A Brief Introduction to Neutrino Oscillations

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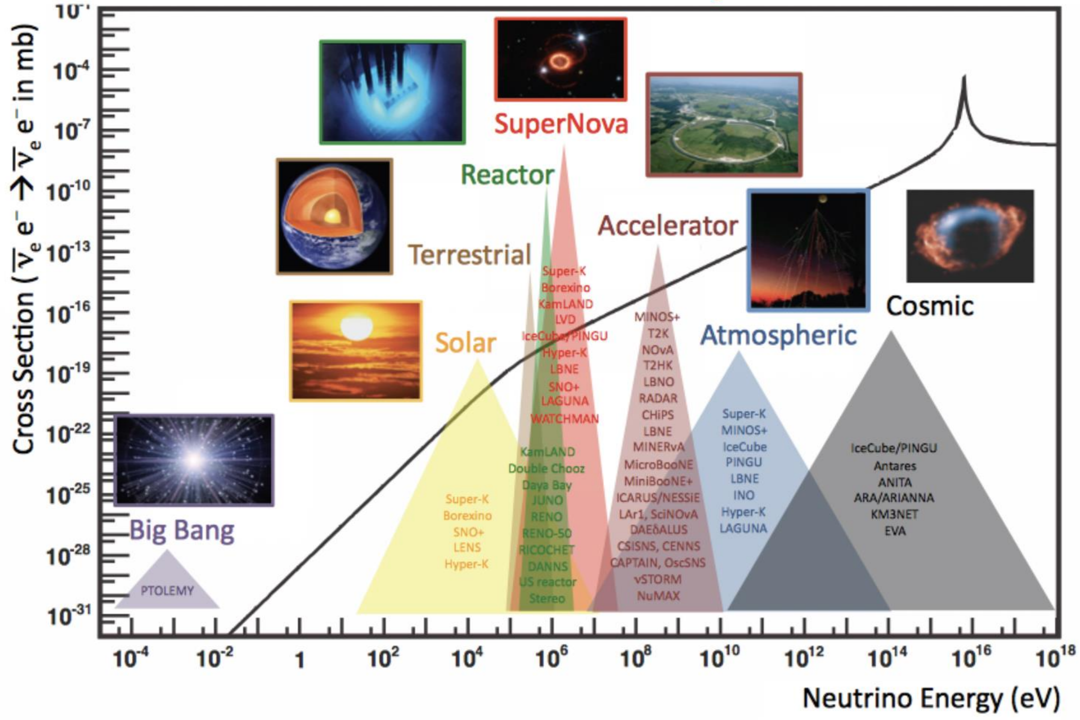
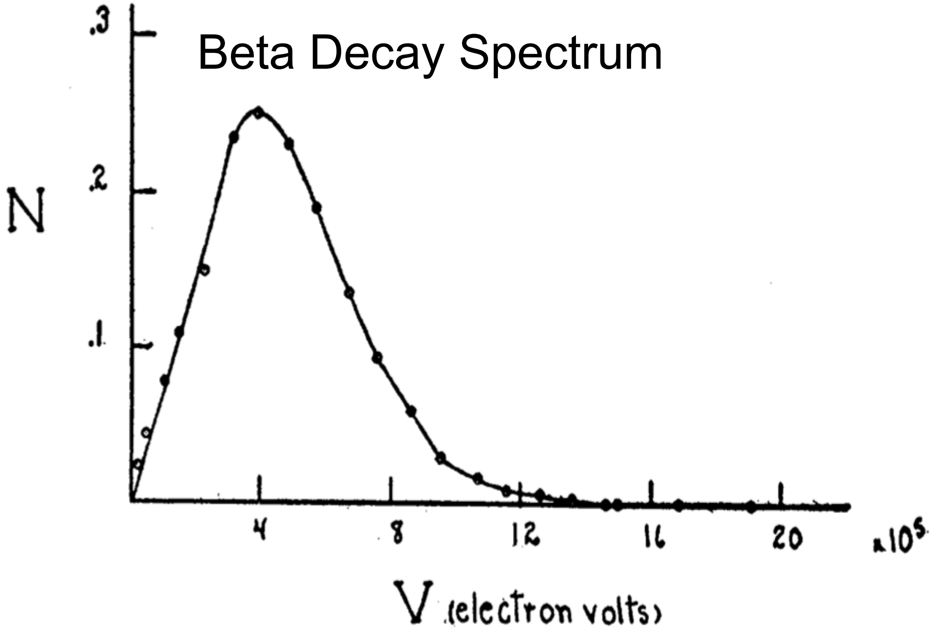
Neutrino Mini-Workshop

07/21/2026

Neutrinos are everywhere!



1930: Neutrino existence is postulated



Scott, F. A. Phys. Rev. 48.5 (1935): 391.

Particle properties:

- no electric charge
- spin 1/2 fermion
- massless or tiny
- Fermi's "weak" interaction

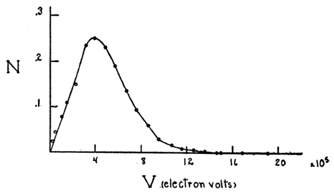
"I have done a terrible thing; I have postulated a particle that cannot be detected."

Neutrino Physics Through the Years

Neutrino postulation

Neutrinos introduced by Pauli to save conservation laws in β decay experiments

1930



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First observation of neutrino

Observation of (anti)neutrinos at Savannah Reactor through IBD with scintillator

1950

Savannah reactor

Solar Neutrino

Homestake experiment observed for the first-time solar neutrinos

1970

Homestake mine

Oscillations (mass)

KamLAND kiloton detector at long baselines, discovery of antineutrino oscillations (mass)

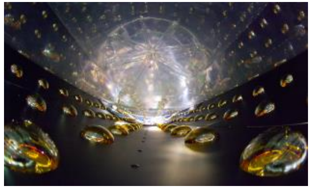
2000

KamLAND

Mixing angle

DYB,DC,RENO use near/far detectors for precision measurement of last mixing angle

2010



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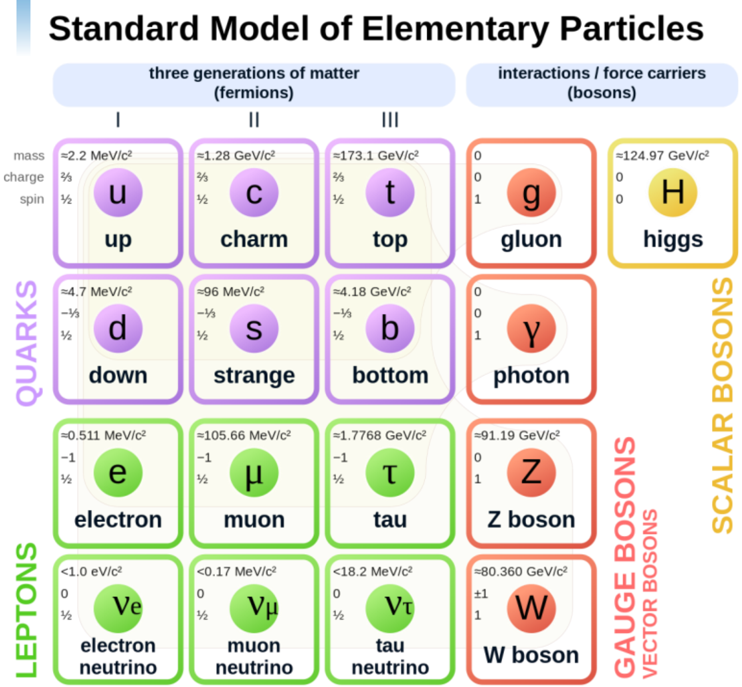
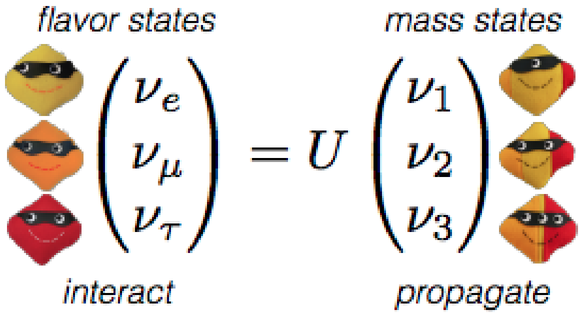
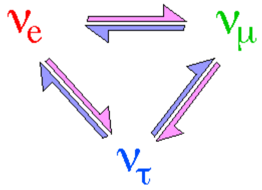
Neutrinos in the Standard Model



