MAKIN' IT ROUND THE BEND: QUANTITATIVE
TEACHER NOTES

DESCRIPTION

This set of three classroom activities allows students to explore some basic physics at the heart of particle physics accelerator design. The activity can serve as a review of concepts in classical mechanics as well as electricity and magnetism. The activity may be done with a qualitative (Open *Makin' It Round the Bend: Qualitative*) or quantitative approach (continue with these instructions) depending on time constraints and the level of your students. Students will study how electric and magnetic fields accelerate, turn and focus the electrically charged particles in the Large Hadron Collider (LHC) beam.

STANDARDS ADDRESSED

Next Generation Science Standards

Science and Engineering Practices

4. Analyzing and interpreting data
5. Using mathematics and analytical thinking
6. Constructing explanations
7. Engaging in arguments from evidence

Crosscutting Concepts

1. Observed patterns . . . guide organization and prompt questions.
2. Cause and effect . . . investigating and explaining causal relationship

Common Core Literacy Standards

Reading

9-12.4 Determine the meaning of symbols, key terms . . .
9-12.7 Translate quantitative or technical information . . .

Common Core Mathematics Standards

MP2. Reason abstractly and quantitatively
MP5. Use appropriate tools strategically.
MP6. Attend to precision.

AP Physics 1 Standards

EK 3.A.2: Forces are described by vectors
EK 3.B.1: If an objects of interest interacts with several other objects…
EK 3.C.3: A magnetic force results from the interaction of a moving….

AP Physics 2 Standards

EK 1.E.6.a: Magnetic dipole moment is a fundamental source…
EK 3.A.2: Forces are described by vectors
EK 3.C.3: A magnetic force results from the interaction of a moving….

IB Physics Standard 1: Measurement and Uncertainty

1.2.6 Describe and give examples of random and systematic errors.
1.2.8 Explain how the effects of random errors may be reduced.
1.2.11 Determine the uncertainties in results.

IB Physics Standard 5: Electricity and Magnetism

5.1 Electric Fields

Aim 7: use of computer simulations would enable students to measure microscopic interactions that are typically very difficult in a school laboratory situation

5.4 Magnetic Effects of Electric Currents

Aims 2 and 9: visualizations frequently provide us with insights into the action of magnetic fields; however, the visualizations themselves have their own limitations

6.1 Circular Motion

Aim 7: technology has allowed for more accurate and precise measurements of circular motion, including data loggers for force measurements and video analysis of objects moving in circular motion

ENDURING UNDERSTANDINGS

Data can be used to develop models based on patterns in the data.

LEARNING OBJECTIVES

Quantitative Treatment:

- Determine the acceleration of a particle given the change in velocity and the change in time
- Construct a graph of electric field strength vs acceleration and determine the physical meaning of the slope
- Construct a graph of electric field strength vs magnetic field strength and determine the physical meaning of the slope
- Determine the conditions in which an electrically charged particle will travel straight through the crossed electric and magnetic fields
- Determine the mathematical model that describes the circular motion of an electrically charged particle in a magnetic field

PRIOR KNOWLEDGE

Students must be able to:

- Define the types of electrically charged particles as positive or negative
- Define the types of magnetic poles as north and south
- Construct graphs given data
- Re-graph data to make a graph that is linear
- Apply the right-hand rule in situations described by vector cross products

BACKGROUND MATERIAL

When an electrically charged particle interacts with an electric field, the particle accelerates in the direction of the electric field vector. The equation $\mathbf{F_e} = q\mathbf{E}$ describes this interaction. When an electrically charged particle interacts with a magnetic field, the particle travels in a circular path and the particle can undergo circular motion. The right-hand rule can be used to determine the direction of the magnetic force of interaction. The equation for this interaction, $\mathbf{F_m} = q\mathbf{v} \times \mathbf{B}$ is a cross product so the direction of the force is determined by using the right hand, sweep the **v** vector direction into the **B** vector direction and the thumb represents the direction for the magnetic force. The more momentum a particle has, the larger the force needed to turn it. Large magnetic fields create larger forces on moving particles. For a review of the use of the right-hand rule in magnetism, see the activity *Mapping the Poles*.

Particles in the CERN accelerators move inside beam pipes about the size of your index finger. Magnets in the curved segments of these accelerators send the particles around the bend. Each revolution of the ring increases the momentum of the particle; each momentum increase requires a stronger turning force; these stronger forces come from adjusting the field of the bending magnets. Electromagnets can do this—sending more current through one of these magnets increases the magnetic field.

