

Quark Model

Mass and Charge Patterns in Hadrons

To “tame” the particle zoo, patterns in the masses and charges can be found that will help lead to an explanation of the large number of particles in terms of just a few fundamental particles.

Spin-1/2 baryons:

Nucleons:

n: 939.57 MeV; p: 938.27 MeV

Sigma:

Σ^- : 1197 MeV; Σ^0 : 1193 MeV; Σ^+ : 1189 MeV

Cascade:

Ξ^- : 1322 MeV; Ξ^0 : 1315 MeV

Spin-3/2 baryons:

Delta(1385):

Δ^- : 1232 MeV; Δ^0 : 1232 MeV; Δ^+ : 1232 MeV; Δ^{++} : 1232 MeV

Sigma(1385):

Σ^- : 1387 MeV; Σ^0 : 1384 MeV; Σ^+ : 1383 MeV

Cascade(1530):

Ξ^- : 1535 MeV; Ξ^0 : 1532 MeV

Spin-0 mesons (pseudoscalars):

Pions:

π^- : 140 MeV; π^0 : 135 MeV; π^+ : 140 MeV

Kaons:

K^+ : 494 MeV; K^0 : 498 MeV

Eta:

η : 549 MeV; η' : 958 MeV

Spin-1 mesons (vectors):

Rho(770):

ρ^- : 770 MeV; ρ^0 : 770 MeV; ρ^+ : 770 MeV

$K^*(892)$:

K^{*+} : 892 MeV; K^{*0} : 892 MeV

Omega, Phi:

ω : 782 MeV; ϕ : 1019 MeV

Isospin and Hypercharge

Interesting diagrams can be made when we define the following quantum numbers that are conserved in strong interactions:

Hypercharge: $Y = B + S$ (+ charm + beauty)

Here B is the baryon number, and S is the strangeness (which can be generalized by adding charmness and beauty)

Isospin: $I_3 = Q - Y/2$

Here Q is the electric charge of the particle (and is computed from $I_3 + Y/2$)

Let's now calculate Y and I_3 for the above listed particles, and plot their coordinates.

Spin-1/2 baryons:

Nucleons: $B=1, S=0 \rightarrow Y = 1$;

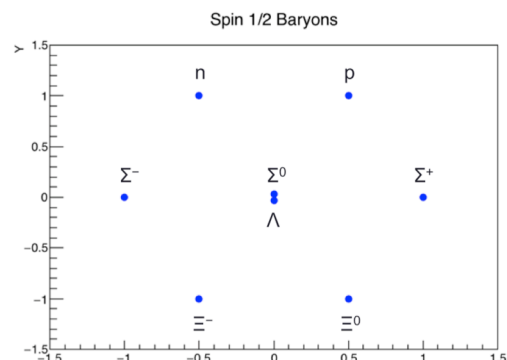
n : $I_3 = -1/2$; p : $I_3 = +1/2$

Sigma: $B=1, S=-1 \rightarrow Y = 0$;

Σ^- : $I_3 = -1$; Σ^0 : $I_3 = 0$; Σ^+ : $I_3 = +1$

Cascade: $B=1, S=-2 \rightarrow Y = -1$;

Ξ^- : $I_3 = -1/2$; Ξ^0 : $I_3 = +1/2$



Spin-3/2 baryons:

Delta(1385): $B=1, S=0 \rightarrow Y = 1$;

Δ^- : $I_3 = -3/2$; Δ^0 : $I_3 = -1/2$; Δ^+ : $I_3 = +1/2$;

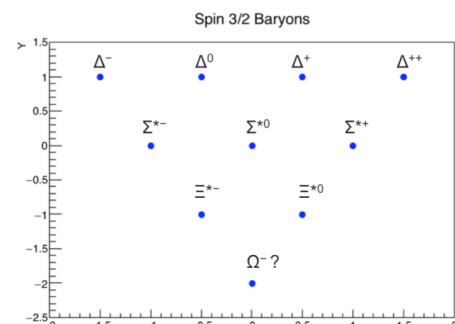
Δ^{++} : $I_3 = +3/2$

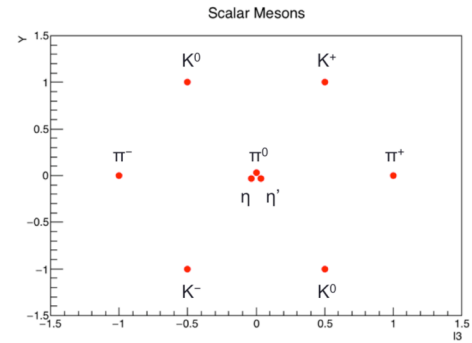
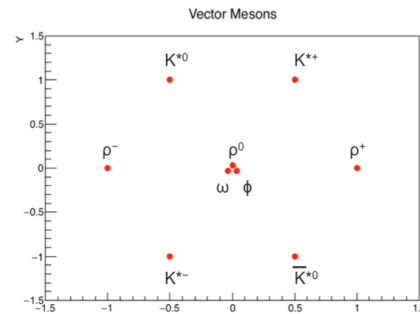
Sigma(1385): $B=1, S=-1 \rightarrow Y = 0$