Current picture

- 3 types of neutrinos corresponding to three lepton generations
- Neutrinos are massless in the SM
- Electrically neutral and weakly interacting
- Neutrino oscillations are evidence for the non-zero mass of the neutrino (BSM physics)





$$P_{\alpha\to\beta} = \delta_{\alpha\beta} - 4\sum_{j>k} \mathcal{R}_e\left\{U^*_{\alpha j}\, U_{\beta j}\, U_{\alpha k}\, U^*_{\beta k}\right\} \sin^2\left(\frac{\Delta_{jk}m^2\, L}{4E}\right)$$

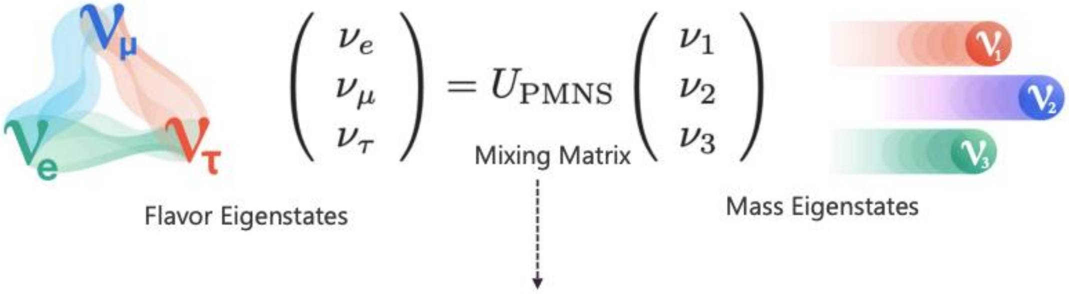
Neutrinos in the Standard Model:
- Open Questions

Many of these questions are studied with oscillation experiments

Image by Symmetry Magazine

Neutrino Oscillations: 3-neutrino picture

- Any quantum state can be written as a superposition — a linear combination — of a complete set of basis states.
- Not an approximation: this is how quantum states work. Flavor (ve,vμ,vτ) and mass (v1,v2,v3) are two different, equally valid bases for describing the same neutrino.
- Same idea as an ordinary vector: written in the standard x-y axes, or in a rotated x′-y′ axes — same vector, different coordinates.
- The two sets of coordinates are related by a matrix — the change-of-basis matrix. That matrix is the PMNS matrix.



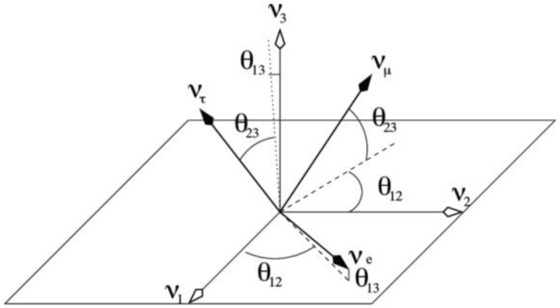


Figure 1.4: Neutrino flavor basis $(\nu_e, \nu_\mu, \nu_\tau)$ relative to mass basis (ν_1, ν_2, ν_3) with mixing angles $(\theta_{12}, \theta_{23}, \theta_{13})$ displayed.

Two state system: Two-neutrino approximation

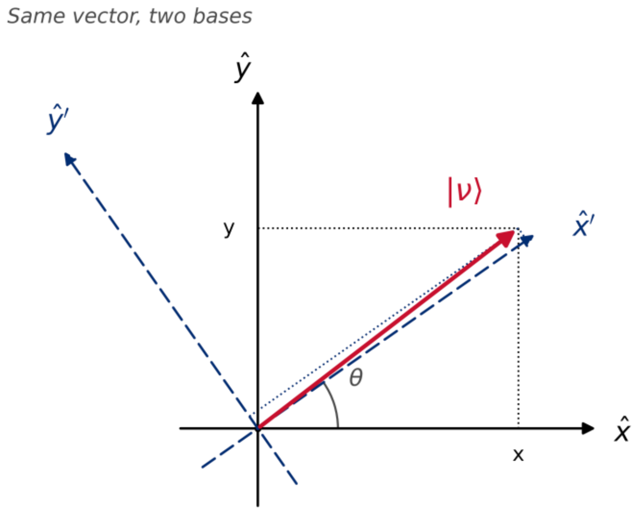
- There are only two types of neutrinos: electron and muon.
- The entire phenomenon takes place in a vacuum during the propagation from the Sun to the Earth

Flavor (weak interaction) eigenstates:

$$|\nu_A\rangle \;, |\nu_B\rangle$$

Mass eigenstates:

$$|\nu_1\rangle \;, |\nu_2\rangle$$



Unitary transformation:

$$\begin{pmatrix} |\nu_A\rangle \\ |\nu_B\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} \\ -\sin\theta_{12} & \cos\theta_{12} \end{pmatrix} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix}$$

Initial state, created by weak interaction (e.g. β-decay):

$$|\psi(0)\rangle = |\nu_A\rangle = \cos\theta_{12}\,|\nu_1\rangle + \sin\theta_{12}\,|\nu_2\rangle$$

e.g., $\nu_A = \nu_e$

Time evolution:

$$|\psi(t)\rangle = e^{-iE_1 t}\cos\theta_{12}\,|\nu_1\rangle + e^{-iE_2 t}\sin\theta_{12}\,|\nu_2\rangle$$

What is probability of being in state $|\nu_A\rangle$ at time t after creation ?

$$\mathcal{P}(\nu_A \to \nu_A)$$

"Survival probability"

Survival amplitude:

$$\langle \nu_A | \psi(t) \rangle = e^{-iE_1 t} \cos\theta_{12} \langle \nu_A | \nu_1 \rangle + e^{-iE_2 t} \sin\theta_{12} \langle \nu_A | \nu_2 \rangle$$

$$= e^{-iE_1 t} \left[\cos^2\theta_{12} + \sin^2\theta_{12} e^{-i(E_2 - E_2)t} \right]$$

Survival probability:

$$\mathcal{P}(\nu_A \to \nu_A) = 1 - 4\cos^2\theta_{12} \sin^2\theta_{12} \sin^2[(E_2 - E_1)t/2]$$

Massive, relativistic neutrinos (why ?)

