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Z MASS SPREADSHEET EXTENSION
STUDENT NOTES

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The Z boson decays in several ways. Physicists do not detect the Z boson directly but rather reconstruct it from its daughter particles. In this activity, the events we will study are $Z \rightarrow e^+ e^-$ (dielectron) or $Z \rightarrow \mu^+ \mu^-$ (dimuon). We learned in the *Calculate the Z Mass* activity that finding the total energy and total momentum for a dimuon or dielectron event provides evidence to determine the mass of a Z boson.

Each particle is governed by the relationship $E^2 = p^2 c^2 + m^2 c^4$. We know that the mass term for the Z boson is comparable to the momentum term. The masses of muons and electrons are very small compared to the mass of the Z boson so we can say E in GeV equals p in GeV/c; that is, the mass term for the electron and muon can be ignored.
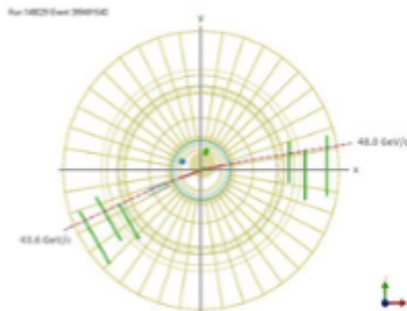
Data from LHC events are displayed in images like the one to the left. For all of our events, the collision yields parent particles which in our case are Z bosons. The event shows the recorded momentum (in GeV/c) of the daughter particles that came from the collision. These muon or electron pairs carry the momentum and mass-energy of the parent Z boson.

Can you identify the muons in this event?
Your class has eight event displays. You will begin by calculating the Z mass by hand using one event.

What do we know?
1. Momentum is conserved. Energy is too.
2. Momentum is a vector. Energy is not.
3. The invariant mass of the Z boson becomes the momentum and mass of the $\mu^+ \mu^-$ pair.
4. The net momentum of the $\mu^+ \mu^-$ pair is the same as the net momentum of the Z boson.
5. The muon mass is small. In these events, we can say that the muon energy (in GeV) and momentum (in GeV/c) are equivalent.
6. Einstein actually wrote $E^2 = p^2 c^2 + m^2 c^4$. The full derivation can be found at the link:
   https://quarknet.org/page/energy-momentum-and-mass
   This allows us to solve for energy, momentum or mass using $E^2 = p^2 + m^2$, where the c value is absorbed into the units.

What are our claims? What is our evidence?
1. **By-Hand Calculation**
   - For each muon:
     - Find the angle from the positive x axis to the direction of the muon track. [angle 1(deg), angle 2 (deg)]
     - Record the magnitude of the momentum. [p1 (GeV/c), p2 (GeV/c)]
     - Record the energy. [E1 (GeV), E2 (GeV)]
     - Determine the x component of the momentum. [p1x (GeV/c) = p1 cos(θ), p2x (GeV/c) = p2 cos(θ)]
     - Determine the y component of the momentum. [p1y (GeV/c) = p2 sin(θ), p2y (GeV/c) = p2 sin(θ)]
   - For the event:

- Find the net momentum in the x direction. [px(net) (GeV/c) = p1x (GeV/c) + p2x (GeV/c)]
- Find the net momentum in the y direction. [py(net) (GeV/c) = p1y (GeV/c) + p2y (GeV/c)]
- Find the net energy of the event [E(net) (GeV) = E1 (GeV) + E2 (GeV)].
- Find the magnitude of the resultant event momentum. $p(net) = \sqrt{(px(net)^2 + py(net)^2)}$
- Finally, determine the mass of the Z boson using $E^2 = p^2 + m^2$.

1. **Setting up the spreadsheet**

    a. Create a spreadsheet that includes all the necessary columns for the calculation of the Z mass. HINT: use the column headings shown in brackets above.

2. **Entering Data**

    a. For each muon, enter the value of the angle, the magnitude of the momentum and the value of the energy.

    b. For calculated columns, enter the "code" for finding each value. Fill down the code for all of the columns.

    c. Is the value for the mass of the Z boson found in your spreadsheet a reasonable match for the value of the mass of the Z boson found in your by-hand calculation? If the values don't agree, troubleshoot your formula for each calculated column.

    d. When you are satisfied that your spreadsheet is yielding correct answers, record the data from the other events into your spreadsheet.

    e. Create a histogram for all eight events using the tools in the spreadsheet.
    - What is the best value for the Z mass using the eight events?
    - What features of the histogram did you use to determine your best value of the Z mass?

3. **Adding More Data**

    a. Follow the link below to find CMS dilepton data. There is a separate tab for dimuon data and dielectron data. The same equations apply for Z boson decay via muons and via electrons.
    Z Activity dilepton CMS student_data:
    https://docs.google.com/spreadsheets/d/1CEg7DIcQ4OW_ETC1v70T3tMR8o42VQzHml5pL0BRpxc/edit#gid=98967810

    b. There are two sheets: *CMS dimuon data* and *CMS dielectron data*. Your teacher will tell you which sheet you should use. Copy the data from your assigned sheet into your spreadsheet. Be sure to fill down the code for the new data. Make a histogram of your data set with mass (GeV/c²) on the horizontal axis.

Questions

1. What is the best value for the Z mass using eight events? Estimate the uncertainty in your answer.
2. Describe how the histogram changed when more data was used.
3. Did the uncertainty in your answer increase or decrease when more data is used?