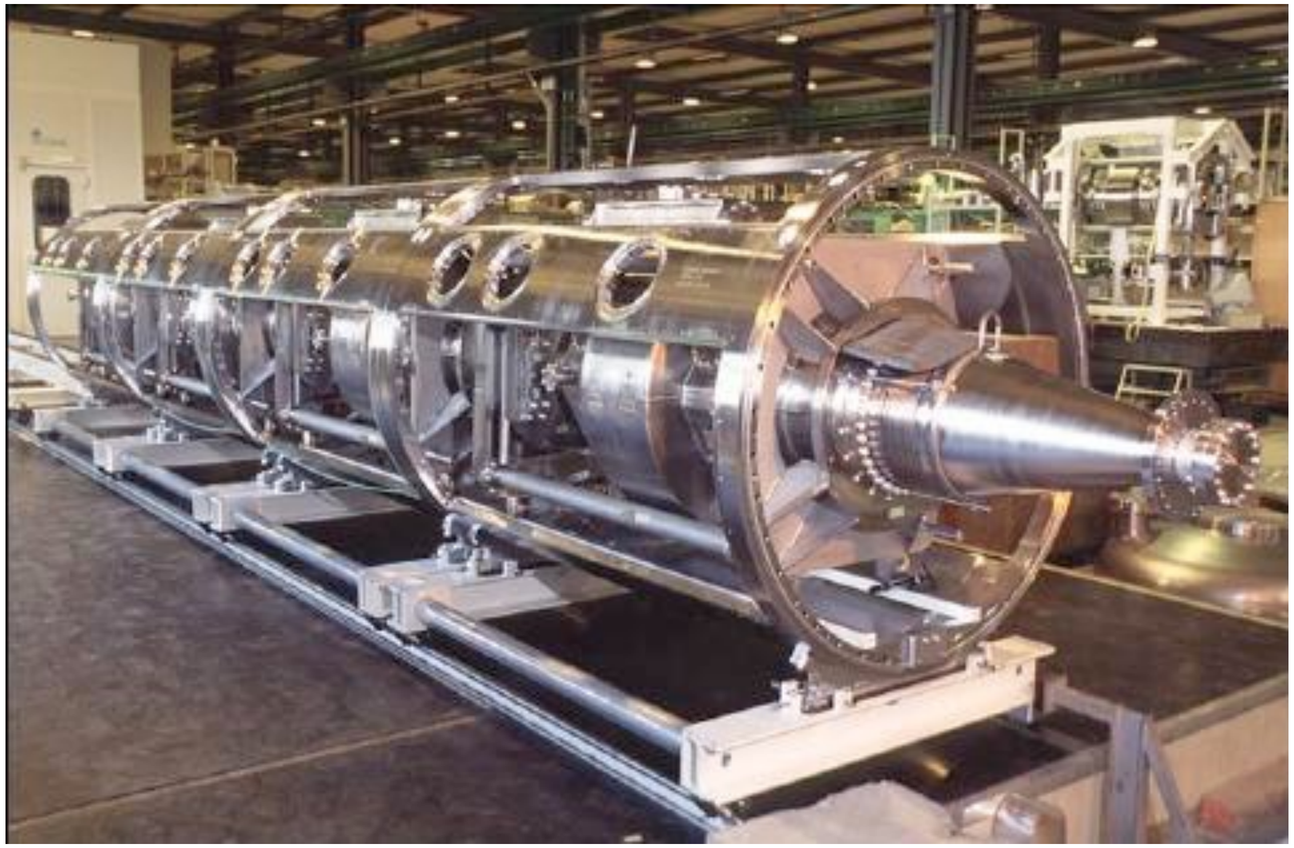


Time the passage of the bunches of beam particles to coincide with maximum positive longitudinal electric field

- high energy applications use Klystron power amplifiers
- lower energy applications [eg medical accelerators] use magnetrons like your microwave oven