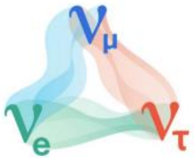
$$E_2 - E_1 = \sqrt{p^2 + m_2^2} - \sqrt{p^2 + m_1^2} \approx \frac{m_2^2 - m_1^2}{2p} \approx \frac{m_2^2 - m_1^2}{2E} \qquad t = L/v \approx L$$

$$\mathcal{P}(\nu_A \to \nu_A) = 1 - 4\cos^2\theta_{12}\sin^2\theta_{12}\sin^2\left[\frac{(m_2^2 - m_1^2)L}{4E}\right]$$

$$= 1 - \sin^2 2\theta_{12}\sin^2\left[\frac{(m_2^2 - m_1^2)L}{4E}\right]$$

- *Two massless neutrinos:* $\theta_{12} = 0$
- *At least one massive neutrino:* $\theta_{12} \neq 0$ *and* $\mathcal{P}(\nu_A \to \nu_A) < 1$
- *Dependence on* Δm^2 *x (L/E)*
- *Transition probability:* $\mathcal{P}(\nu_A \to \nu_B) = 1 - \mathcal{P}(\nu_A \to \nu_A)$

Neutrino Oscillations: 3-neutrino picture



Flavor Eigenstates

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{\mathrm{PMNS}} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Mixing Matrix

Mass Eigenstates

$$U_{\mathrm{PMNS}} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}$$

$c_{ij} = \cos\theta_{ij}$
$s_{ij} = \sin\theta_{ij}$

Measured from the following neutrino sources

Solar

Reactor

Accelerator

Atmospheric

Image by Symmetry Magazine

Oscillation probability

$$P_{\alpha\to\beta} = \delta_{\alpha\beta} - 4\sum_{j>k} \mathcal{R}_e\left\{U^*_{\alpha j}\, U_{\beta j}\, U_{\alpha k}\, U^*_{\beta k}\right\} \sin^2\left(\frac{\Delta_{jk}m^2\, L}{4E}\right) + 2\sum_{j>k} \mathcal{I}_m\left\{U^*_{\alpha j}\, U_{\beta j}\, U_{\alpha k}\, U^*_{\beta k}\right\} \sin\left(\frac{\Delta_{jk}m^2\, L}{2E}\right)$$

where $\Delta_{jk}m^2 \equiv m_j^2 - m_k^2$.

parameter	best fit $\pm\, 1\sigma$	2σ range	3σ range
$\Delta m^2_{21}\,[10^{-5}\mathrm{eV}^2]$	$7.50^{+0.22}_{-0.20}$	7.12–7.93	6.94–8.14
$\lvert\Delta m^2_{31}\rvert\,[10^{-3}\mathrm{eV}^2]$ (NO)	$2.55^{+0.02}_{-0.03}$	2.49–2.60	2.47–2.63
$\lvert\Delta m^2_{31}\rvert\,[10^{-3}\mathrm{eV}^2]$ (IO)	$2.45^{+0.02}_{-0.03}$	2.39–2.50	2.37–2.53
$\sin^2\theta_{12}/10^{-1}$	3.18 ± 0.16	2.86–3.52	2.71–3.69
$\theta_{12}/^\circ$	34.3 ± 1.0	32.3–36.4	31.4–37.4
$\sin^2\theta_{23}/10^{-1}$ (NO)	5.74 ± 0.14	5.41–5.99	4.34–6.10
$\theta_{23}/^\circ$ (NO)	49.26 ± 0.79	47.37–50.71	41.20–51.33
$\sin^2\theta_{23}/10^{-1}$ (IO)	$5.78^{+0.10}_{-0.17}$	5.41–5.98	4.33–6.08
$\theta_{23}/^\circ$ (IO)	$49.46^{+0.60}_{-0.97}$	47.35–50.67	41.16–51.25
$\sin^2\theta_{13}/10^{-2}$ (NO)	$2.200^{+0.069}_{-0.062}$	2.069–2.337	2.000–2.405
$\theta_{13}/^\circ$ (NO)	$8.53^{+0.13}_{-0.12}$	8.27–8.79	8.13–8.92
$\sin^2\theta_{13}/10^{-2}$ (IO)	$2.225^{+0.064}_{-0.070}$	2.086–2.356	2.018–2.424
$\theta_{13}/^\circ$ (IO)	$8.58^{+0.12}_{-0.14}$	8.30–8.83	8.17–8.96
δ/π (NO)	$1.08^{+0.13}_{-0.12}$	0.84–1.42	0.71–1.99
$\delta/^\circ$ (NO)	194^{+24}_{-22}	152–255	128–359
δ/π (IO)	$1.58^{+0.15}_{-0.16}$	1.26–1.85	1.11–1.96
$\delta/^\circ$ (IO)	284^{+26}_{-28}	226–332	200–353

Anatomy of a Neutrino Oscillation Experiment

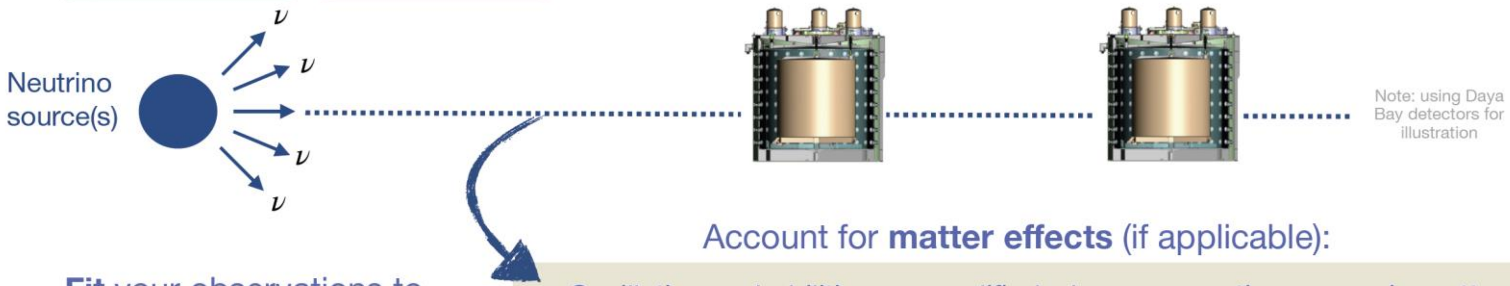
- How do you make a neutrino oscillation experiment?

Use a strong neutrino **source(s)**:

Artificial:
- Nuclear reactors
- Accelerators
- Radioactive sources

Natural:
- Sun
- Earth
- Supernovae
- Atmospheric (cosmic rays)

Sample the neutrino flux in at least one location:
- Sampling at multiple baselines → reduce flux uncertainties
- Using identical (or functionally-identical) detectors → reduce correlated detection systematics (e.g. efficiency and cross-section)

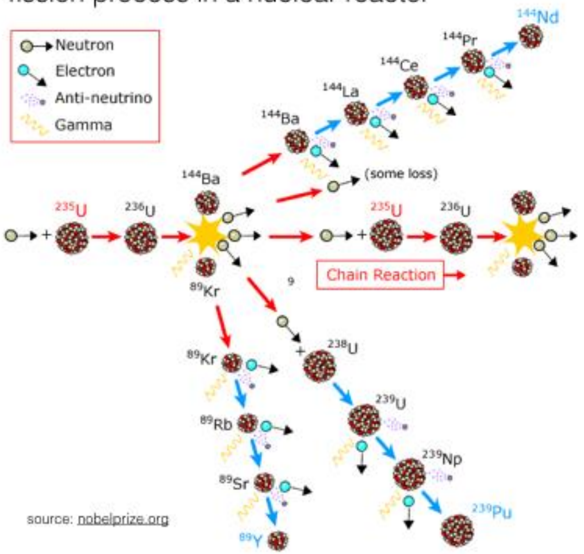


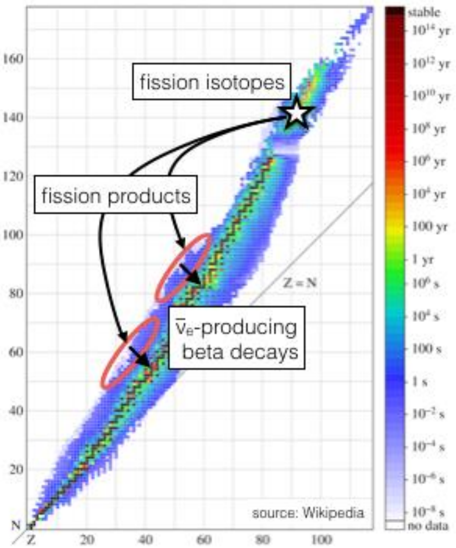
Account for **matter effects** (if applicable):
- Oscillation probabilities are modified when propagation occurs in matter
- Arises from ν_e's ability to experience CC scattering with electrons in addition to the NC interactions available to all flavors
- Provides sensitivity to **mass ordering**!

