MEAN LIFETIME PART 1: DICE
TEACHER NOTES

DESCRIPTION

Often physics students have experience with the concept of half-life from lessons on nuclear decay. Teachers may introduce the concept using M&Ms to model a decaying object. When students begin their study of decaying fundamental particles, their understanding of half-life may be shaky. The introduction of mean lifetime as used by particle physicists can cause more confusion. This activity uses dice as the model for decaying particles.

Students represent “particles” with six-sided dice, and determine the “decay time” using a histogram of the number of dice remaining after each roll of the dice. Dice are removed if their value matches a “decay” criterion. The dice serve as a model to help students visualize particle decay. The students find half-life and mean lifetime from the plot.

STANDARDS ADDRESSED

Next Generation Science Standards

Science and Engineering Practices
- 2. Developing and using models
- 4. Analyzing and interpreting data
- 5. Using mathematics and computational thinking

Crosscutting Concepts
- 1. Patterns
- 2. Cause and Effect: Mechanism and Explanation
- 3. Scale, Proportion, and Quantity
- 4. Systems and System Models
- 7. Stability and Change

Common Core Literacy Standards

Reading
- 9-12.7 Translate quantitative or technical information . . .

Common Core Mathematics Standards
- MP5. Use appropriate tools strategically.
- MP6. Attend to precision.

IB Physics Standards

Topic 7.1
- Understandings: Half-life
- Application: Investigation of half-life experimentally (or by simulation)
- Utilization: Exponential functions

Topic 7.3
- Understandings: Quarks, leptons, and their antiparticles

Topic 12.2 (AHL)
- Understandings: The law of radioactive decay and the decay constant
- Applications and Skills: Solving problems involving the radioactive decay law for arbitrary time intervals

ENDURING UNDERSTANDINGS

- Particles that decay do so in a predictable way, but the time for any single particle to decay and the identity of the decay products are both probabilistic in nature.
- Scientists can use data to develop models based on patterns in the data.

LEARNING OBJECTIVES

Students will know and be able to:

- Using a decay curve, describe how half-life and mean lifetime can explain how particles decay randomly yet decrease in number in a predictable way.
- Explain the difference in the mathematical models used to determine half-life and mean lifetime.
- Determine the half-life and mean lifetime using a decay curve of a system of particles.
- Make a claim supported by evidence for the choice of mean lifetime to describe particle decay.
- Provide evidence to refute the claim that "All particles of a particular type decay in exactly a time described by the particle mean lifetime."

PRIOR KNOWLEDGE

Students must be able to:

- Keep careful records of observations and add integers.
- Make and interpret graphs.
- Understand exponential functions.
- Distinguish between a curve which is exponential, a power of e, and a curve that is quadratic, a power of x.

BACKGROUND MATERIAL

When elementary particles decay into daughter particles, each particle takes a different amount of time to decay. The process is governed by probability—different kinds of particles have different probable rates of decay. For example, a π^+ or π^- meson might have a mean lifetime on the order of tens of nanoseconds while a muon might have a mean lifetime in the microsecond range. This means that in the case of the π^+ meson, an initial sample N_0 will reduce to N_0/e after one mean lifetime, N_0/e^2 after two mean lifetimes, etc. For any one of these mesons, we cannot predict when it will decay; we can only predict the most likely time it will take to decay.

While particle physicists use particle mean lifetime, other scientists often use half-life, a related term with the same mathematics as above except that it is defined as the time (or number of rolls) for N_0 dice to "decay" to $N_0/2$ dice, etc. There are several similarities between finding half-life and particle mean lifetimes, so it makes sense to build on the foundation from the students' earlier experiences in chemistry class. Students will measure half-life and mean lifetime in this activity and compare these values with the mean number of rolls for all the dice.

We can probabilistically predict the decay behavior and the typical mean lifetime of each type of "particle" using the analysis of an exponential decay curve. We use dice in this activity.

- The *half-life* of the muon (not generally used by particle physicists but useful to compare with radioactive half-life) is the time for ½ the sample to decay according to the mathematical model
$$N = N_0 2^{-t/T_{1/2}}$$
where N is the number of muons in the sample, N_0 is the initial number of muons, t is time, and $T_{1/2}$ is the half-life.
- The *mean lifetime* of the muon is the time for 1/e of the sample to decay according to the mathematical model
$$N = N_0 (e^{-t/\tau})$$

where N is the number of muons in the sample, N_0 is the initial number of muons, t is time, and τ is the mean lifetime.

A fair die has an equal probability of producing any of the available numbers. We can use a die to model a single particle. We model this with dice by each of N_0 students having one die and rolling until he or she gets, say, a 6, at which point we say the die has "decayed." The number of rolls will vary. The mean lifetime is defined as the time for a sample of N_0 dice to "decay" to N_0/e dice; it takes two mean lifetimes to decay down to N_0/e^2 dice, etc.

The mathematical model for half-life and mean lifetime must describe the same exponential decay curve. For this to be true, the following equality must hold:

$$N_o 2^{-t/T_{1/2}} = N_o e^{-t/\tau}.$$

Notice that N_0 cancels. Now operate on both sides with ln().

$$\frac{-t}{T_{1/2}} \ln(2) = \frac{-t}{\tau}$$

The equation relating mean lifetime to half-life simplifies to

$$\tau = \frac{T_{\frac{1}{2}}}{\ln(2)}.$$

IMPLEMENTATION

We do not provide a student handout with this activity.

In this activity, students model the behavior of a population of particles that decay.

All students can be issued a few dice. Make a data table on the board with the following columns:

Roll Number	**Number of Dice Remaining**
0	{Enter N_0 here}
1	
2	

Extend the table to include the number of rolls needed for all "particles" to decay. Be sure to start the table with Roll 0 and enter the value of N_0 in Number of Dice Remaining column.