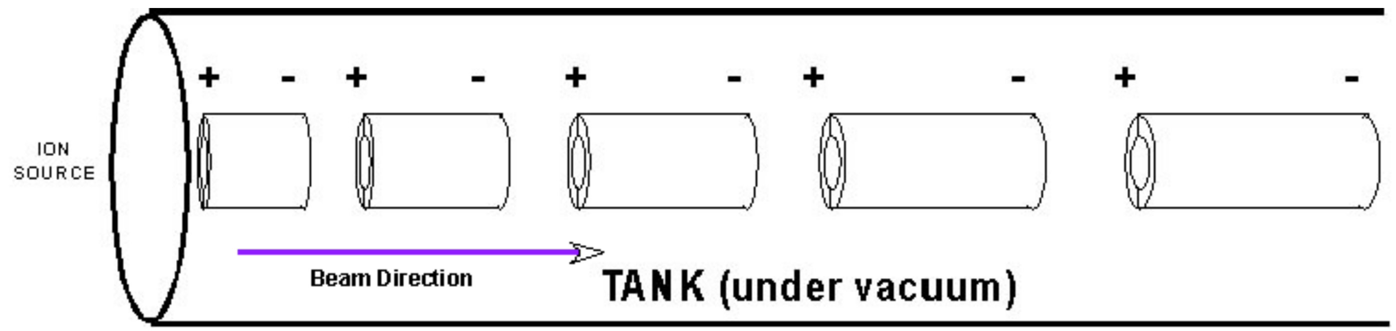



Each LHC proton beam passes through 2x4=8 RF cavities in each orbit

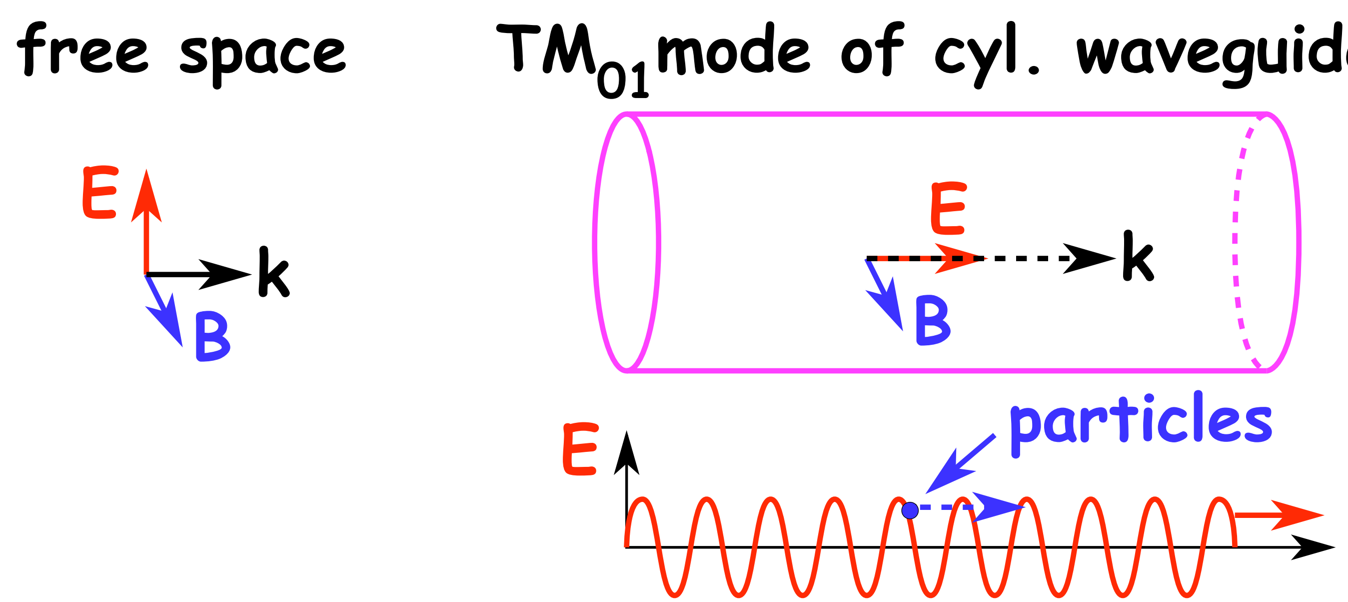
- each cavity: longitudinal e-field = 5 MV/m, eff len = 0.4m, f=400MHz
- gains T=8x2 Mev = 16 MeV/orbit

# Linear Accelerators

- Wideroe's original idea is still in use to accelerate low energy protons and ions,