Fit your observations to the model and extract the parameter(s) of interest

Neutrino oscillations with reactors

fission process in a nuclear reactor

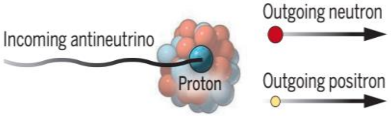




- Nuclear reactors are the most intense terrestrial sources of neutrinos
- They produce an immense flux of antineutrinos in the MeV range.
- Flavor pure, only electron antineutrinos are produced
- Allow for proximity to the source, and it is usually paid by others

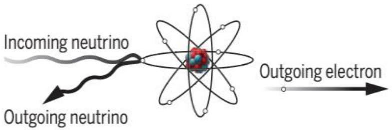
Inverse beta decay

An incoming antineutrino (neutrino) exchanges charge with a proton (neutron) in the nucleus, converting it to a neutron (proton) and becoming a positron (electron).



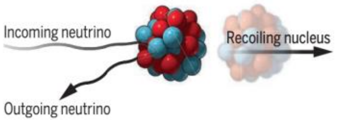
Neutrino electron elastic scattering

An incoming neutrino scatters off of an electron. It may exchange charge with the electron or it may not. In either case, a recoiling electron emerges in a direction that is very close to that of the incoming neutrino.



Coherent elastic neutrino-nucleus scattering (CEνNS)

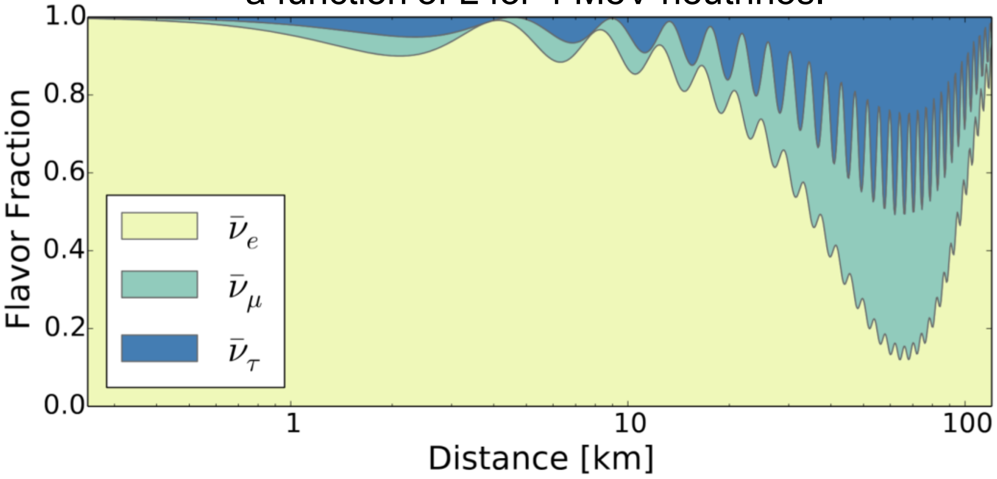
The entire nucleus recoils as a solid body off of an incoming neutrino whose quantum wavelength is comparable to the diameter of the nucleus. No charge or internal energy is transferred to the nucleus.



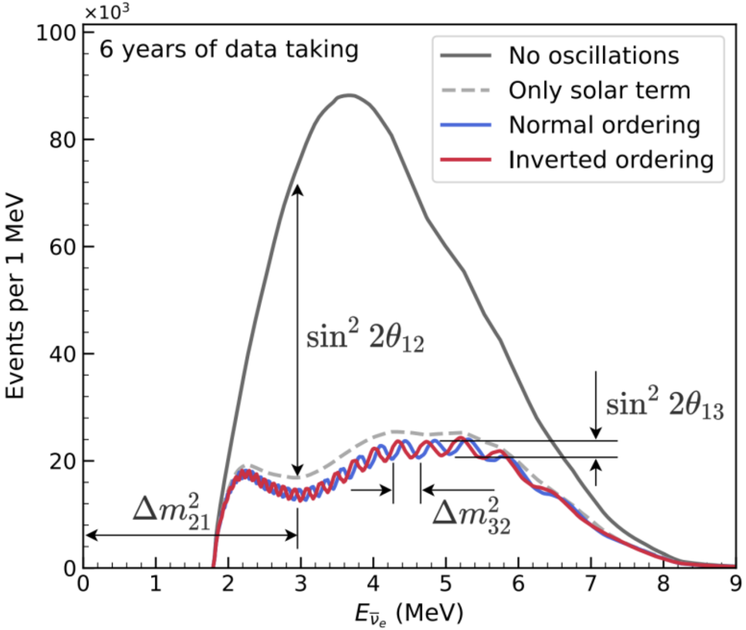
Link J M 2017 Scattering neutrinos caught in the act Science 357 1098–1099 URL https://www.science.org/doi/abs/10.1126/science.aao4050

Neutrino oscillations with reactors

Expected flavor composition of the Rx flux as a function of L for 4 MeV neutrinos.



Expected spectrum for JUNO (L = 53 km)



$$P_{\bar{\nu}_e \to \bar{\nu}_e} = 1 - 4|U_{e1}^2||U_{e3}^2| \sin^2 \Delta_{31} - 4|U_{e2}^2||U_{e3}^2| \sin^2 \Delta_{32} - 4|U_{e1}^2||U_{e2}^2| \sin^2 \Delta_{21}$$
$$= 1 - \sin^2 2\theta_{13}(\cos^2\theta_{12} \sin^2 \Delta_{31} + \sin^2 \theta_{12} \sin^2 \Delta_{32}) - \cos^4 \theta_{13} \sin^2 2\theta_{12} \sin^2 \Delta_{21}$$

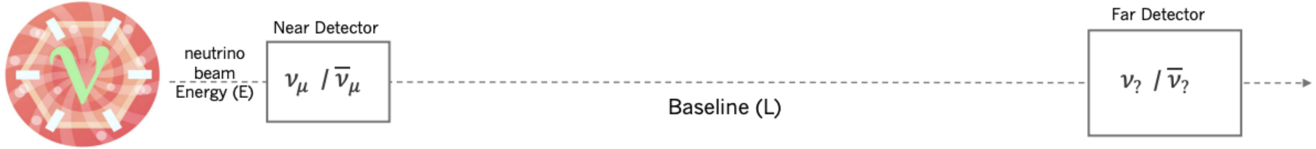
- Only disappearance channel available at current distances for MeV neutrinos

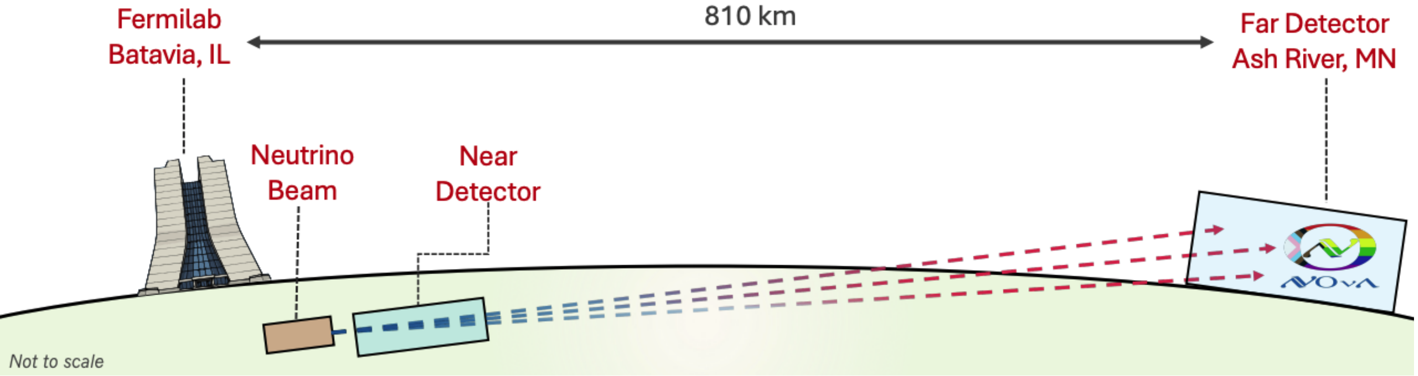
Neutrino oscillations with accelerators

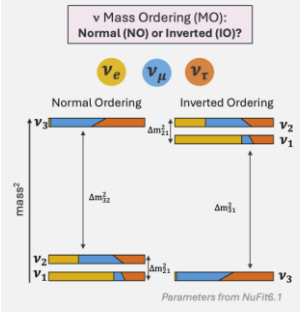
NOvA is a long-baseline neutrino oscillation experiment.