RESOURCES/MATERIALS

The links below provide useful background material.
https://home.cern/about/how-accelerator-works
http://lhc-machine-outreach.web.cern.ch/lhc-machine-outreach/components/magnets.htm
http://www.lhc-closer.es/taking_a_closer_look_at_lhc/0.magnetic_multipoles

IMPLEMENTATION

Each part below will use the *Makin' it Round the Bend* simulation. *Link to Simulation*. Open the simulation to introduce your students to the features of the simulation. Along the bottom there are buttons for *Play, Pause, Step left (Step <<), Step right (Step >>),* and *Reset*. There is no Stop button so the students must use *Pause* to stop the simulation. At the very bottom, that there are three tabs along the bottom:

1. *Accelerator*
   - Notice that along the bottom, the values for t in x10^{-6} s, x in mm, and v_x in km/s are given. Below the parallel plates there are sliders to select *Electric field* in N/C, and *mass/charge ratio* in x10^{-9} kg/C.
2. *Velocity Selector*
   - Notice that along the bottom, the values for t in x10^{-6} s is given. Just under the parallel plates, the values for x in mm, y in mm, v_X in km/s and v_Y in km/s are given. Below the parallel plates there are sliders to select *Electric field* in N/C, *Magnetic field* in mT, Charge Attributes: *mass/charge ratio* in x10^{-9} kg/C, and *Initial* v_x in km/s.
3. *Mass Spectrometer*
   - Notice that under the velocity selector plates, the values for t in x10^{-6} s, the values for x in mm, y in mm, v_X in km/s and v_Y in km/s are given. The sliders control each region of the mass spectrometer:
     - *Velocity Selector* Controls: *Electric field* in N/C, *Magnetic field* in m T
     - *Mass Deflector* Controls: *Magnetic field 2* in mT
     - *Charge Attributes*: *mass/charge ratio* in x10^{-9} kg/C
     - *Initial* v_x in km/s

In each section, the force vector resulting from the interaction between the electrically charge particle and the field is the same color as the field vector.

PART 1: PICKING UP SPEED

Select the tab *1. Accelerator* to begin this activity. Make sure that students know how to start and stop the simulation to collect data for change in velocity and change in time. These data can then be used to determine the acceleration of the electrically charged particle for a chosen setting of the *Electric field* slider.

<u>Quantitative Treatment:</u>

Make sure that students know how to start and stop the simulation to collect data for change in velocity and change in time. These data can then be used to determine the acceleration of the electrically charged particle for a chosen setting of the *Electric field* slider. You can either let the students organize their data into a data table or you can suggest the following table structure:

t_1 (10^{-6}s)	v_{x1} (km/s)	t_2 (10^{-6}s)	v_{x2} (km/s)	a (km/s^2)	E (N/C)

Encourage your students to collect data for at least five values of *Electric field* strength. Students then construct a graph of *Electric field* vs. *acceleration*. The graph should be straight. The units of the slope are

$$\frac{N/C}{km/s^2}$$

which reduces to kg/C. The students should recognize these units are the same as the units for the mass/charge ratio. Therefore, the equation of their graph is

$$E = \frac{m}{q}a$$

which reduces to $qE = ma$. The equation is in SI units and the data is in N/C and km/s^2; therefore, the units for the slope will be g/C. Since the data slider allows the students to select their *mass/charge* ratio in units of kg/C, the slope value converted to units of kg/C which should then match the slider value. This provides the evidence for the claim that the interaction between an electrically charged particle and an electric field causes the positively charged particle to accelerate in the direction of the electric field vector.

PART 2: CHOOSE YOUR RACER

Select the tab *2. Velocity Selector* to begin this activity. Notice that the initial setting results in the electrically charged particle traveling straight through the region of crossed electric and magnetic fields.

Quantitative Treatment:

Instruct the students to select a new initial velocity in the x direction and a new *mass/charge* ratio. Select a value for *Electric field* and adjust the *Magnetic field* value until the electrically charged particle continues straight through the *Velocity Selector*. Change the *Electric field* value and repeat for at least five data points.

You can either let the students organize their data into a data table or you can suggest the following table structure:

Mass/charge ________________(kg/C) *Initial* v_x ___________________km/s

E (N/C)	B (mT)

Encourage your students to collect data for at least five values of *Electric field* strength. Instruct the students to construct a graph of E vs B. The units of the slope will be

$$\frac{N/C}{T}$$

A good exercise for the students is to show that this ratio reduced to m/s.

$$\frac{\frac{kg * m}{s^2 C}}{\frac{kg}{s^2 A}}$$

Since A = C/s, this equation reduces to

$$\frac{m}{s}$$

The equation is in SI units and the slope of the graph will be km/s; therefore, the slope value must be converted to SI units for comparison. The equation for this graph is $E = vB$.