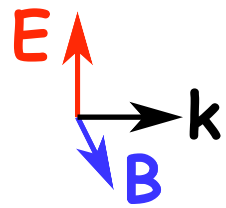

- mid 1950's, the traveling wave linear accelerator is developed



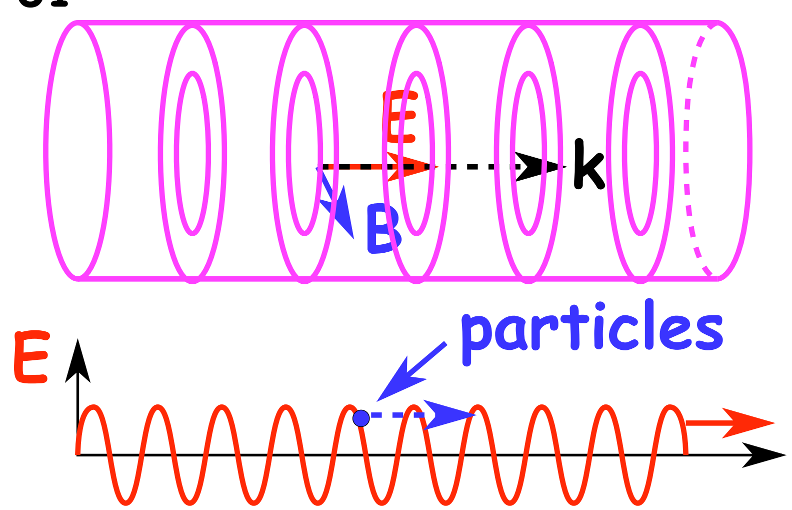

  - uses traveling axial electric field of waveguide
  - particles "surf" along leading edge of traveling field
  - works only for highly relativistic (v=c) particles [electrons]
  - full acceleration in a single pass [not repeated transits of cavity]

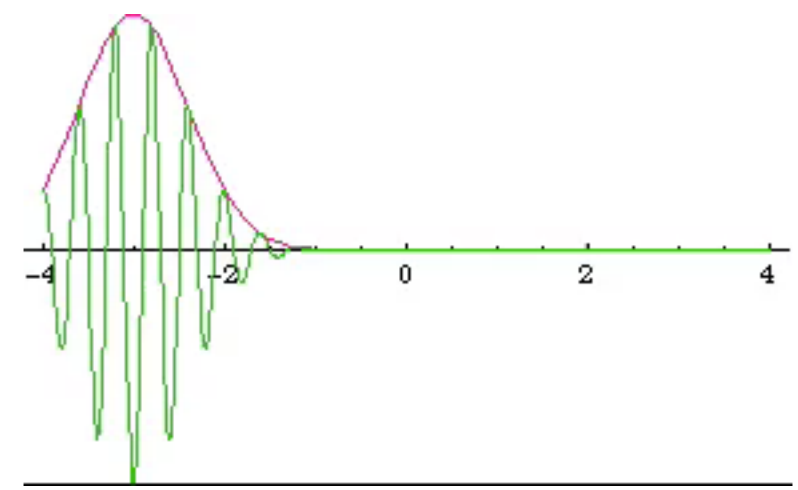
# An unmodified cylindrical waveguide cannot accelerate a highly relativistic particle

free space




TM$_{01}$ mode of cyl. waveguide




- the Phase Velocity $v_{ph} > c$
  - E-field won't stay synchronized with the traveling wave
  - fix by "loading" the waveguide with disks: slow $v_{ph}$ to c
- the Group Velocity of the RF pulse $v_{gr} = 0.03c$
  - particles quickly "outrun" the RF pulse
  - feed the waveguide every few meters with a "fresh" pulse

# Cross Section and Luminosity

Describing the rates of particle collisions relies on the concepts of luminosity and cross section

"Beam" particles                "Target" particles

$$\frac{N_{beam}}{\text{Area x Time}}$$

Effective area of one Target = $\sigma$

$$\text{Collision rate} = \frac{N_{beam}}{\text{Area x Time}} \times \sigma \times N_{target} = \underbrace{\frac{N_{beam} \times N_{target}}{\text{Area x Time}}}_{\text{Luminosity } \mathcal{L}} \times \underbrace{\sigma}_{\text{Cross Section}}$$

- Luminosity is instantaneous "brightness" of the accelerator
  - measured in inverse area per second
  - the design luminosity of the LHC is $1\text{x}10^{34}cm^{-2}s^{-1}$
  - design energy of the LHC is 14 TeV
  - actual energy/luminosity of the LHC are 13 TeV/$1.2\text{x}10^{34}cm^{-2}s^{-1}$

# Center of Mass Energy

The center of mass energy of the beam and target particle determine how much energy is available to produce new (interesting) states. The $E_{cm}$ of a collision


$$E_{cm} = (2ME_b + M^2)^{1/2} \simeq (2ME_b)^{1/2}$$

$$E_{cm} = 2E_b$$

- for fixed target collisions, the $E_{cm}$ scales as $(E_b)^{1/2}$
  - for $E_b$ = 6500 GeV, $E_{cm}$ = 110 GeV
- for collisions of 2 accelerated beams, the $E_{cm}$ scales as $E_b$
  - for $E_b$ = 6500 GeV, $E_{cm}$ = 13000 GeV