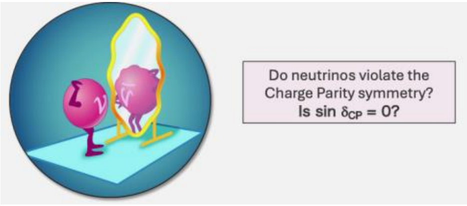
Primary goals: $\sin^2\theta_{23}$, $|\Delta m^2_{32}|$, mass ordering, δ_{CP} (and $\sin^2\theta_{13}$ to a lesser degree)

Strategy: use ν_μ disappearance and ν_e appearance in neutrino and antineutrino beams to disentangle the effects from CP violation, mass ordering, and oscillation parameters

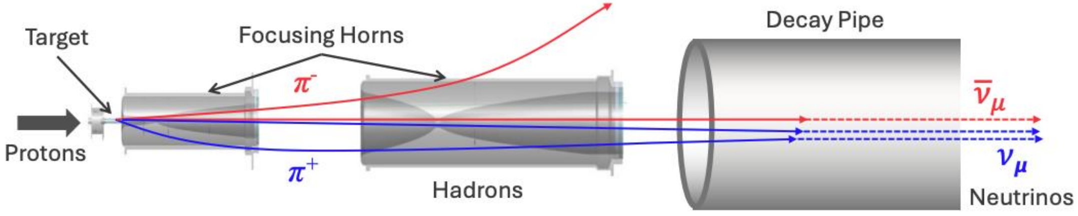


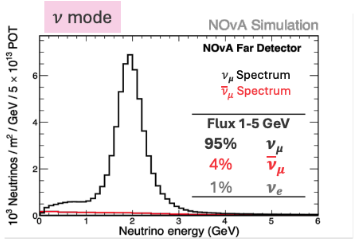


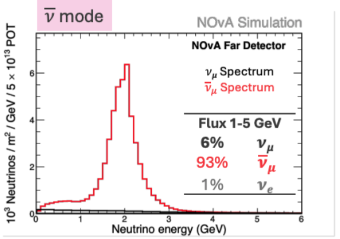




Neutrinos in NOvA:



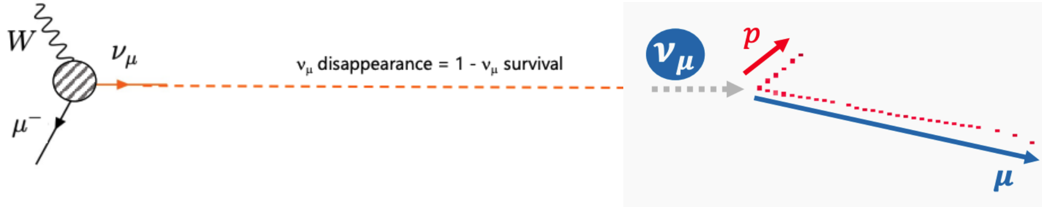




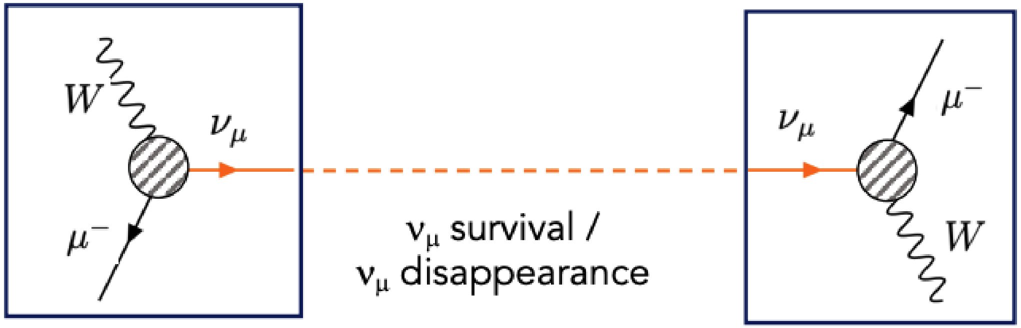
Two complementary oscillation channels at the FD:

- Muon (anti)neutrino **disappearance**

- Electron (anti)neutrino **appearance**

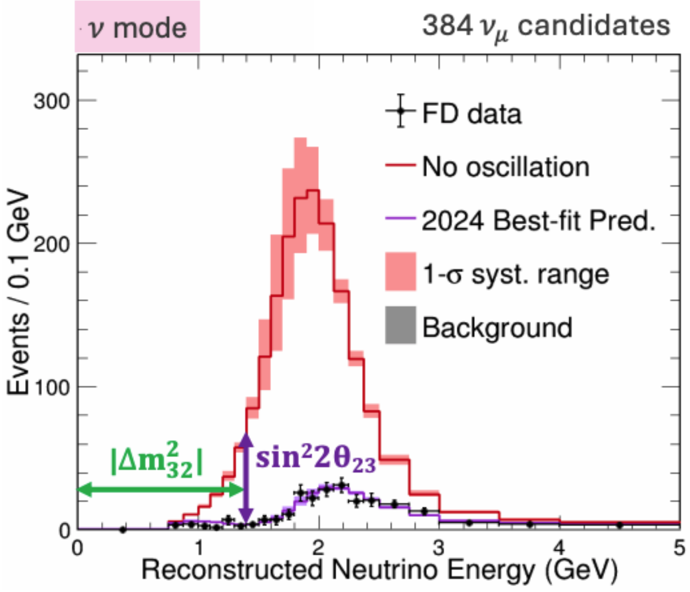


Neutrinos in NOvA: Muon (anti)neutrino **disappearance**

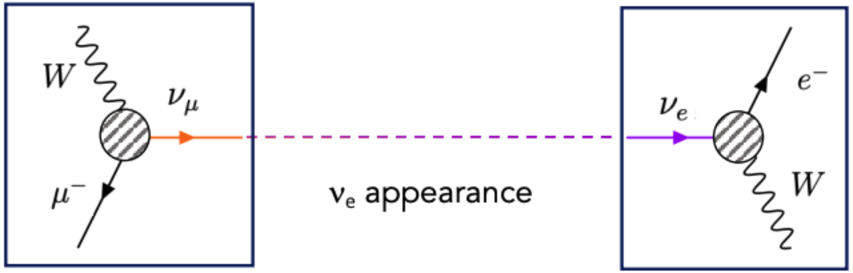


$$P\left(\overset{(-)}{\nu}_\mu \rightarrow \overset{(-)}{\nu}_\mu\right) \approx 1 - \sin^2 2\theta_{23} \sin^2\left(\Delta m^2_{32}\frac{L}{4E}\right)$$

- Leading order dependence on Δm^2_{32} and $\sin^2\theta_{23}$
- Aims to answer the question of **maximal mixing in θ_{23}**



