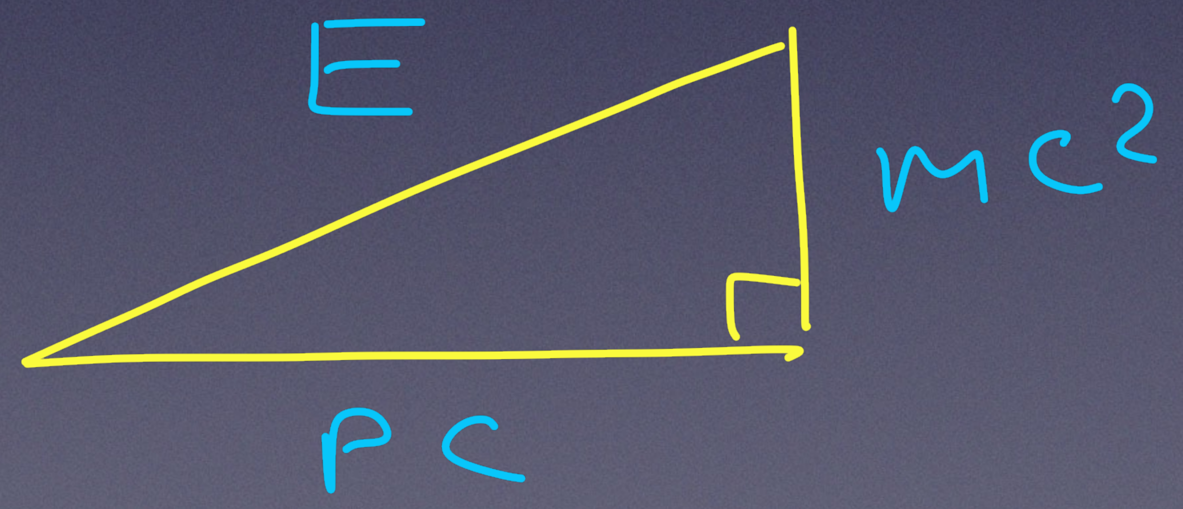
Relationship Between Energy and Momentum

- In Newtonian physics: $K = \frac{1}{2}mu^2$ and $p = mu$, where $p^2 = p_x^2 + p_y^2 + p_z^2$
- So $K = \frac{p^2}{2m}$ (= E in absence of any other potential energies)
- In Relativity: $E = \gamma mc^2$ and $p = \gamma mu$ (in magnitude)
- $p^2c^2 = \gamma^2 m^2 u^2 c^2 = \gamma^2 m^2 c^4 \frac{u^2}{c^2} = \gamma^2 m^2 c^4 \left(1 - \frac{1}{\gamma^2}\right)$
- $\Rightarrow p^2c^2 = E^2 - m^2c^4$
- $E^2 = p^2c^2 + m^2c^4$
- $m^2c^4 = E^2 - p^2c^2$



Since mass is constant, this is an example of an invariant quantity. Another example of **Lorentz Invariance**

Invariant Mass

- While mass does not need to conserved in interactions, the invariant mass of a system of objects <u>is</u> conserved since energy and momentum are each conserved:
  - $E_i = E_f$, where $E_i = \sum_{k=1}^{N_{\text{initial}}} E_k$ is the initial energy, and $E_f = \sum_{k=1}^{N_{\text{final}}} E_k$ is final energy
  - $\vec{p}_i = \vec{p}_f$, where $\vec{p}_i = \sum_{k=1}^{N_{\text{initial}}} \vec{p}_k$ for initial momenta, and $\vec{p}_f = \sum_{k=1}^{N_{\text{final}}} \vec{p}_k$ for final
  - $\Rightarrow E_i^2 - p_i^2 c^2 = m^2 c^4$ = constant
  - = $E_f^2 - p_f^2 c^2$
- So whatever this is initially, even if the initial particles disintegrated and new particles were created, is what it is afterward

Invariant Mass Example

An electron and a positron (an anti-electron) annihilate with equal and opposite momentum of magnitude1.55 GeV/*c* (note the new unit of momentum!) The collision produces a new particle called the J/ψ in the reaction $e^- + e^+ \rightarrow J/\psi$. What is the mass of this new particle? [Note that the rest mass energy of the electron is 0.511 MeV]