Σ^- : $I_3 = -1$; Σ^0 : $I_3 = 0$; Σ^+ : $I_3 = +1/2$

Cascade(1530): $B=1, S=-1 \rightarrow Y = -1$

Ξ^- : $I_3 = -1/2$; Ξ^0 : $I_3 = +1/2$



Spin-0 mesons (pseudoscalars):Pions: $B=0, S=0 \rightarrow Y=0$; $\pi^-: I_3 = -1$; $\pi^0: I_3 = 0$; $\pi^+: I_3 = +1$ Kaons: $B=0, S=+1 \rightarrow Y=+1$; $K^0: I_3 = -1/2$; $K^+: I_3 = +1/2$ $B=0, S=-1 \rightarrow Y=-1$; $K^-: I_3 = -1/2$; $\bar{K}^0: I_3 = +1/2$ **Spin-1 mesons (vectors):**Rho(770): $B=0, S=0 \rightarrow Y=0$; $\rho^-: I_3 = -1$; $\rho^0: I_3 = 0$; $\rho^+: I_3 = +1$ $K^*(892): B=0, S=-1 \rightarrow Y=-1$; $K^{*+}: I_3 = -1/2$; $K^{*0}: I_3 = +1/2$ 

We see that the isospin number I_3 is like the L_z component of spin or angular momentum. It can take on integer or half-integer values. And it must satisfy $|I_3| \leq I$.

Eightfold Way

It was Murray Gell-Mann, and independently Yuval Ne'eman, who developed what is called the "eight-fold way" and first constructed these patterns in what are known as multiplets in 1961. The "eight" comes from the octet particles that make up the spin-1/2 baryon multiplet. But there are other patterns as well. The mesons form nonets of 9 particles, and the spin-3/2 baryons form a decuplet of 10. In fact it turned that one particle was missing from the decuplet at the time of the eightfold way, the particle with strangeness = -3. Gell-Mann was able to predict this particle and its mass based on the mass splitting between the three known rows of particles. The Ω^- was discovered in 1964, and has a mass of 1672 MeV.

Quark Model

It only took until 1964 for Gell-Mann and George Zweig to propose a theory for the apparent structure in the particle multiplets and explain the particle zoo. Their theory is that hadrons, i.e. baryons and mesons, are not elementary but are composed of smaller constituents. Just as atoms are not elementary and are instead comprised of combinations of electrons, protons, and neutrons; the hadrons known at the time could be explained as combinations of just three fundamental particles, dubbed “quarks” by Gell-Mann (from a line in James Joyce’s Finnegan’s Wake).

We call those quarks up, down, and strange (u, d, and s). These quarks are spin $\frac{1}{2}$ fermions, and have fractional electric charge ($+\frac{2}{3} e$ for up, $-\frac{1}{3} e$ for down and strange). The strange quark has strangeness = -1 .

The baryons are composed of 3 quarks (and antibaryons by 3 antiquarks, which have the opposite charges of quarks):

Spin-1/2 baryons:

Nucleons:

n: udd; p: uud

Sigma:

Σ^- : ddd; Σ^0 : udd; Σ^+ : uud

Cascade:

Ξ^- : dds; Ξ^0 : uds

The mesons, however, are composed from a quark-antiquark pair:

Spin-0 mesons (pseudoscalars):

Pions:

π^- : $d \bar{u}$; π^0 : $(u \bar{u} - d \bar{d})/\sqrt{2}$; π^+ : $\bar{d} u$

Kaons:

K^+ : $\bar{s} u$; K^0 : $\bar{s} d$

K^- : $s \bar{u}$; \bar{K}^0 : $s \bar{d}$

Eta:

η : $(u \bar{u} + d \bar{d})/\sqrt{2}$; η' : $s \bar{s}$

Note that that the π^+ is the antiparticle of the π^- , and that the π^0 is its own antiparticle. However, the K^0 has a separate antiparticle, the \bar{K}^0 .