Neutrinos in NOvA: Electron (anti)neutrino **appearance**



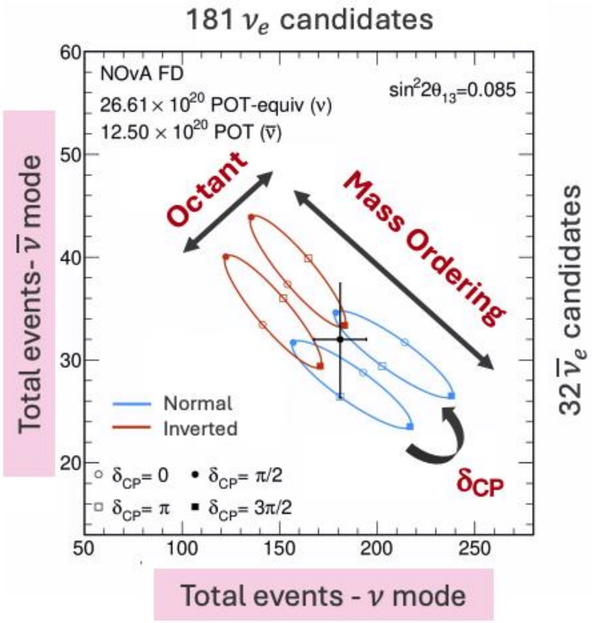
Mass Ordering

$$P\left(\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e\right) \simeq \sin^2\theta_{23}\,\sin^2 2\theta_{13}\,\frac{\sin^2(\Delta_{31} \mp aL)}{(\Delta_{31} \mp aL)^2}\Delta_{31}^2 \mp [\text{CP-odd}] + [\text{CP-even}] + [\text{solar}]$$

sin δ_{CP}

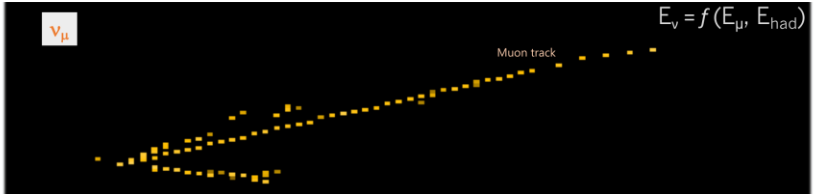
- Dependence on θ_{13}, θ_{23}, δ_{CP} and the neutrino mass ordering through the matter effect. Constraint on θ_{13} via the reactor neutrino experiments.

- Comparing the rate of ν_e appearance with $\bar{\nu}_e$ appearance provides a measurement of δ_{CP} and mass ordering.



Neutrinos in NOvA: CC vs NC

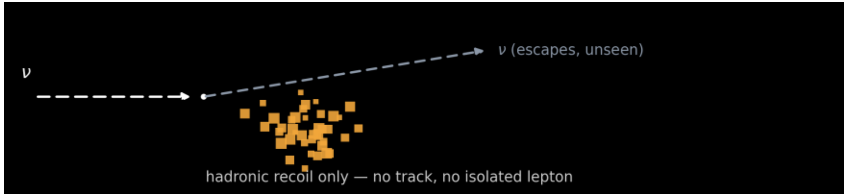
CC — real event display



Source: Vallari, NDM22 (2022)

- W boson exchange
- Produces a charged lepton matching flavor
- The lepton is the flavor tag: μ → long track, e → shower

NC — schematic (illustrative)



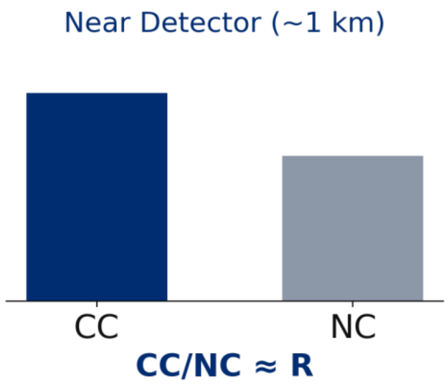
No real NC display in source material — illustrative

- Z boson exchange
- No charged lepton — just a hadronic shower
- Same rate for every flavor — flavor-blind

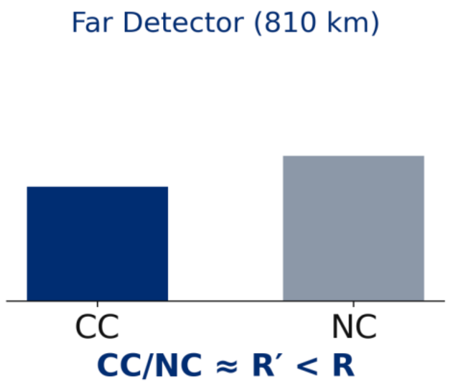
Key idea: the charged lepton (or its absence) is what tells CC and NC apart

Reconstructing Oscillations: CC/NC Ratio, Near vs Far

Near Detector (~1 km) — before oscillation acts



Far Detector (810 km) — after oscillation



A different ratio at Far than Near ⇒ oscillation occurred — no absolute flux prediction needed

Summary

Neutrinos: nearly massless, electrically neutral, and barely interacting yet essential to understanding matter, stellar energy loss, and physics beyond the Standard Model.

Oscillations: the direct proof that neutrinos have tiny but nonzero mass. Flavor states are superpositions of mass states that quantum interference lets a neutrino born as one flavor arrive as another.

NOvA: turns that effect into a measurement: comparing CC and NC rates at the Near and Far detectors reveals muon-neutrino disappearance over 810 km — exactly what you will do during the workshop.

The math you saw today, superposition, change of basis, interference, is the same math behind every result in this field. Enjoy the workshop, and thank you for bringing this into your classrooms!

Backup

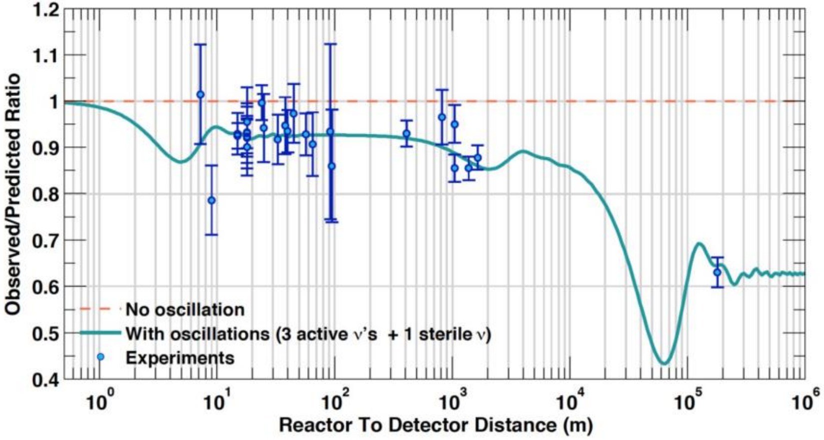
PROSPECT Motivations and Goals: Sterile Neutrinos?