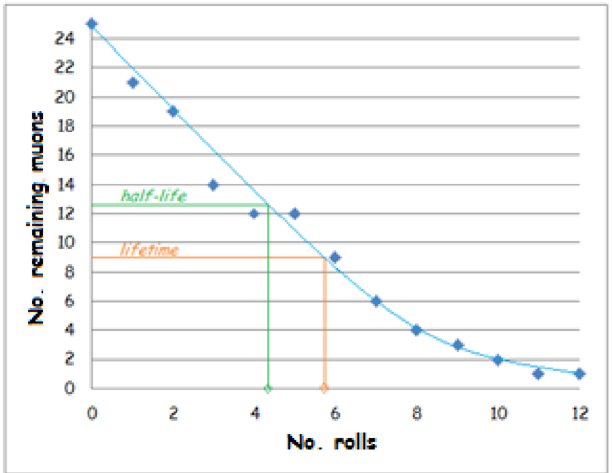
Indicate the time to roll. The students roll all the dice. If a student rolls the desired number on one or more dice, he or she removes each die that "decayed" and places those dice in the graveyard. Enter the sum of all remaining dice in the table for that roll. Repeat for roll 2, roll 3, roll 4, etc., until all of the dice are in the dice graveyard.

When the last "particle" has decayed, make a histogram on the board using Number of Dice Remaining on the y-axis and Roll Number on the x-axis.

Here is an example of a table and plot from rolling 25 dice:

Finding half-life and lifetime from decay of dice

No. rolls	No. muons
0	25
1	21
2	19
3	14
4	12
5	12
6	9
7	6
8	4
9	3
10	2
11	1
12	1



N_o = 25 rolls.

Approximations from plot:

$N_o/2$ = 12.5.
Half-life = 4.2 rolls.

N_o/e = 9.1
Lifetime = 5.8 rolls.

ANALYSIS

We use the sample plot to answer the following questions:

1. Find the half-life using the number of rolls to drop from 25 dice to 12.5 dice.
2. Find the mean lifetime using the number of rolls to drop from 25 dice to 25/e or 9.19 dice.
3. In the example above, the first half-life is found using the number of rolls to drop from 25 dice to 12.5 dice. What is the half-life using the drop from 12.5 dice to 6.25 dice? This can be called the second half-life.
4. In the example above, the first mean lifetime is found using the number of rolls to drop from 25 dice to 9.19 dice. What is the mean lifetime using the drop from 9.19 dice to 9.19/e or 3.3 dice? This can be called the second mean lifetime.

Answer questions 1–4 using the plot of your class data.

EXTENSIONS

1. Use dice with a different number of sides, as in d20. Compare the half-life and mean lifetime with the results from standard d6 dice.
2. If there are multiple classes doing this activity, collect the data from all classes in a table and have students plot and analyze that data on their own.
3. Have students repeat the plot creation and analysis using a spreadsheet and exponential trendline with an equation like $N = N_0e^{-kt}$. The mean lifetime should be 1/k and the half-life should be (ln 2)/k. See Background Material above.
4. Have students make their own plot from classroom data by hand, make a best fit curve, and then mathematically derive the equation, the mean lifetime, and the half-life.

ASSESSMENT

Have students discuss the plots and analysis in small groups and then report. They should address these questions as well as their own:

- How well does each plot fit using two half-lives? With two mean lifetimes? What does this say about the reliability of the plot?

  The number of rolls needed for the sample size to drop by one half is the same for any chosen number of particles. The number of rolls needed to drop the sample size by 1/e is the same for any chosen number of particles. Students should cite evidence from their graph to provide evidence for their answer.

- What would happen if the experiment were repeated? Would students get the same plot or would the plot be different?

  Each time the experiment is repeated, the shape of the plot should be the same. The value for N_0 *may be different, but the half-life and mean lifetime should come out the same.*
- Given the half-life of a particle, find the mean lifetime of the particle.

  Use the equation $\tau = \frac{T_{\frac{1}{2}}}{\ln(2)}$.
- Given the mean lifetime of a particle, find the half-life of the particle.

  Use the equation $\tau = \frac{T_{\frac{1}{2}}}{\ln(2)}$. *Solving for* $T_{1/2}$ *yields* $T_{1/2} = \tau \ln(2)$.
- Using a decay curve, describe how half-life and mean lifetime can explain how particles decay randomly yet decrease in number in a predictable way.

  If particle decay was not random, every particle would decay in exactly the same amount of time. That plot would be a single spike at the lifetime value. Particle decay at a variety of times is shown by the data that followed an exponential decay curve. This provides evidence for the particle decay to be best described using probability. The half-life is predictable because every time the sample size dropped by one half, the number of rolls was the same. The lifetime is predictable because every time the sample size dropped by 1/e, the number of rolls was the same.
- Explain the difference in the mathematical models used to determine half-life and mean lifetime.

  Follow the derivation in the background materials.
- Determine the half-life and mean lifetime using a decay curve of a system of particles.

  Check for the following process: Selection of N_0; *identify* $N_0/2$ *for half-life or* N_0/e *for mean lifetime; determine the number of rolls for that interval. The number of rolls represents the half-life or mean lifetime.*
- Make a claim supported by evidence for the choice of mean lifetime to describe particle decay.

  The mean lifetime value using N_0/e *is very close to the value for the mean lifetime by finding the average of all of the data. Therefore, it makes sense for particle physicists to prefer mean lifetime as the descriptor of particle decay.*
- Provide evidence to refute the claim that "All particles of a particular type decay in exactly a time described by the particle's mean lifetime."

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Students can also do one of the extensions for evaluation by the teacher. These extensions can be evaluated using the questions listed above.