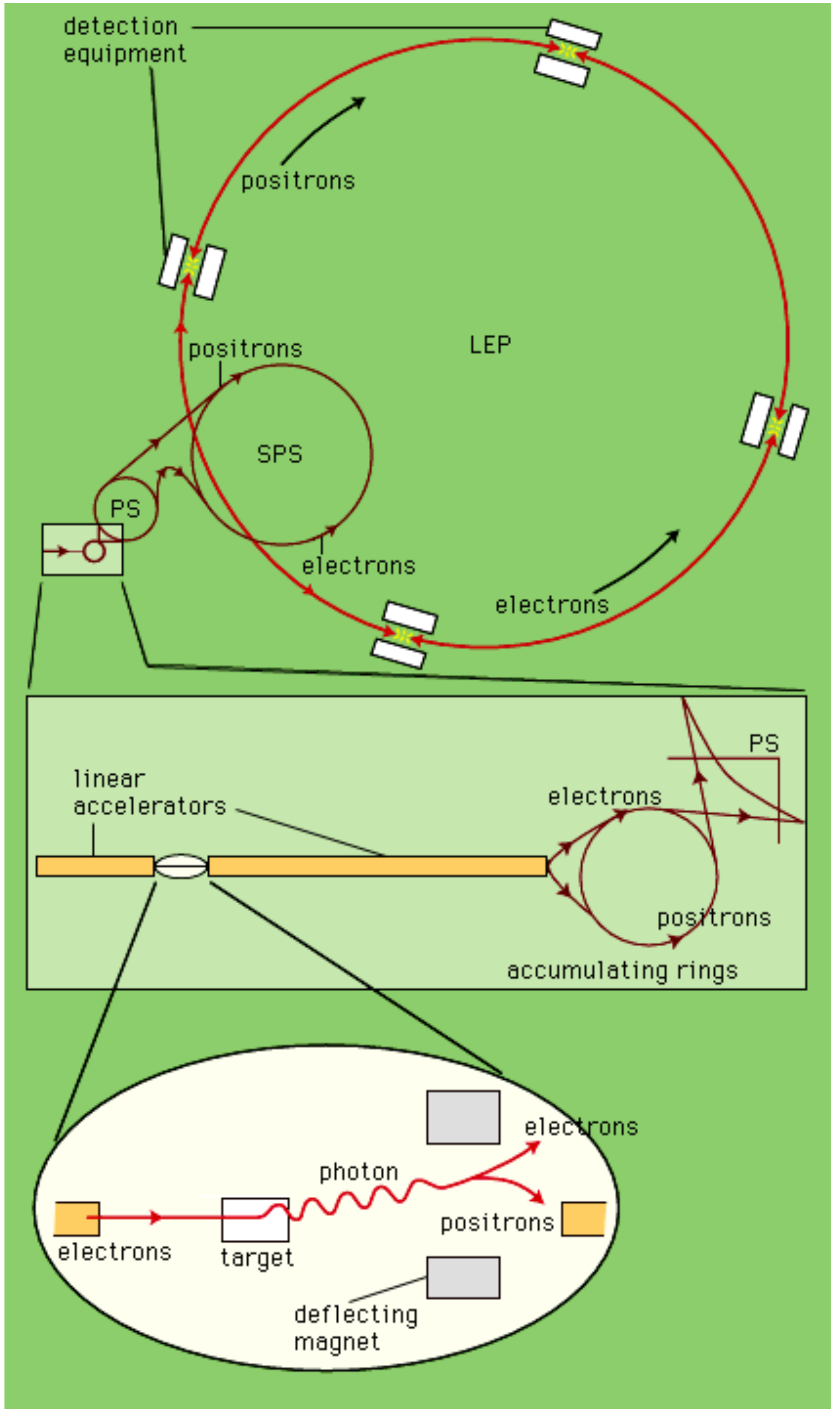
Huge energy advantages when colliding 2 accelerated beams!

Huge luminosity penalty due to low target density (~$10^5$)!