Sterile Neutrino, 3+1 Scenario

- Expanding PMNS matrix to include sterile neutrino components
- Sterile Neutrino searches become simpler at short baseline
- $\mathrm{Sin}^2(2\theta)$ modulates the oscillation amplitude
- Δm^2 is the oscillation phase
- Δm^2 assumed to be ~1eV^2

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \nu_s \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \mu_4 \end{pmatrix}$$

$$P(\bar{\nu}_e \longrightarrow \nu_e) = 1 - \sin^2 2\theta_{14} \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right)$$



https://doi.org/10.1016/j.physrep.2021.06.002

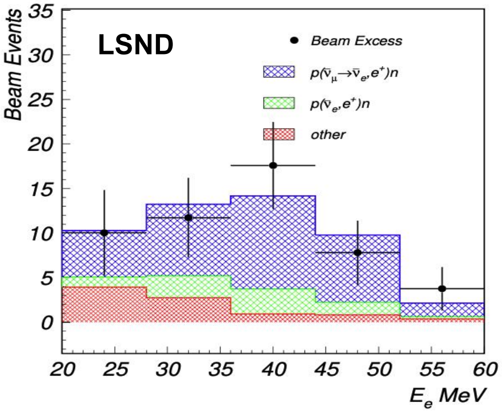
Motivation for an SBL program: Experimental Anomalies

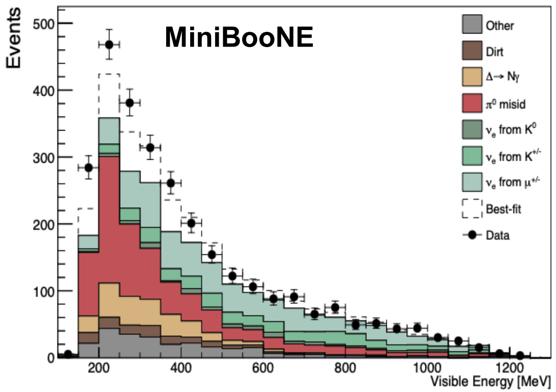
Exploring the Quantum Universe

From P5

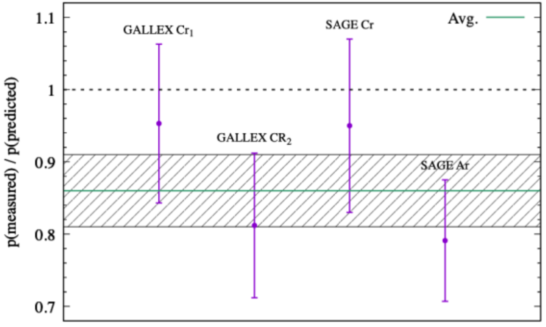
Over the past decades neutrino oscillation searches at length/distance scales of 1 MeV/m have found a number of anomalous results: The liquid scintillator neutrino detector (LSND) anomaly, the reactor antineutrino anomaly, the MiniBooNE low-energy excess and the gallium anomaly. These anomalies have not been confirmed, and the reactor antineutrino anomaly has been recently resolved.

Electron (anti)neutrino appearance

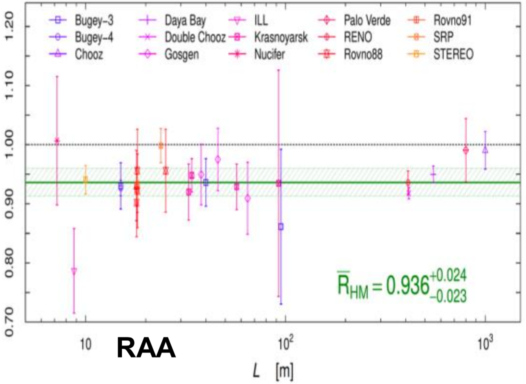




Electron (anti)neutrino disappearance



Gallium



RAA

Global context for the RAA: What now?

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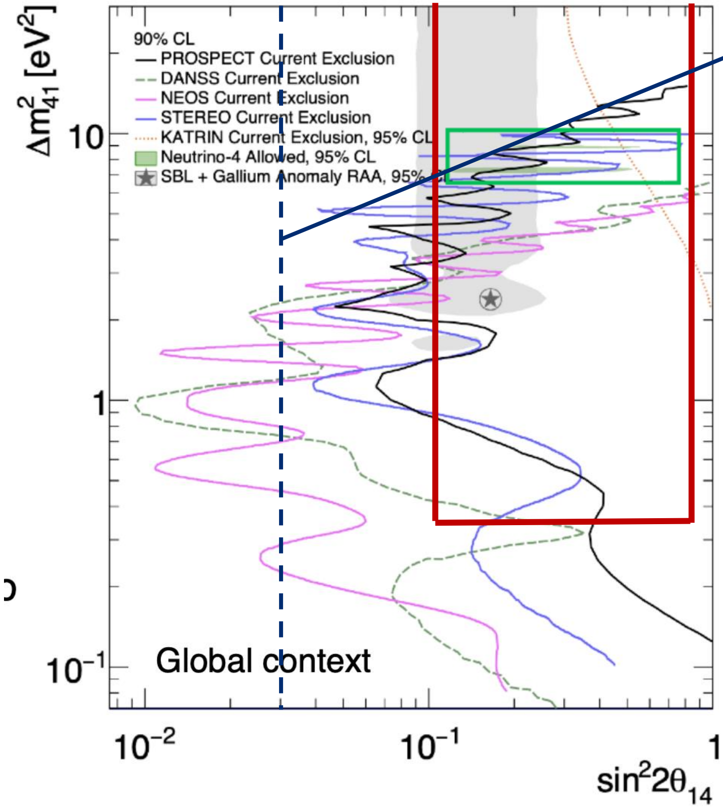
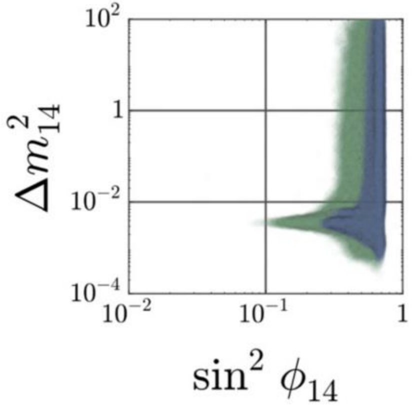
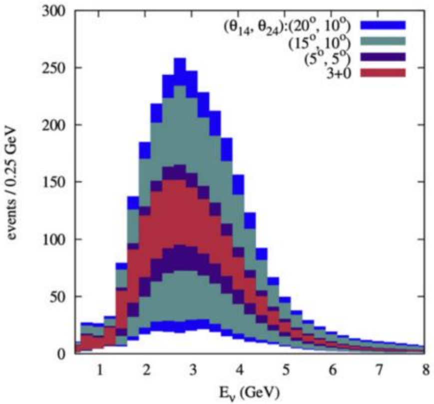


Ambiguities interpreting LBL results

- Need to nail down θ_{14} and θ_{24} to better than ~5 degrees to correctly interpret CPV results.





- CPV active+sterile sector can mimic CP-conserved LBL signatures
- CP-conserved ones can look like CPV at DUNE
- Can reduce mass hierarchy sensitivity
- Need to constrain θ_{14} in the region from 1 to more than 100 eV^2

Need for more reactor-based data sets

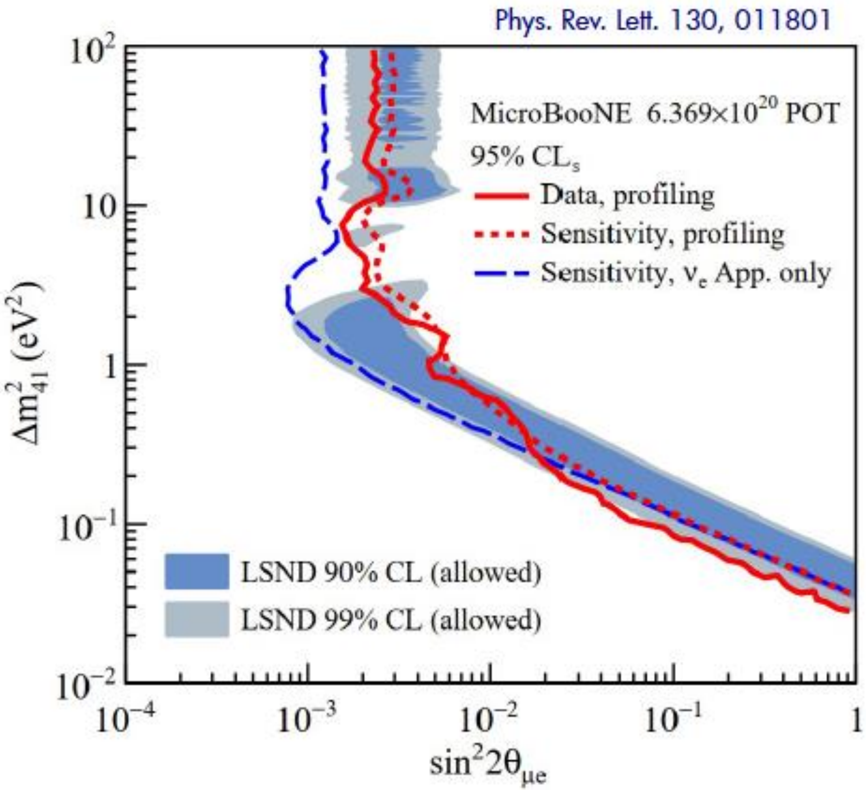
- Sterile oscillations remain an important plank in phenomenological discussions of the neutrino anomalies