Question: What is the quark composition for the baryons comprising the spin 3/2 baryon decuplet, and for the meson comprising the spin 1 nonet?

Heavier Quarks

It turns out that the quark model does not stop with the strange quark. In 1974 a new particle now called the J/ψ was discovered by two groups, one led by B. Richter at a SLAC e^+e^- storage ring and another led by C. Ting in a fixed-target experiment at Brookhaven National Lab. The mass of this particle is 3.1 GeV and it had an anomalously long lifetime indicative of the weak nuclear force. The interpretation is that the particle, a meson, is bound state of a new quark and anti-quark, called charm. Like strangeness, it is conserved in all interactions except the weak force. Its discovery was so momentous to the field that this period was referred to as the November Revolution.

The bound state of charm and anticharm is the quark equivalent of positronium, the bound state of an electron and antielectron. In other words, it is similar to hydrogen. And like hydrogen, there are different energy levels for such bound states. For the charm-anticharm state, one can label these states in a similar fashion as for hydrogen. The J/ψ corresponds to the 1S state. The 2S state is the ψ' , which has a mass of 3.69 GeV. Higher energy states are mesons with higher mass.

Aside from bound states of charm-anticharm, meson and baryon particles can be made with charm quarks as well. As production of such particles is usually through the strong or electromagnetic force, two such particles generally need to be produced, one with the charm quark and one with the anticharm, since the “charmness” quantum number is conserved in such productions. Mesons and baryons with charm also form multiplets as was the case for particles with charm. The charm quark has electric charge of $+2/3 e$ like the up quark.

Quarks did not stop there, however. In 1977 the beauty quark, now referred to as the bottom quark, was discovered in fixed-target collisions at Fermilab by a team led by Leon Lederman. Such particles also had anomalously long lifetimes as well, and thus decayed only via the weak force. Bottom-antibottom bound states form what is known as the Upsilon family. The Upsilon (1S) is the lightest and has a mass of 9.46 GeV. Bottom quarks also can be included into mesons and baryons with other quark flavors, forming multiplets with bottom quarks. The bottom quark has electric charge of $-1/3 e$ like the down quark.

With the discovery of charm and bottom, as well as the discovery of the tau lepton, it became clear that quarks and leptons are grouped into “generations” that replicate some of the quantum numbers of the lightest generation (formed by the up and down quark). Thus by discovering a third down-type quark (bottom), there seemed to be a good reason to expect a third up-type quark. Eventually this top quark was discovered in 1995 at Fermilab in proton-antiproton collisions. However, its mass is far above the others, 173 GeV, which is why previous searches at lower energy machines were unsuccessful. Also, by virtue of its high mass, it decays extremely quickly via the weak force into a W boson and bottom quark. So quickly it is not expected to form bound states with other quarks.

Complications

Despite the apparent success of the quark model to explain the patterns and proliferation of hadrons, there are two nagging issues. First, no experiment has ever observed a bare quark. Only mesons and baryons have been observed. Apparently quarks are confined to composite particles. As we shall see in a later module, there is experimental evidence for quarks inside the hadrons, however, from deep inelastic scattering off of protons for example. The other complication has to do with particles comprised of three identical quarks, like the Ω^- that is made from three s quarks. Quantum Mechanics tells us that you cannot have identical fermions in the same state. With two spin-1/2 quarks, one can be in a spin up state and the other in a spin down state, but what about the third? The solution is to introduce another quantum number for quarks, called color, which comes in 3 varieties (red, green, blue, for example). Then the quarks would become distinguishable. This will form the basis for the theory for the strong nuclear force, known as Quantum Chromodynamics and which we will discuss later.

Extra:**Spin-3/2 baryons:**

Delta(1385):

 Δ^- : ddd; Δ^0 : udd; Δ^+ : uud; Δ^{++} : uuu

Sigma(1385):

 Σ^- : dds; Σ^0 : uds; Σ^+ : uus

Cascade(1530):

 Ξ^- : dss; Ξ^0 : uss

Omega(1672)

 Ω^- : sss**Spin-1 mesons (vectors):**

Rho(770):

 ρ^- : $d \bar{u}$; ρ^0 : $(u \bar{u} - d \bar{d})/\sqrt{2}$; ρ^+ : $\bar{d} u$

K*(892):

 K^{*+} : $\bar{s} u$; K^{*0} : $\bar{s} d$ K^{*-} : $s \bar{u}$; \bar{K}^{*0} : $s \bar{d}$

Omega, Phi:

 ω : $(u \bar{u} + d \bar{d})/\sqrt{2}$; ϕ : $s \bar{s}$