Since the y direction forces must be balanced for the electrically charged particle to pass through the *Velocity Selector*, the forces in the y direction must balance to zero.

$$qE = qvB$$

After algebraic simplifications, the equation reduces to

$$E = vB$$

Notice that the data provided for magnetic field strength is in mT (mT = 10^{-3} T) so that when the students make their plot, care must be taken to include the powers of ten for mT. This provides the evidence for the claim that the interaction between an electrically charged particle and crossed electric and magnetic fields results in x direction motion only when the magnitudes of the electric field and magnetic field are equal.

PART 3: MAKIN' IT ROUND THE BEND

Select the tab *3. Mass Spectrometer* to begin this activity. The values for the Charge Attributes, mass/charge ratio and Initial v_x can be set at the beginning of each run. Notice that the initial setting of v_x allows the electrically charged particle to travel straight through the *Velocity Selector* when the *Electric field 1* and *Magnetic field* sliders are set appropriately. The electrically charged particle then enter the Mass Deflector which is controlled by *Magnetic field 2*.

<u>Quantitative Treatment:</u>

The students set the *Initial* v_x value as the electrically charged particle enters the *Velocity Selector*. Then the students must adjust the *Electric field 1* and the *Magnetic field* in the *Velocity Selector* to be sure the electrically charged particle passes into the *Mass Spectrometer*. The magnetic field in the *Mass Deflector* is controlled by *Magnetic field 2*. Let the simulation run and press *Pause* while the particle is still in the *Mass Deflector*. Record the necessary data

You can either let the students organize their data into a data table or you can suggest the following table structure:

mass/charge ratio ________________(x 10^{-9} kg/C) *Initial* v_x ______________(km/s)
Magnetic Field 2 ___________________mT

Mass Deflector			
Δx (mm)	*Δy* (mm)	*r* = ½ *Δy* (mm)	*B* (mT)

Encourage your students to collect data for at least five values of *Magnetic field* strength.. Remind the students that the radius value (*r*) is found using ½ Δy. Instruct the students to construct a graph of *r* vs *B*. Check to be sure that the students graph B with units of T, not mT and *r* in m not mm. This graph will not be linear. The students should linearize the data by plotting *r* vs *1/B*. The units of the slope of this graph will be

$$m\,T$$

Substituting the SI units for T

$$\frac{m\,kg}{s^2\,A}$$

The SI units for A are C/s so the slope units reduce to

$$\frac{m}{s}\,\frac{kg}{C}$$

Therefore, the slope should represent

$$v * \frac{m}{q}$$

Students must compare their slope value with the value obtained when multiplying the velocity and the mass/charge ratio. You should check with the students to make sure they are handling the units correctly since the simulation is not using standard SI units.

The equation of this graph then becomes

$$r = v\,\frac{m}{q} * \frac{1}{B}$$

After algebraic rearrangement, the equation becomes

$$qB = \frac{mv}{r}$$

When you multiply both sides by *v*, the equation becomes

$$qvB = \frac{mv^2}{r}$$

which is the equation for a magnetic force causing circular motion.

This provides the evidence for the claim that the interaction between an electrically charged particle passing through a magnetic field results in a magnetic force directed radially inward. This results in the electrically charged particle moving in a circular path governed by the strength of the magnetic field that provided the radially inward force.

ASSESSMENT

You might ask students the following questions:

Quantitative Treatment:

A suggested rubric is provided:

Part 1:

- Plot graph of *E* vs *a* including linear scale, units on axes, correctly plotted points, best fit line.
- Find the slope of the best fit line not just two data points.
- Correctly determine the SI units of the slope.
- Use the slope units to determine the physical meaning of the slope.
- Write the equation of this graph.
- Relate this result to theoretical equations.
- Write a paragraph providing claim, evidence and reasoning to justify the mathematical model suggested by the value and units of the slope of the linear graph.

Part 2:

- Plot graph of *E* vs *B* including linear scale, units on axes, correctly plotted points, best fit line.
- Find the slope of the best fit line not just two data points.
- Correctly determine the SI units of the slope.
- Use the slope units to determine the physical meaning of the slope.
- Write the equation of this graph.
- Relate this result to theoretical equations.
- Write a paragraph providing claim, evidence and reasoning to justify the mathematical model suggested by the value and units of the slope of the linear graph.

Part 3:

- Plot graph of *r* vs *B* including linear scale, units on axes, correctly plotted points, best fit line.
- Correctly linearize the data by plotting *r* vs *1/B*
- Find the slope of the best fit line not just two data points.
- Correctly determine the SI units of the slope.
- Use the slope units to determine the physical meaning of the slope.
- Write the equation of this graph.
- Relate this result to theoretical equations.
- Write a paragraph providing claim, evidence and reasoning to justify the mathematical model suggested by the value and units of the slope of the linear graph.