# Colliding Beam Accelerators

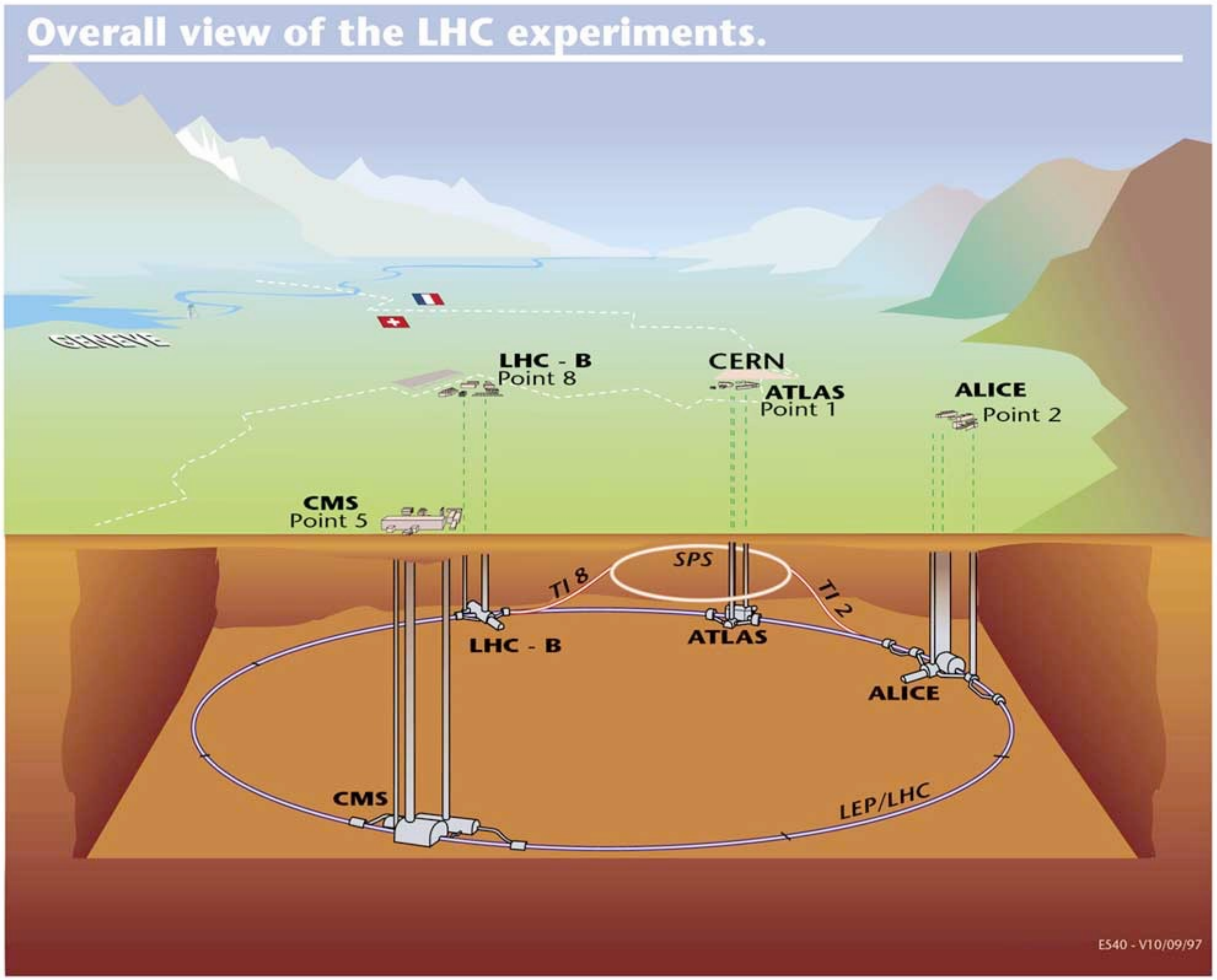
Beginning in the 1960s, colliding beam machines were developed to push the energy frontier

- earliest machines collided e-e- and e+e- beams
  - e+e- is probably the most valuable technique ever developed in HEP
  - lower energy but "clean" initial state
- 1970s-1980s pp and p-pbar colliding beam accelerators are developed
  - Tevatron at Fermilab is p-pbar @ $E_{cm}$ = 2 TeV
  - LHC operating at CERN is p-p @ $E_{cm}$ = 13 TeV
  - higher energy but "dirty" initial state ... more reach but higher backgrounds



# The Present

~14 TeV Large Hadron Collider is now operating at CERN near Geneva.




- chain of 5 accelerators: cockroft-walton, linac, 3 synchrotrons
- supports 2 major colliding beam experiments: Atlas + CMS
- supports 2 smaller experiments: Alice + LHCb

# The Present

# The Longer Future

There is international interest in building a ~1 TeV linear e+e- collider called the International Linear Collider (ILC)

- would help understand the complexities of new physics discovered at the LHC
- support 1-2 experiments
- timescale unknown (probably not before 2025)
- cost: $5-10 billion??