- Reactor experiments play a key role in an integrated global effort to understand these anomalies

- Diversify dataset:
  - **MeV** and GeV regimes
  - **Electron** and Muon
  - **Disappearance** and appearance

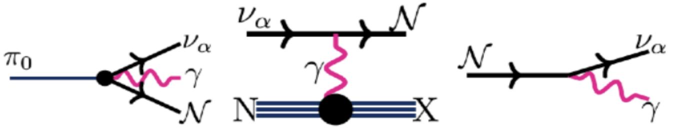
$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & \boxed{U_{e4}} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} & \boxed{U_{\mu4}} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} & U_{\tau4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \end{pmatrix}$$

- Extend model to more complex sterile neutrino scenarios: 3+1+NSI, 3+1+decays, ..



Extensions of the 3+1 formalism: 3+1+Decay

- Global 3+1 models fail to fit the low energy excess reported by MiniBooNE
- Considering more exotic models in which a sterile neutrino decays into active states could help explain this anomaly



Heavy neutral leptons only relevant at higher energies O(100) MeV

- Currently, this model predicts no excess in MiniBooNE data
- As reactor data will be insensitive to these decays, more 3+1 disappearance data will help constrain this model

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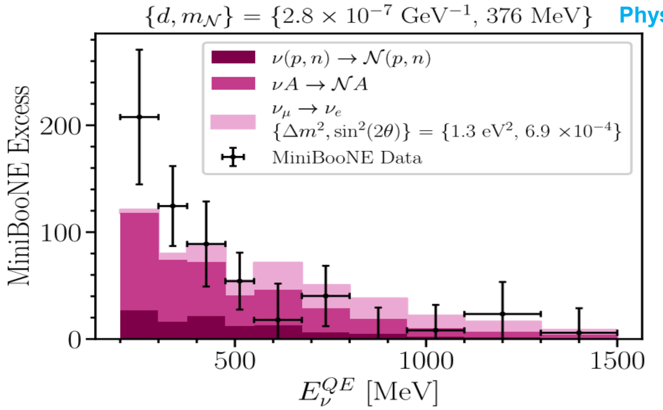
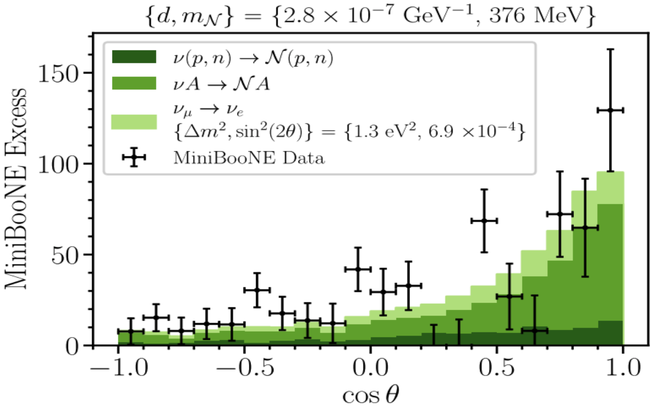
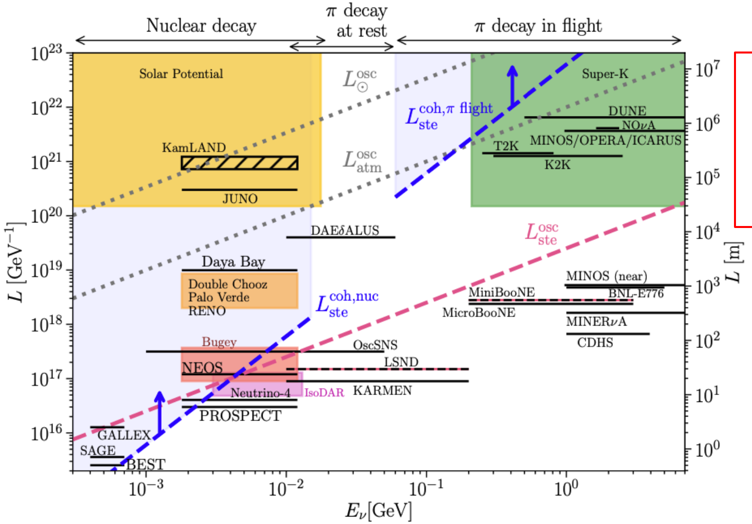
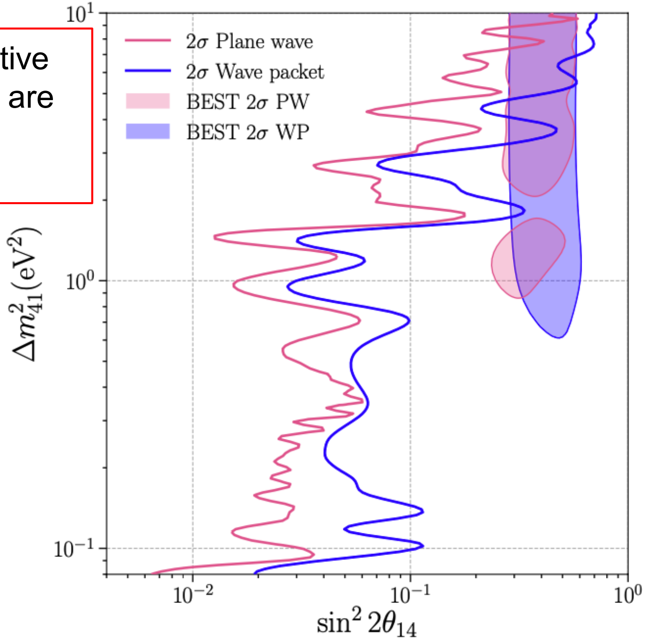




Extensions of the 3+1 formalism: 3+1+Decoherence





Reactor and radioactive sources experiments are ideal to study decoherence effects

- Simple 3+1 picture assumes that neutrinos propagate as plane waves (PW)
- Corrections accounting for finite-size wave packet (WP) have an impact on the dampening of the oscillation

$$P_{\alpha\beta} = \sum_{i=1}^{n} |U_{\alpha i}|^2|U_{\beta i}|^2 + 2\mathrm{Re}\sum_{j>i} U_{\alpha i}U^*_{\alpha j}U^*_{\beta i}U_{\beta j}\exp\left\{-2\pi i \frac{L}{L^{ij}_{\mathrm{osc}}} - 2\pi^2\left(\frac{\sigma_x}{L^{ij}_{\mathrm{osc}}}\right)^2 - \left(\frac{L}{L^{ij}_{\mathrm{coh}}}\right)^2\right\}$$

$$L^{ij}_{\mathrm{osc}} = \frac{4\pi E}{\Delta m^2_{ji}} \quad \text{and} \quad L^{ij}_{\mathrm{coh}} = \frac{4\sqrt{2}E^2\sigma_x}{\Delta m^2_{ji}},$$

- Solid lines correspond to the combination of Daya Bay, NEOS and PROSPECT data

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